Phys102 Lecture 18/19
Ampere's Law

Key Points

- Ampère’s Law
- Magnetic Field Due to a Straight Wire
- Magnetic Field of a Solenoid and a Toroid

References

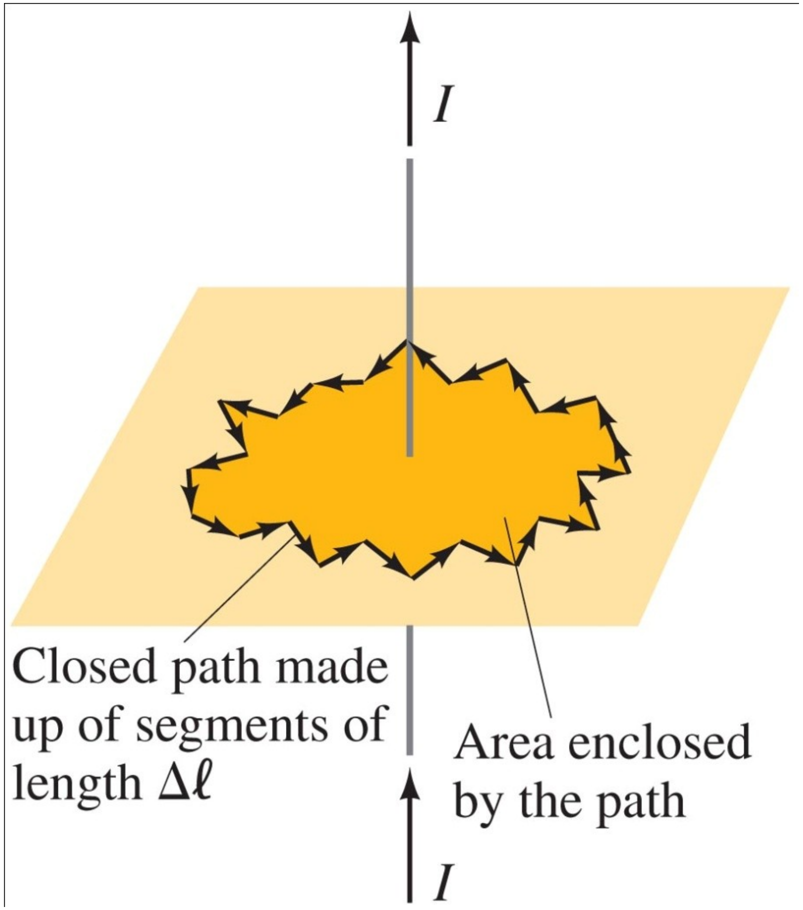
SFU Ed: 28-1,2,3,4,5.

6^{th} Ed: 20-5,6,7.

Ampère's Law

Ampère's law relates the magnetic field around a closed loop to the total current flowing through the loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}.$$



Meaning

$$\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$$

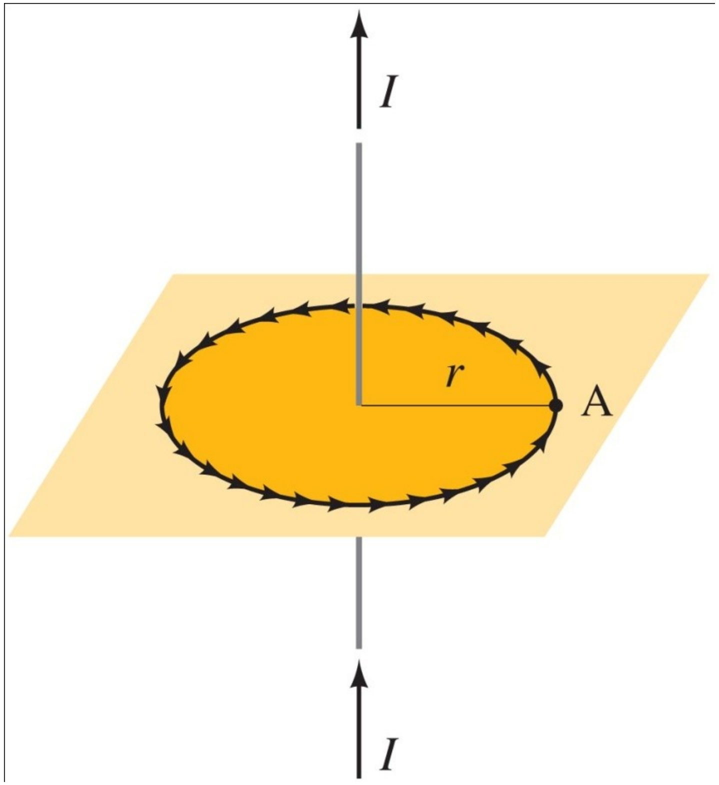
where

$$\vec{B} \cdot \Delta\vec{l} = B\Delta l \cos\theta, \qquad \theta - \text{angle between } \vec{B} \text{ and } \Delta\vec{l}$$

$\mu_0 = 4\pi \times 10^{-7}$ T·m/A is the magnetic permeability of free space.

Using Ampère's law to find the field around a long straight wire:

Use a circular path with the wire at the center; then $\vec{B}$ is tangent to $d\vec{\ell}$ at every point. Then



$$\sum \vec{B} \cdot \Delta \vec{l} = \mu_0 I_{encl.}$$

$$B \sum \Delta l = \mu_0 I_{encl.}$$

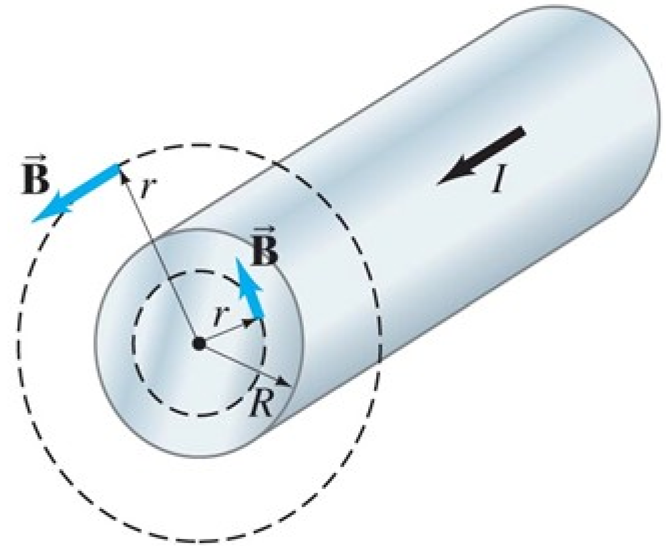
$$B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

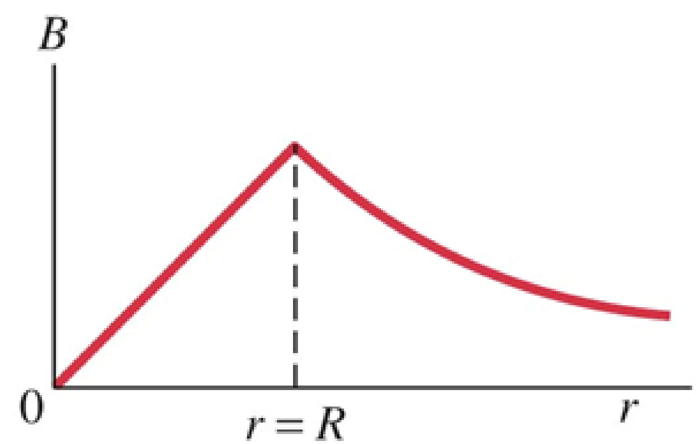
Similar to Gauss's law, we need symmetry.
e.g, we need to choose a closed path so that B has a constant magnitude.

Example 28-6: Field inside and outside a wire.

A long straight cylindrical wire conductor of radius R carries a current I of uniform current density in the conductor. Determine the magnetic field due to this current at (a) points outside the conductor ($r > R$) and (b) points inside the conductor ($r < R$). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R = 2.0 mm and I = 60 A, what is B at r = 1.0 mm, r = 2.0 mm, and r = 3.0 mm?



(a)



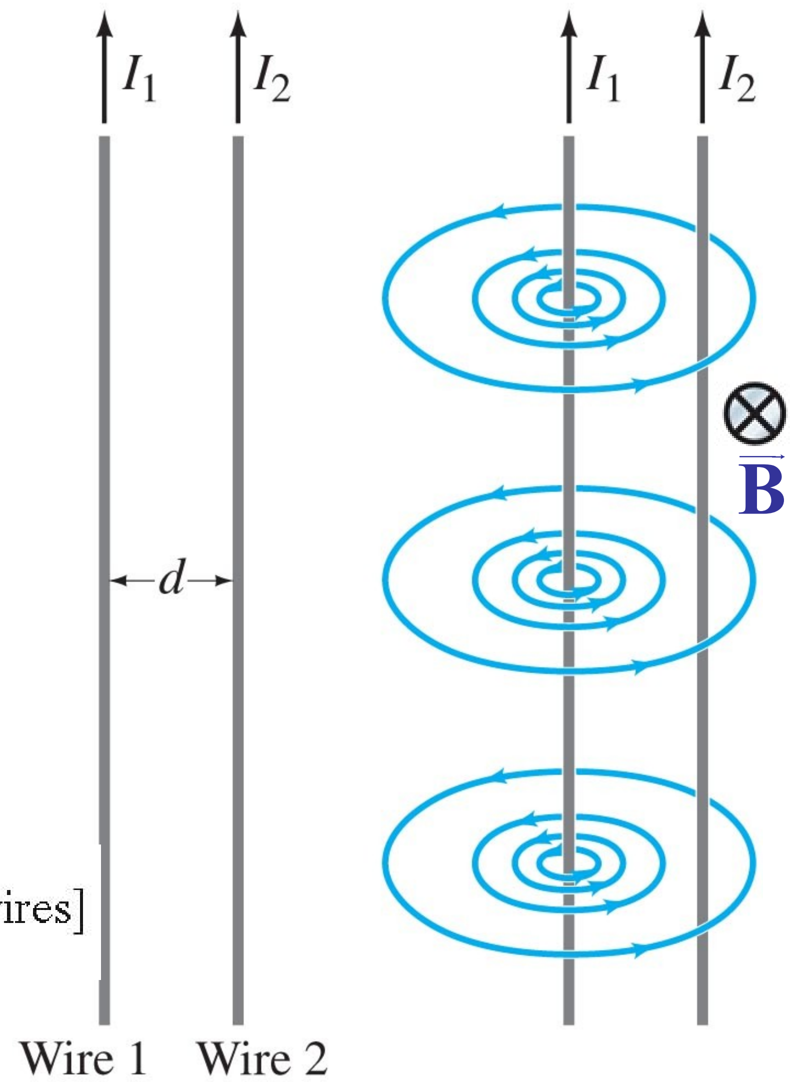
Force between Two Parallel Wires

The magnetic field produced at the position of wire 2 due to the current in wire 1 is

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}.$$

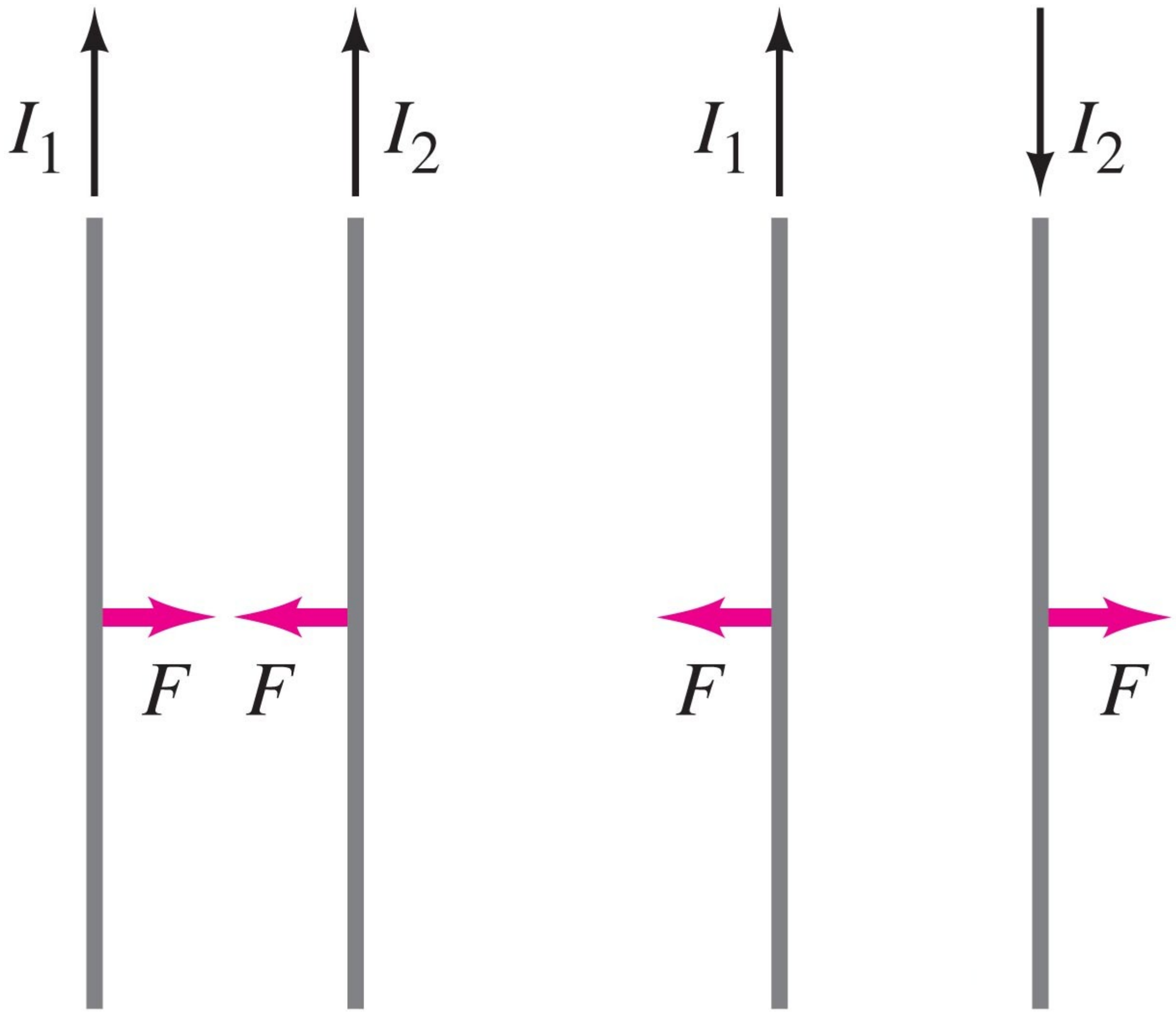
The force this field exerts on a length ℓ_2 of wire 2 is

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 . \quad \text{[parallel wires]}$$



Force between Two Parallel Wires

Parallel currents attract; antiparallel currents repel.



Example 28-4. Force between two current-carrying wires.

The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2$$

$$F_2 = \frac{\left(4\pi\times10^{-7}\right)(8.0)(8.0)(2.0)}{2\pi(0.003)} = 8.5\times10^{-3}\,N$$

Definitions of the Ampere and the Coulomb

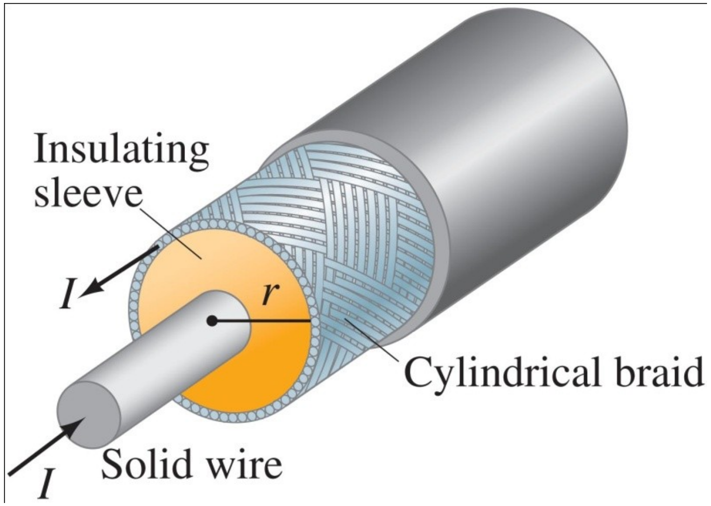
The ampere is officially defined in terms of the force between two current-carrying wires:

One ampere is defined as that current flowing in each of two long parallel wires 1 m apart, which results in a force of exactly 2×10^{-7} N per meter of length of each wire.

The coulomb is then defined as exactly one ampere-second.

Conceptual Example 28-7: Coaxial cable.

A coaxial cable is a single wire surrounded by a cylindrical metallic braid. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors, and (b) outside the cable.



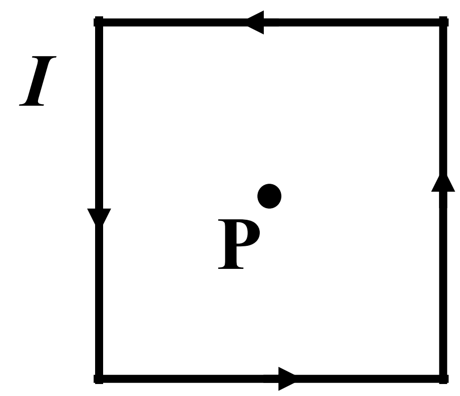
(a) $\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$

$$B = \frac{\mu_0 I}{2\pi r}$$

i-clicker 18-1 Current Loop

What is the direction of the magnetic field at the center (point P) of the square loop of current?

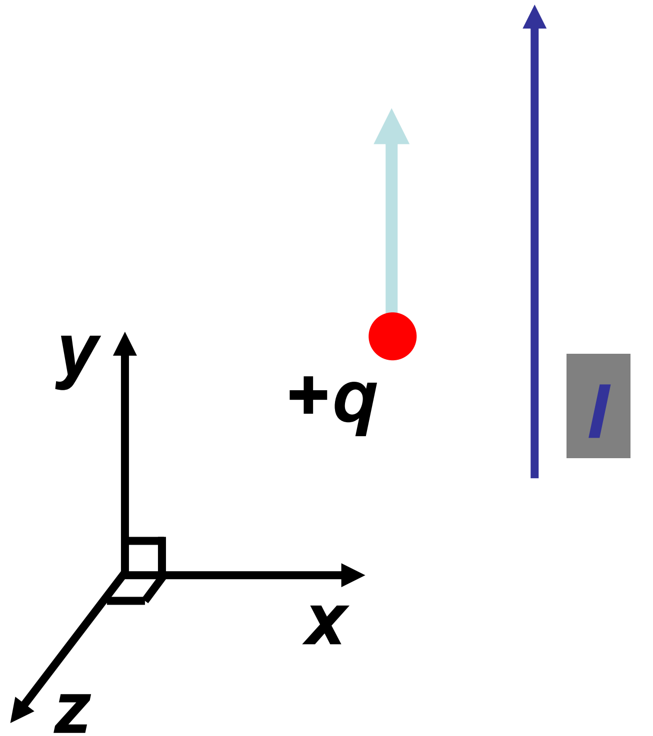
A) left
B) right
C) zero
D) into the page
E) out of the page



I-clicker 18-2 Field and Force

A positive charge moves parallel to a wire. If a current is suddenly turned on, in which direction will the force act?

A) +*z* (out of page)
B) -*z* (into page)
C) +*x*
D) -*x*
E) -*y*



Magnetic Field of a Solenoid

i-clicker 18-3:

For a solenoid with closely packed coils, the magnetic field inside

A) Varies dramatically.

B) Is nearly uniform.

C) Is nearly zero.

Magnetic Field of a Solenoid

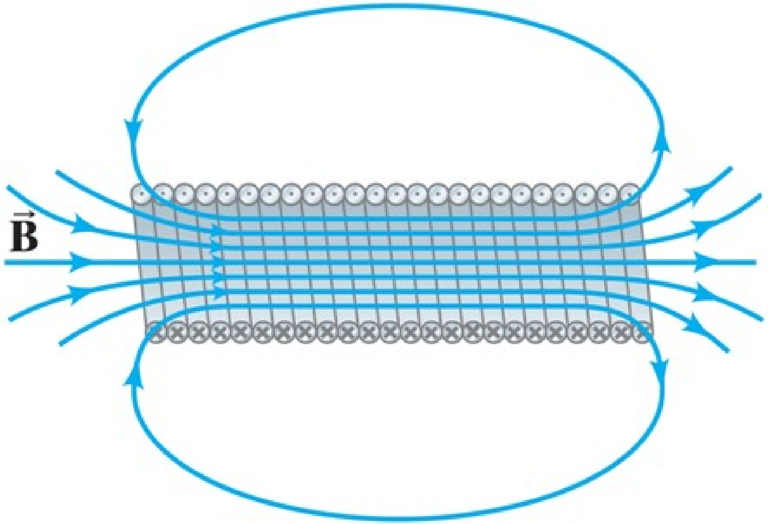
i-clicker 18-4:

For a solenoid with closely packed coils, the magnetic field outside the solenoid

A) Varies dramatically.

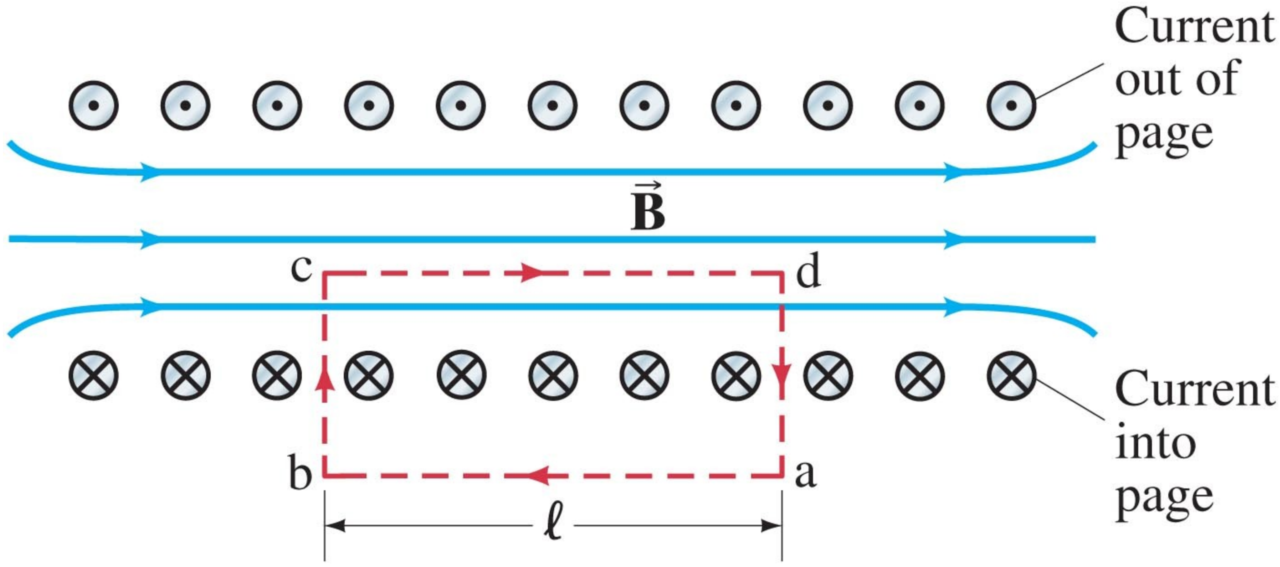
B) Is nearly uniform.

C) Is nearly zero.



Magnetic Field of a Solenoid and a Toroid

A solenoid is a coil of wire containing many loops. To find the field inside, we use Ampère's law along the path indicated in the figure.



$$\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$$

$$Bl = \mu_0 NI$$

$$B = \mu_0 nI$$

$n = N / l$ number of loops per unit length

Magnetic Field of a Solenoid and a Toroid

The field is zero outside the solenoid, and the path integral is zero along the vertical lines, so the field is (n is the number of loops per unit length)

$$B = \mu_0 n I. \qquad \text{[solenoid]}$$

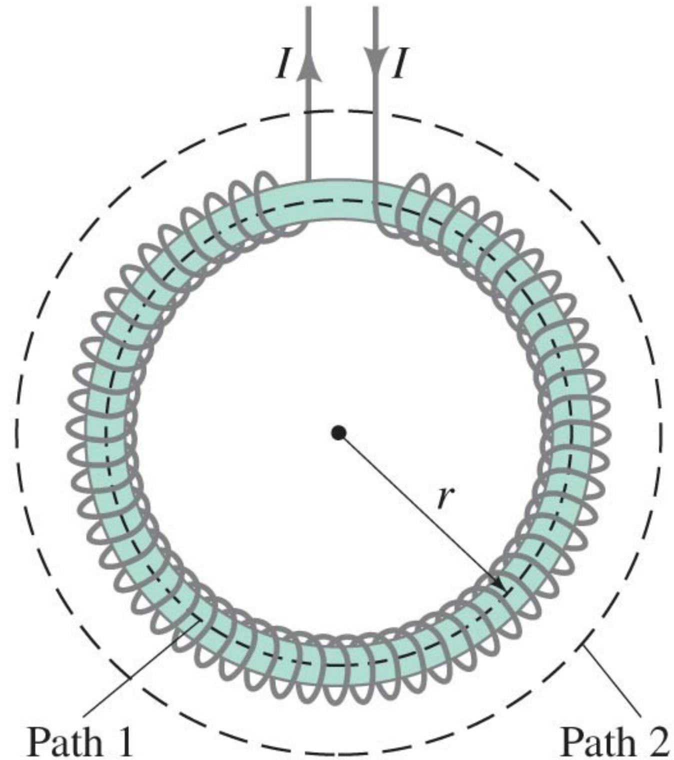
Example 28-9: Field inside a solenoid.

A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

$$B = \mu_0 n I = \left(4\pi \times 10^{-7}\right)\left(\frac{400}{0.10}\right)(2.0) = 0.01\ T$$

Example 28-10: Toroid.

Use Ampère's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle as shown.



(a) Inside (path 1):

$$\sum \vec{B} \cdot \Delta \vec{l} = \mu_0 I_{encl.}$$

$$B 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I$$

(b) Outside (path 2): B=0

28-9* Magnetic Fields in Magnetic Materials

If a ferromagnetic material such as iron is placed in the core of a solenoid or toroid, the magnetic field is enhanced by the field created by the ferromagnet itself. This is usually much greater than the field created by the current alone.

The magnetic field can be written as

$$B = \mu n I$$

where μ is the magnetic permeability, ferromagnets have $\mu >> \mu_0$ (e.g. $\mu \approx 2000\ \mu_0$ for electrical steel), while all other materials have $\mu \approx \mu_0$.