Complex Numbers

Complex numbers have a *real* part and an *imaginary* part:

$$z = a + ib, \qquad i = \sqrt{-1}$$

$$\mathrm{Re}\{z\} = a, \qquad \mathrm{Im}\{z\} = b$$

Basic Algebra:

$$(a+ib)+(c+id) = (a+c)+i(b+d)$$

$$(a+ib)(c+id) = (ac-bd)+i(bc+ad)$$

Complex Conjugate: $z^* = (a+ib)^* = a - ib$

$$z + z^* = 2a$$

$$z - z^* = 2ib$$

Absolute Value: $|z|$ modulus, amplitude, magnitude

$$zz^* = a^2 + b^2$$

$$|z| = \sqrt{zz^*} = \left(a^2+b^2\right)^{1/2}$$

Polar Form: $z = r[\cos\theta + i\sin\theta] = r\,\mathrm{cis}\,\theta = r\,\mathrm{e}^{i\theta}$

$$\left.\begin{aligned} \mathrm{e}^{i\theta} &= \cos\theta + i\sin\theta \\ \mathrm{e}^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned}\right\} \qquad \left.\begin{aligned} \cos\theta &= \tfrac{1}{2}\left(\mathrm{e}^{i\theta} + \mathrm{e}^{-i\theta}\right) \\ \sin\theta &= \tfrac{1}{2i}\left(\mathrm{e}^{i\theta} - \mathrm{e}^{-i\theta}\right) \end{aligned}\right\}$$

If $z_1 = r_1\,\mathrm{e}^{i\theta_1}$ and $z_2 = r_2\,\mathrm{e}^{i\theta_2}$

$$z_1 z_2 = r_1 r_2\,\mathrm{e}^{i(\theta_1+\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{e}^{i(\theta_1-\theta_2)}$$

$$z^m = \left(r\mathrm{e}^{i\theta}\right)^m = r^m \mathrm{e}^{im\theta}$$