

PHYS 102 Midterm examination #1 (Version 1A)

October 15, 2004

Name _____

Time: 50 minutes

Student No. _____

Please show complete solutions and explain your reasoning, stating any principles that you have used.

1(5/20 marks). Two charges $q_1 = +2.0\text{nC}$ and $q_2 = -4.0\text{nC}$ are fixed to a baseline at $x_1=0.0\text{m}$ and $x_2=1.00\text{m}$, respectively.

(a) Where on the baseline is the electric potential equal to zero? Show your result in a schematic diagram.

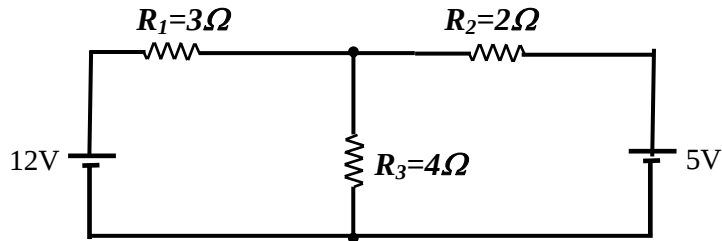
(b) What is the magnitude of the electric field at that location?

2(5/20 marks). Two parallel-plate capacitors of $C_1=1.0\mu\text{F}$ and $C_2=3.0\mu\text{F}$ are separately charged to $Q_1=10.0\mu\text{C}$ and $Q_2=16.0\mu\text{C}$, respectively. They are then attached so that the (+) plate of one is connected to the (-) plate of the other, and vice versa.

(a) Find the final voltage across the parallel combination of the two capacitors after the charges are redistributed.

(b) Now the space between the plates of the second capacitors is filled with dielectric material whose dielectric constant is 10.0. What is the voltage across the parallel combination of the two capacitors?

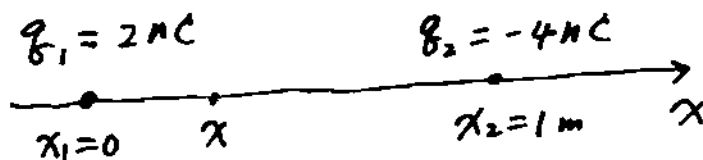
3(5/20 marks). Find all the currents (through R_1 , R_2 and R_3) in the network shown.



4(5/20 marks). A very long solid cylinder with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ . Use Gauss's law to find the expression of the magnitude of electric field inside and outside the cylinder.

1.

3/5(a). Since the two charges has opposite signs, the zero-potential point should be between the two charges, at x .



Potential at x :

$$V = V_1 + V_2$$

$$0 = \frac{k q_1}{x} - \frac{k |q_2|}{1-x}$$

V_1 - potential due to q_1
 V_2 - potential due to q_2

(2/3)

$$\frac{q_1}{x} = \frac{|q_2|}{1-x}$$

$$q_1 - q_1 x = x |q_2|$$

$$x = \frac{q_1}{q_1 + |q_2|} = \frac{2 \text{ nC}}{2 \text{ nC} + 4 \text{ nC}} = \frac{1}{3} = 0.333 \text{ m}.$$

2/5(b). Both $\vec{E}_1$ and $\vec{E}_2$ are pointing to the $+x$ direction ($\rightarrow$).

$$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E = \frac{k q_1}{x^2} + \frac{k |q_2|}{(1-x)^2} = k \left(\frac{q_1}{(\frac{1}{3})^2} + \frac{|q_2|}{(\frac{2}{3})^2} \right) \quad (V_2)$$

$$= 8.99 \times 10^9 \left(9 \times 2 \times 10^{-9} + \frac{9}{4} \times 4 \times 10^{-9} \right)$$

$$= 243 \text{ N/C}$$

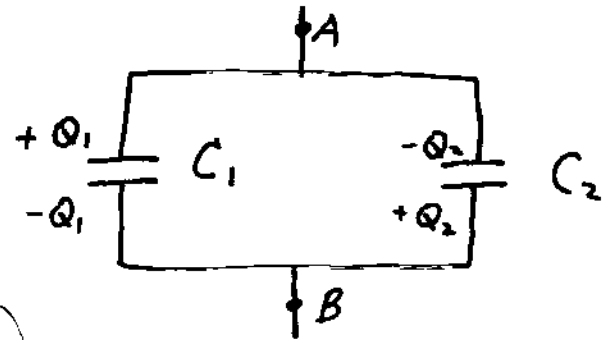
$$C_1 = 1 \mu F, C_2 = 3 \mu F.$$

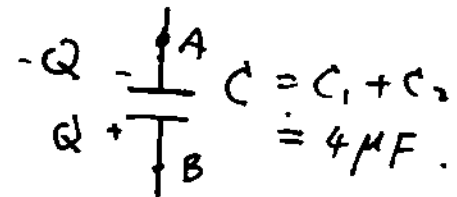
$$2. \quad Q_1 = 10 \mu C, Q_2 = 16 \mu C.$$

Total capacitance of the parallel combination:

$$C = C_1 + C_2 = 4 \mu F$$

(1/5)



$$\Downarrow$$


$$Q = Q_2 - Q_1 = 6 \mu C.$$

(1/5)

$$(a). \quad V = \frac{Q}{C} = \frac{Q_2 - Q_1}{C_1 + C_2}$$

(1/5)



$$V = \frac{16 \mu C - 10 \mu C}{1 \mu F + 3 \mu F}$$

$$= 1.5 \text{ Volts.}$$

(b). Now,

$$C_2' = K C_2 = 10 \cdot C_2 = \cancel{10 \mu F} 30 \mu F.$$

(1/5)

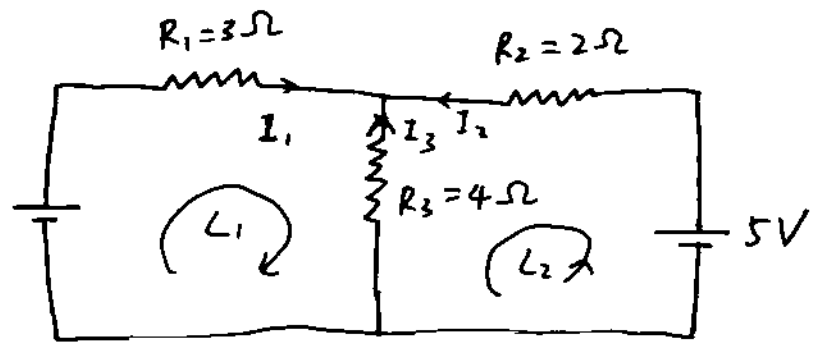
$$C' = C_1 + C_2' = 1 \mu F + 30 \mu F = 31 \mu F.$$

$$V' = \frac{Q}{C'} = \frac{6 \mu C}{31 \mu F} = 0.194 \text{ V.}$$

3

Kirchhoff's Law:

$$\begin{aligned}
 \text{KVL} \left\{ \begin{aligned} I_1 + I_2 + I_3 &= 0 & (1) \\ 12 - I_1 R_1 + I_3 R_3 &= 0 & (2) \\ 5 - I_2 R_2 + I_3 R_3 &= 0 & (3) \end{aligned} \right.
 \end{aligned}$$



Rearrange:

$$I_1 + I_2 + I_3 = 0 \quad (1)$$

$$I_1 R_1 - I_3 R_3 = 12 \quad (2)$$

$$I_2 R_2 - I_3 R_3 = 5 \quad (3)$$

$\frac{1}{5}$
idea of
eliminating unknowns
 $(1) \times R_1 - (2)$:

$$I_2 R_1 + I_3 R_1 + I_3 R_3 = -12$$

$$I_2 R_1 + I_3 (R_1 + R_3) = -12 \quad (4)$$

$(4) \times R_2 - (3) \times R_1$:

$$I_2 R_1 R_2 + I_3 (R_1 + R_3) R_2 = -12 R_2$$

$$I_2 R_2 R_1 - I_3 R_3 R_1 = 5 R_1$$

final result. $\frac{1}{5}$

$$I_3 (R_1 R_2 + R_3 R_2 + R_3 R_1) = -12 R_2 - 5 R_1$$

i.e.:

$$I_3 = \frac{-12 R_2 - 5 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3} = \frac{-24 - 15}{6 + 8 + 12} = -1.5 \text{ A}$$

(5):

$$I_2 = \frac{5 + I_3 R_3}{R_2} = \frac{5 - 1.5 \times 4}{2} = -0.5 \text{ A}$$

(1):

$$I_1 = -I_2 - I_3 = \text{~~2.0~~} = 2.0 \text{ A}$$

4. Construct a gaussian surface

as a cylindrical surface with a radius r ,

The flux through the Gaussian surface:

$$V_5. \quad \bar{\Phi} = \bar{\Phi}_{\text{top}} + \bar{\Phi}_{\text{bottom}} + \bar{\Phi}_{\text{side}}.$$

$\vec{E}$ is always perpendicular to the side of the Gaussian surface.

$$V_5. \quad \therefore \bar{\Phi}_{\text{top}} = 0 \quad \bar{\Phi}_{\text{bottom}} = 0$$

$$\bar{\Phi} = E \cdot A_{\text{side}}.$$

$$= E \cdot 2\pi r h.$$

Gauss's Law: $\bar{\Phi} = \frac{q}{\epsilon_0}$

$$q = \rho \cdot \pi R^2 h. \quad \text{--- for } r \geq R$$

$$\therefore E 2\pi r h = \rho \pi R^2 h / \epsilon_0$$

$$E = \frac{\rho R^2}{2 \epsilon_0 r}$$

for $r \leq R$, $q = \rho \pi r^2 h$

$$\therefore E = \frac{\rho r}{2 \epsilon_0}$$

