**PHYS 102 Midterm examination #1 (Version 1B)**

October 15, 2004 Name________________________________

Time: 50 minutes Student No.___________________________

Please show complete solutions and explain your reasoning, stating any principles that you have used.
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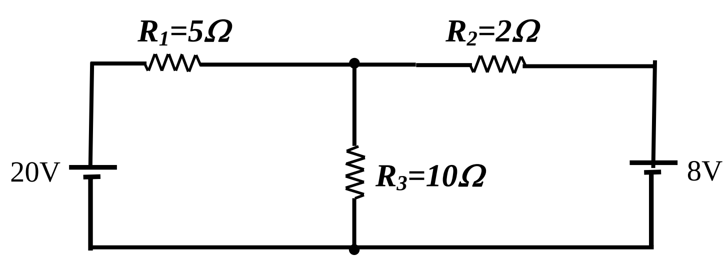
1(5/20 marks). Two charges $q_1$ = +2.0nC and $q_2$ = -4.0nC are fixed to a baseline at $x_1$=0.0m and $x_2$=1.00m, respectively.
(a) Where on the baseline is the electric field equal to zero? Show your result in a schematic diagram.
(b) What is the electric potential at that location?

2(5/20 marks). Two parallel-plate capacitors of $C_1$=1.0μF and $C_2$=3.0μF are separately charged to $Q_1$=10.0μC and $Q_2$=16.0μC, respectively. They are then attached so that the (+) plate of one is connected to the (-) plate of the other, and vice versa.
(a) Find the final voltage across the parallel combination of the two capacitors after the charges are redistributed.
(b) Now the space between the plates of the first capacitors is filled with dielectric material whose dielectric constant is 10.0. What is the voltage across the parallel combination of the two capacitors?

3(5/20 marks). Find all the currents (through $R_1$, $R_2$ and $R_3$) in the network shown.

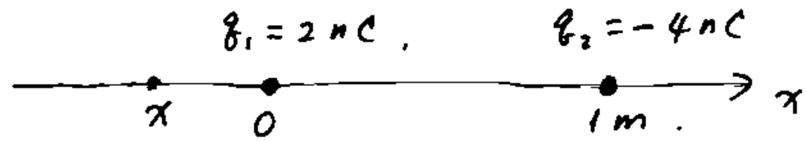



4(5/20 marks). A very long solid cylinder with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Use Gauss's law to find the expression of the magnitude of electric field inside and outside the cylinder.

# Physics 102.

## Midterm 1B. Solutions.

1.




3/5 (a). Since $q_1$ and $q_2$ have the opposite signs, the zero-field point should be on the left to the origin. ( $\vec{E}_1 : \longleftarrow$ ; $\vec{E}_2 : \longrightarrow$
$\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ )

at $x$:

$$\boxed{E = \frac{k q_1}{x^2} - \frac{k|q_2|}{(1-x)^2}} \quad 2/3.$$

$$\boxed{0 = \frac{q_1}{x^2} - \frac{|q_2|}{(1-x)^2}}$$

$$\frac{(1-x)^2}{x^2} = \frac{|q_2|}{q_1}$$

$$\frac{1-x}{x} = \pm\sqrt{\frac{|q_2|}{q_1}} = \pm\sqrt{2}.$$

$$\frac{1}{x} - 1 = \pm\sqrt{2}, \quad \frac{1}{x} = 1 \pm \sqrt{2}, \quad x = \frac{1}{1\pm\sqrt{2}}.$$

$x = \begin{cases} 0.414 & \text{rejected. } \because \vec{E}_1 \text{ and } \vec{E}_2 \text{ are both } \longrightarrow \text{ and won't cancell.} \\ -2.414\text{ m} & \text{— answer.} \end{cases}$

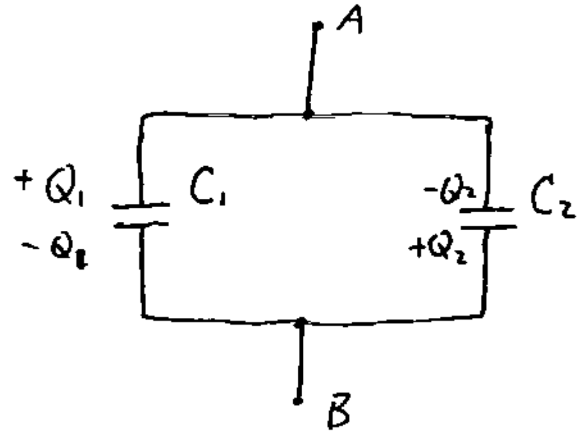
2/5 (b). $$\boxed{V = \frac{k q_1}{|x|} + \frac{-k|q_2|}{1-x}} = 8.99\times10^9\left(\frac{2\times10^{-9}}{2.414} - \frac{4\times10^{-9}}{3.414}\right)$$ 1/2.

$$= -3.08\ V.$$

2 .

$C_1 = 1\mu F$ ,  $C_2 = 3\mu F$ .

$Q_1 = 10\mu C$ .  $Q_2 = 16\mu C$




Parallel combination .

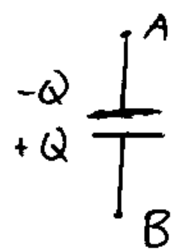
$\boxed{C = C_1 + C_2} = 4\mu F$ .  1/5

(1) : $\boxed{V = \frac{Q}{C} = \frac{Q_2 - Q_1}{C_1 + C_2}}$  1/5

$= \frac{16\mu C - 10\mu C}{1\mu F + 3\mu F}$

$= 1.5\ V$ .

⇓




$C = C_1 + C_2$

$\boxed{Q = Q_2 - Q_1}$  1/5

$= 16\mu C - 10\mu C$

$= 6\mu C$ .

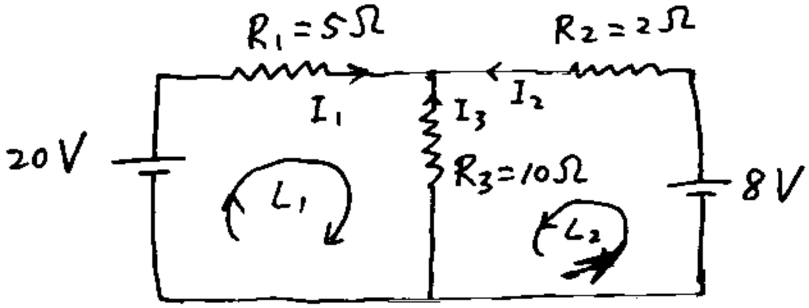
(b) . Now , $\boxed{C_1' = \kappa C_1} = 10 \cdot C_1 = 10\mu F$ .  1/5

$C' = C_1' + C_2 = 10\mu F + 3\mu F = 13\mu F$ .

$V' = \frac{Q}{C'} = \frac{6\mu C}{13\mu F} = 0.462\ V$ .

3.

Kirchoff's Laws:




1/5
1/5
1/5

$$\begin{cases} I_1 + I_2 + I_3 = 0 & (1) \\ 20V - I_1 R_1 + I_3 R_3 = 0 & (2) \\ 8V - I_2 R_2 + I_3 R_3 = 0 & (3) \end{cases}$$

Rearrange:

$$\begin{cases} I_1 + I_2 + I_3 = 0 & (1) \\ I_1 R_1 - I_3 R_3 = 20 & (4) \\ I_2 R_2 - I_3 R_3 = 8 & (5) \end{cases}$$

$(1) \times R_1 - (4)$: $\quad I_2 R_1 + I_3 (R_1 + R_3) = -20 \quad (6)$

$(5) \times R_1 - (6) \times R_2$:

$$I_2 R_1 R_2 - I_3 R_1 R_3 = 8 R_1$$

$$I_2 R_1 R_2 + I_3 (R_1 + R_3) R_2 = -20 R_2$$

1/5 Idea of variable elimination

$$-I_3 (R_1 R_3 + R_1 R_2 + R_2 R_3) = 8 R_1 + 20 R_2$$

1/5 answer

$$I_3 = \frac{-(8R_1 + 20R_2)}{R_1 R_3 + R_1 R_2 + R_2 R_3}$$

$$= \frac{-(40 + 40)}{50 + 10 + 20}$$

$$= -1 \text{ A}.$$

(5): $\quad I_2 = \dfrac{(8 + I_3 R_3)}{R_2} = \dfrac{8 - 10}{2} = -1 \text{ A}.$

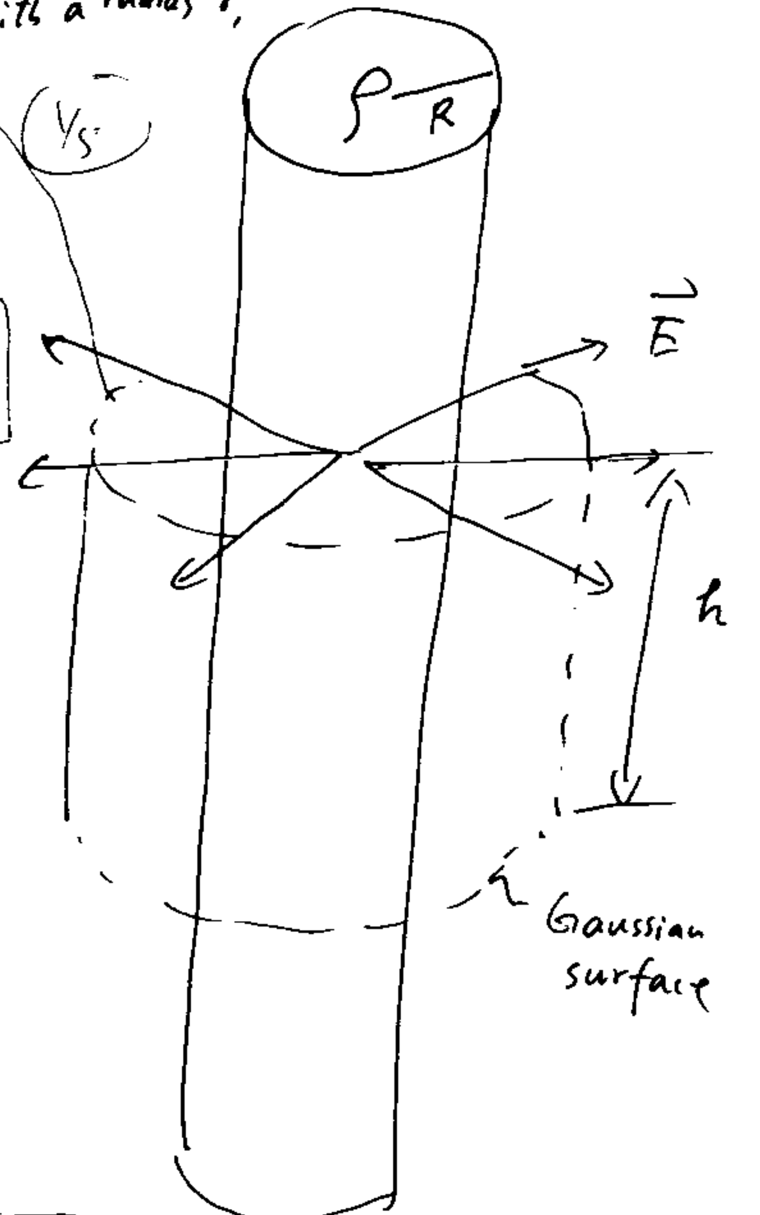
(1): $\quad I_1 = -(I_2 + I_3) = 2 \text{ A}.$

4. construct a gaussian surface

as a cylindrical surface with a radius r,

The flux through the Gaussian surface:

1/5. $\boxed{\Phi = \Phi_{Top} + \Phi_{bottom} + \Phi_{side}.}$




$\vec{E}$ is always perpendicular to the side of the Gaussian surface.

1/5 $\therefore \Phi_{Top} = 0 \quad \Phi_{bottom} = 0$

$$\Phi = E \cdot A_{side} = E \cdot 2\pi r h$$

Gauss's Law: $\Phi = \frac{q}{\varepsilon_0}$

1/5 $q = \rho \cdot \pi R^2 h$ — for $r \geq R$

$\therefore E 2\pi r h = \rho \pi R^2 h / \varepsilon_0$

$$E = \frac{\rho R^2}{2 \varepsilon_0 r}$$

1/5 for $r \leq R$, $q = \rho \pi r^2 h$

$$\therefore E = \frac{\rho r}{2 \varepsilon_0}.$$

Top View