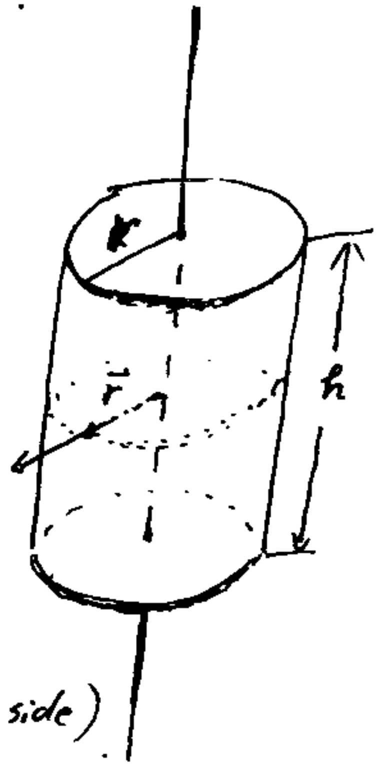
Physics 102

Hand-written Assignment #1.

① Ch. 19. #55 (p.643)

Gauss's Law : $\Phi = \dfrac{q}{\varepsilon_0}$



where the flux is :

$$\Phi = \Phi_{Top} + \Phi_{Bottom} + \Phi_{side}.$$

$$= 0 + 0 + \sum_{side} \vec{E}_i \cdot \Delta\vec{A}.$$

$$= \sum_{side} E(r) \cdot \Delta A \qquad (\Delta\vec{A} \,/\!/\, \vec{E}_i \text{ on side}).$$

$$= E \sum \Delta A$$

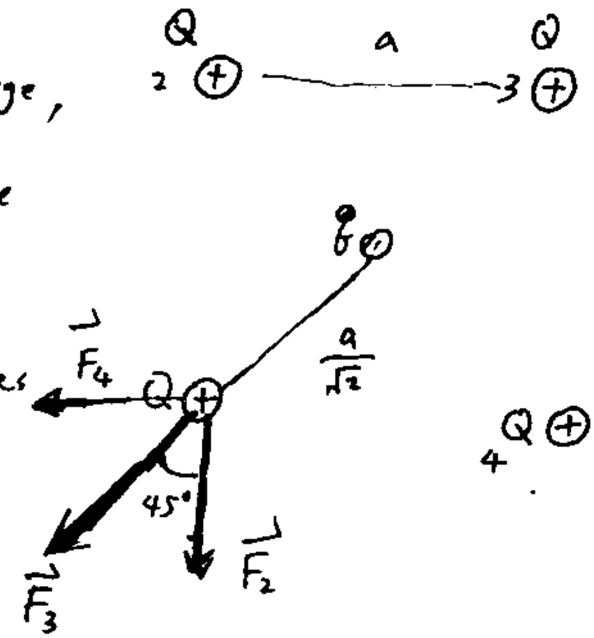
$$= E \cdot 2\pi r \cdot h.$$

The enclosed charge : $q = \lambda h$.

$$\therefore \quad E \cdot 2\pi r \cdot h = \frac{\lambda h}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}.$$

② ch. 19. #75.



Before introducing the fifth charge, each charge experiences the forces due to the other 3 charges.

For example, charge 1 experiences 3 forces. $\vec{F}_2$, $\vec{F}_3$, $\vec{F}_4$.

$$F_2 = F_2 = \frac{KQ^2}{a^2}.$$

$$F_4 = \frac{KQ^2}{2a^2}.$$

The resultance force : $\vec{F}_R = \vec{F}_2 + \vec{F}_3 + \vec{F}_4$ along $\vec{F}_3$ direction

$$F_R = F_3 + F_2 \cos 45^\circ + F_4 \cos 45^\circ$$

$$= \frac{KQ^2}{2a^2} + \frac{\sqrt{2}}{2} \cdot \frac{KQ^2}{a^2} + \frac{\sqrt{2}}{2} \frac{KQ^2}{a^2}$$

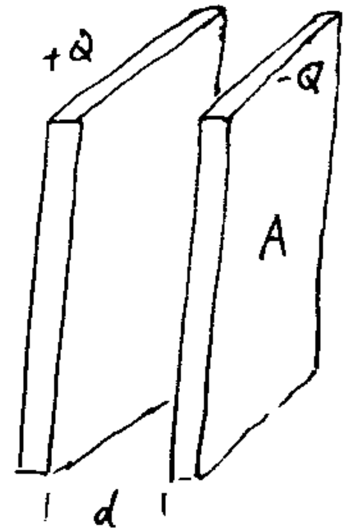
$$= \frac{KQ^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right)$$

The fifth charge, q, should be negative, to cancel with $\vec{F}_R$. It should be placed at the centre, so that works the same for all 4 charges.

$$\frac{KQq}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{-KQ^2}{a^2}\left(\frac{1}{2} + \sqrt{2} \right) \Rightarrow \frac{2q}{a^2} = \frac{-Q}{a^2}\left(\frac{1}{2} + \sqrt{2} \right)$$

$$q = -Q \left(\frac{1}{4} + \frac{\sqrt{2}}{2} \right)$$

③. ch.20. #60 . p.675



Given: $A = 405\ cm^2 = 0.0405\ m^2$

$d_0 = 2.25\ mm = 2.25 \times 10^{-3}\ m$.

Initial voltage $V_0 = 575\ V$.

charge on a plate: $Q_0 = C_0 V_0$

where $C_0 = \dfrac{A\varepsilon_0}{d_0}$

$$\therefore \quad Q_0 = \frac{A\varepsilon_0 \cdot V_0}{d_0} = \frac{0.0405 \times \varepsilon_0 \times 575}{2.25 \times 10^{-3}} = 9.16 \times 10^{-8}\ C.$$

a). $$U_0 = \frac{1}{2} Q_0 V_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \frac{A\varepsilon_0}{d_0} \cdot V_0^2.$$

$$= \frac{1}{2} \frac{0.0405 \times 8.85 \times 10^{-12}}{2.25 \times 10^{-3}} \times 575^2$$

$$= \frac{5.27 \times 10^{-5}}{2}$$

$$= 2.63 \times 10^{-5}\ \text{Jouls}.$$

b). Now. $d = 4.5\ mm = 2d_0$.

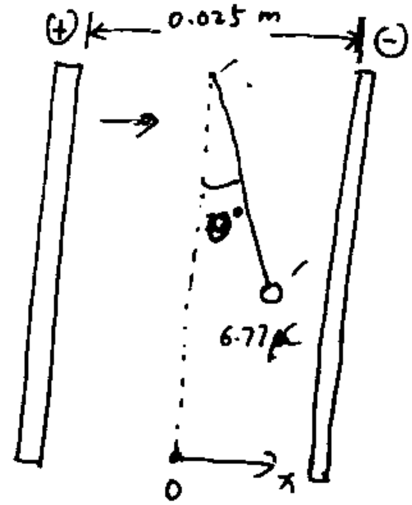
$C = C_0/2$ since $U = \frac{1}{2} C V^2$. $(U \propto C)$

$$\therefore \quad U = U_0/2 = \cancel{5.27 \times 10^{-4}\ \text{Jouls}} = 1.32 \times 10^{-5}\ \text{Jouls}.$$

c). $$W = \Delta U = U - U_0 = -1.32 \times 10^{-5}.$$

The capacitor will do work of 1.32×10^{-4} (J) when the gap is increased from 2.25 mm to 4.5 mm

④ . ch. 20. #79. p. 676.



$m = 0.071\ Kg$. $Q = 6.77 \times 10^{-6}\ C$.

$L = 0.022\ m$.

$E = 4.16 \times 10^{4}\ V/m$.

a). We are interested in how U_e varies with θ: (U_e — electric potential energy of the charge)

$U_e(\theta)$: we define $U_e(0) = 0$.

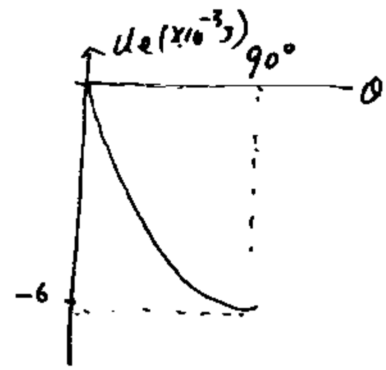
i.e. when $\theta = 0$, $U_e = 0$.

Then, electric potential $V_e = -E \cdot x = -E \cdot L \cdot \sin\theta$

$$U_e = QV_e = -QEL \cdot \sin\theta$$

$$= -6.77 \times 10^{-6} \times 4.16 \times 10^{4} \times 0.022 \cdot \sin\theta$$

$$= -6.12 \times 10^{-3} \cdot \sin\theta \quad \text{(Jouls)}$$



b). define $U_g(\theta = 0) = 0$.

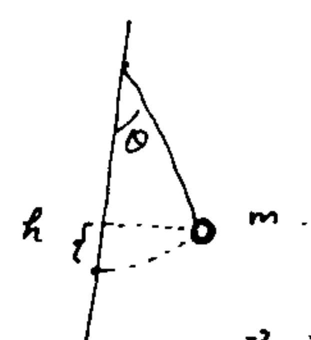
$$U_g = mgh$$

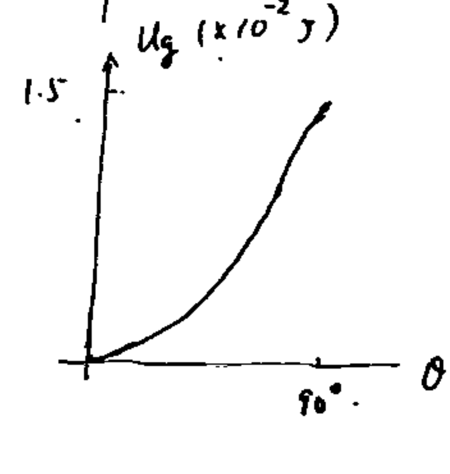
$$= mg(L - L\cos\theta)$$

$$= mgL(1 - \cos\theta)$$

$$= 0.071 \times 9.81 \times 0.022\ (1 - \cos\theta)$$

$$= 1.53 \times 10^{-2}\ (1 - \cos\theta)$$





c) $U_T = U_e + U_g$

$$= -QEL\cdot\sin\theta + mgL(1-\cos\theta)$$

To find U_T min,

let: $\frac{dU_T}{d\theta} = 0$: $\qquad 0 = -QEL\cdot\cos\theta + mgL\cdot\sin\theta$

$$QE\cos\theta = mg\sin\theta$$

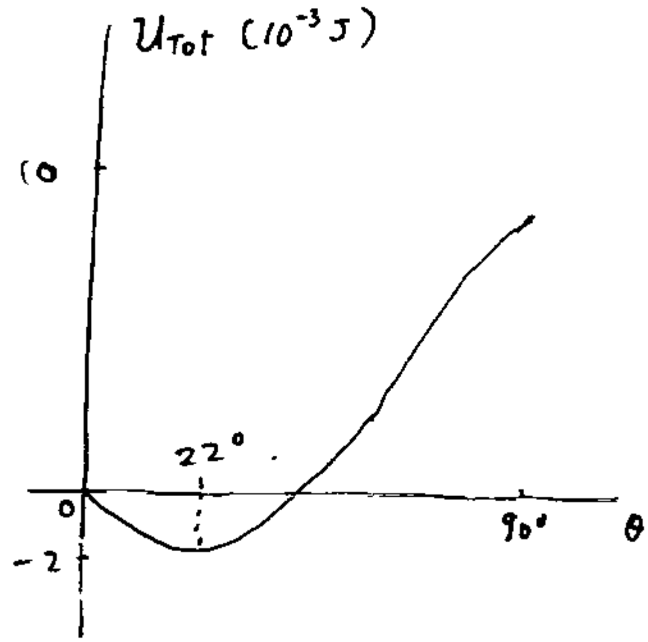
$$\frac{QE}{mg} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \frac{QE}{mg} = \frac{6.77\times10^{-6}\times 4.16\times10^{4}}{0.071\times9.81} = 0.404$$

$$\theta = \tan^{-1}0.404 = 22^\circ$$

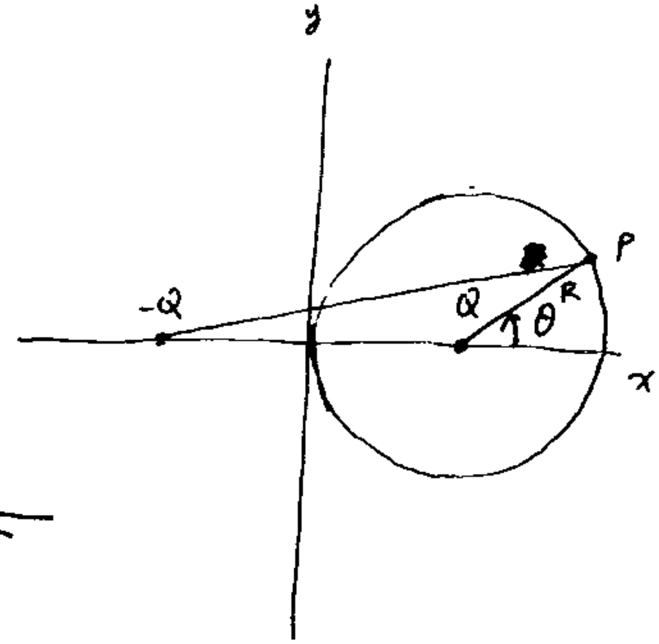
$$\frac{d^2U_T}{d\theta^2} = QEL\cdot\sin\theta + mgL\cdot\cos\theta > 0$$

$\therefore$ $U_T(\theta = 22^\circ)$ is indeed a minimum.



(5). ch.21. #81. p.676

$R = 0.50\,m \quad Q = 1.2\times10^{-6}\,C$.



electric potential at P:

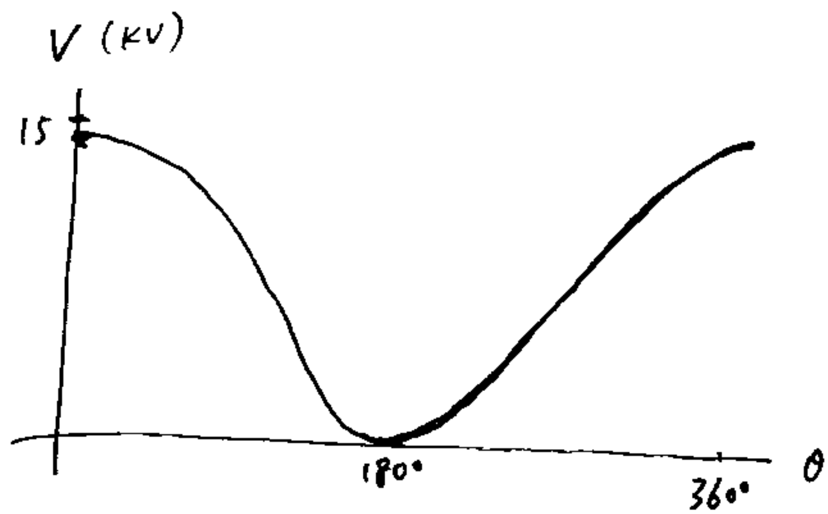
$$V = V_Q + V_{-Q}$$

$$= \frac{kQ}{R} + \frac{-kQ}{\sqrt{R^2+4R^2+4R^2\cos\theta}}$$

$$= \frac{kQ}{R}\left[1 - \frac{1}{\sqrt{5+4\cos\theta}}\right]$$

$$= \frac{8.99\times10^{9}\times1.2\times10^{-6}}{0.50}\left[1 - \frac{1}{\sqrt{5+4\cos\theta}}\right]$$

$$= 2.16\times10^{4}\left[1 - \frac{1}{\sqrt{5+4\cos\theta}}\right]$$



Note: Law of cosine:

$$c^2 = a^2 + b^2 - 2ab\cos C.$$