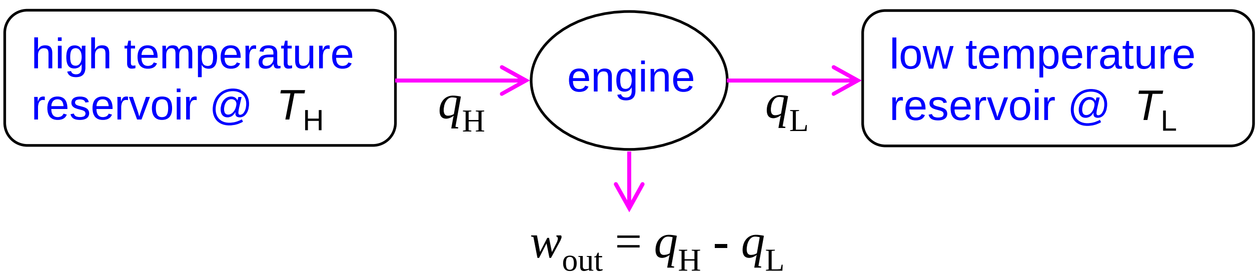
Heat Engines

A heat engine is a system capable of transforming heat into work by some cyclic process.

The 2nd Law states that an isothermal cyclic process can not produce net work.

The efficiency of a heat engine is defined as the ratio of the work produced to the heat input:

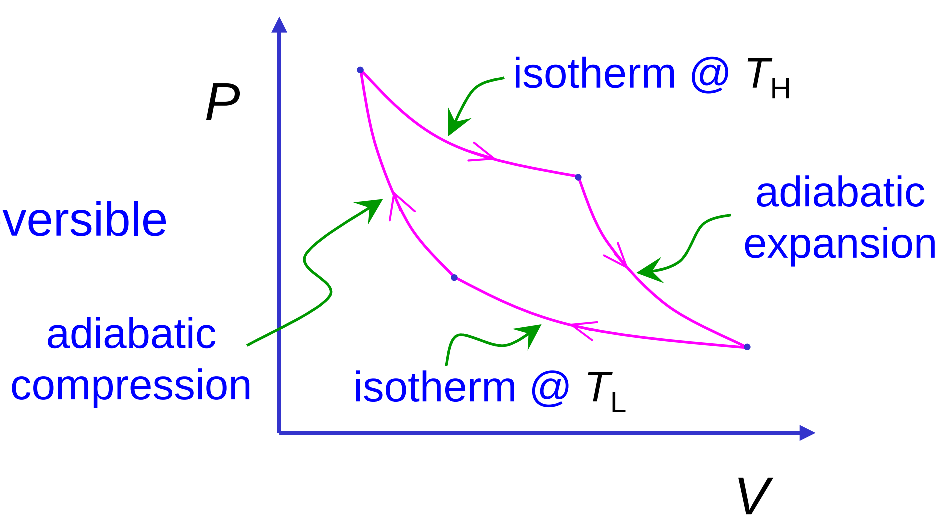
$$\varepsilon = \frac{w}{q_{\mathrm{H}}} = \frac{q_{\mathrm{H}} - q_{\mathrm{L}}}{q_{\mathrm{H}}} = 1 - \frac{q_{\mathrm{L}}}{q_{\mathrm{H}}}$$



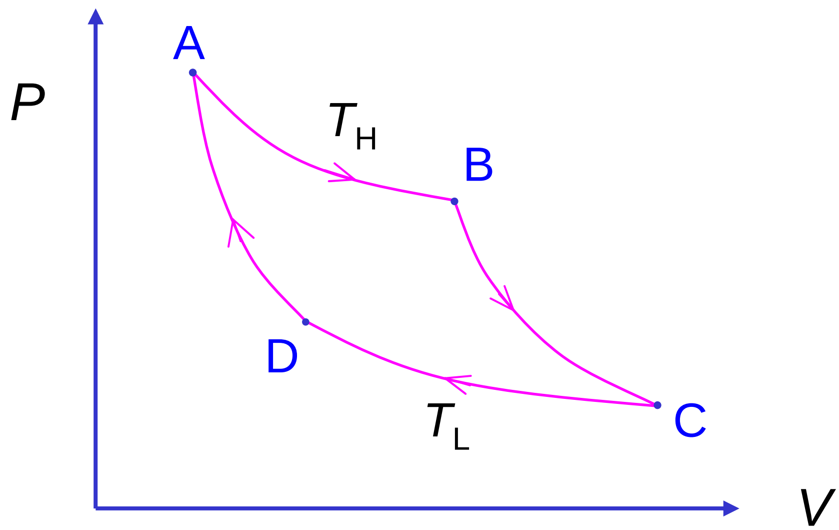
A heat pump is a heat engine in reverse. Work is needed to transfer heat from a lower to a higher temperature reservoir.

Carnot Cycle

- ideal gas
- all steps reversible



The Carnot Cycle



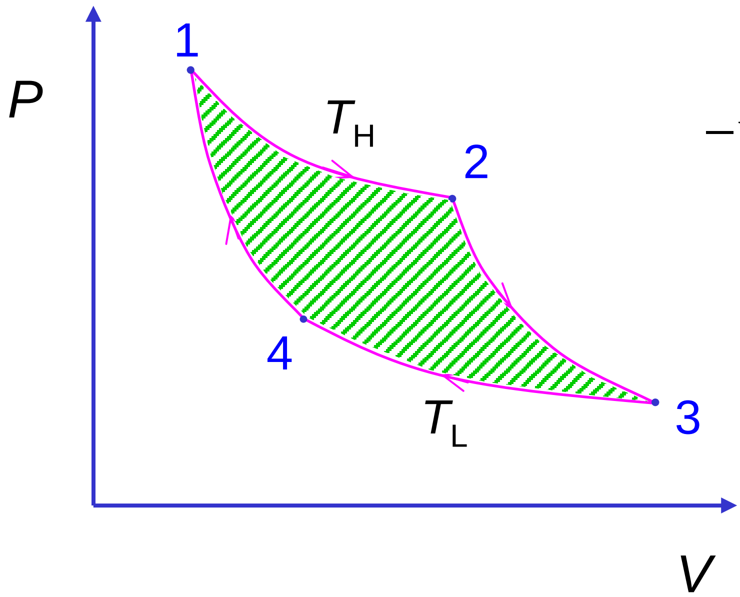
step	w	q	ΔU
A → B	$-nRT_H \ln(V_B/V_A)$	$-w_{AB}$	0
B → C	ΔU_{BC}	0	$-C_V(T_H - T_L)$
C → D	$-nRT_L \ln(V_D/V_C)$	$-w_{CD}$	0
D → A	ΔU_{DA}	0	$C_V(T_H - T_L)$
Total	$nR(T_H - T_L)\ln(V_A/V_B)$	$-w_{cyc}$	0

$$\varepsilon = \frac{w_{out}}{q_{in}} = \frac{nR(T_H - T_L)\ln(V_A/V_B)}{nRT_H \ln(V_A/V_B)} = 1 - \frac{T_L}{T_H}$$

$$\varepsilon = \frac{(T_H - T_L)}{T_H}$$

for best efficiency,
maximize T_H
minimize T_L

Entropy Changes in the Carnot Cycle



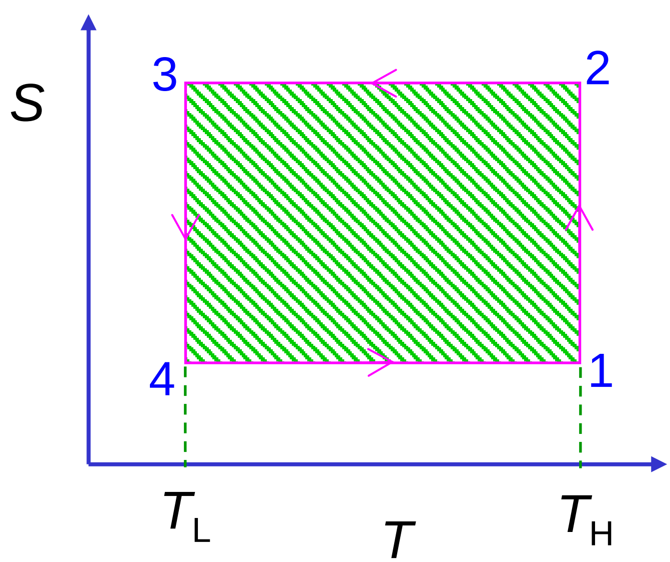
$$-w_{cycle} = \oint P\,dV$$
$$= \text{area of } PV \text{ plot}$$
$$= q_{cycle}$$

Isothermal expansion (1 → 2):

$$\Delta U = 0 \Rightarrow -w = q > 0 \Rightarrow \Delta S = \frac{q}{T} > 0$$

Isothermal compression (4 ← 3): $\Delta S < 0$

Adiabatic steps (2 → 3 and 1 ← 4): $q = 0 \Rightarrow \Delta S = 0$



$$q_{tot} = T_H(S_2 - S_1) + T_L(S_4 - S_3)$$
$$= (T_H - T_L)(S_2 - S_1)$$
$$= \text{area of } ST \text{ plot}$$
$$= -w_{cycle}$$