# **T** Dependence of Complex Reactions

Assume some complex reaction

$$A + B \xrightarrow{k_1} C + D \xrightarrow{k_2} \cdots$$

for which the overall reaction rate constant can be expressed in terms of the elementary steps:

$$k_{\text{overall}} = \frac{k_1^{n_1} k_2^{n_2} \dots}{k_3^{n_3} \dots}$$

If each rate constant obeys the Arrhenius expression,  $k_i = A_i e^{-E_i/RT}$ 

$$k_{\text{overall}} = \left(\frac{A_1^{n_1} A_2^{n_2} \dots}{A_3^{n_3} \dots}\right) \exp\left\{-\left(n_1 E_1 + n_2 E_2 - n_3 E_3 - \dots\right)/RT\right\}$$

i.e. The Arrhenius parameters are

$$A_{\text{overall}} = \prod_{i} A_{i}^{n_{i}}$$
$$E = \sum_{i} n_{i} E_{i}$$

The overall "activation energy" may be negative, if  $n_i$  is negative and the corresponding  $E_i$  is large enough. Also, for a pre-equilibrium reaction where  $k_{overall} = K_1 k_2$ 

$$E_{\rm a} = RT^2 \frac{d\ln k_{\rm overall}}{dT} = RT^2 \frac{d\ln K_1}{dT} + E_2 = \Delta H_1 + E_2$$

 $E_2$  is positive but  $\Delta H$  can be negative.

## **Chain Reactions**

$$\begin{array}{cc} A \rightarrow R_{1} & \text{initiation} \\ \\ R_{1} + B \rightarrow R_{2} + P_{1} \\ \\ R_{2} + C \rightarrow R_{1} + P_{2} \end{array} \end{array} \begin{array}{c} \text{propagation} \\ \\ \\ 2R_{1} \rightarrow P_{3} \end{array} \end{array}$$

R is often, but not always, a free radical. Initiation may be thermal, photochemical, radiolysis, ... The overall reaction is determined by adding the propagation steps:

$$\mathrm{B} + \mathrm{C} \rightarrow \mathrm{P}_{\! 1} + \mathrm{P}_{\! 2}$$

Important examples include polymerization, combustion, photochemical smog production and the depletion of stratospheric ozone by CFCs.

$$CF_2Cl_2 \xrightarrow{hv} \cdot CF_2Cl + Cl \cdot Cl \cdot +O_3 \longrightarrow ClO \cdot +O_2$$
$$ClO \cdot +O \cdot \longrightarrow Cl \cdot +O_2$$

The net effect is catalysis by the CFC of the reaction

$$O \cdot + O_3 \rightarrow 2O_2$$

## **Rice-Herzfeld Mechanism**

e.g. for the thermal decomposition of acetaldehyde			
1	$CH_{3}CHO \rightarrow \cdot CH_{3} + \cdot CHO$	initiation	
2	$\cdot CHO \rightarrow \cdot H + CO$		
3	$\cdot H + CH_3CHO \rightarrow H_2 + CH_3\dot{C}O$		
4	$\cdot CH_3 + CH_3CHO \rightarrow CH_4 + CH_3\dot{C}O$		
5	$CH_3\dot{C}O \rightarrow \cdot CH_3 + CO$	J	
6	$2 \cdot CH_3 \rightarrow C_2H_6$	termination	
Write steady-state equations for [CHO], [H], [CH <sub>3</sub> ], [CH <sub>3</sub> CO]:			

$$\frac{d}{dt}[CHO] = k_1[CH_3CHO] - k_2[CHO] = 0 \quad [CHO] = (k_1/k_2)[CH_3CHO]$$
$$\frac{d}{dt}[H] = k_2[CHO] - k_3[H][CH_3CHO] = 0 \quad [H] = (k_1/k_3)$$
$$\frac{d}{dt}[CH_3CO] = k_4[CH_3][CH_3CHO] + k_3[H][CH_3CHO] - k_5[CH_3CO] = 0$$
$$\frac{d}{dt}[CH_3] = k_1[CH_3CHO] - k_4[CH_3][CH_3CHO] + k_5[CH_3CO] - 2k_6[CH_3]^2 = 0$$
$$[CH_3]^2 = (k_1/k_6)[CH_3CHO]$$
$$rate = \frac{d}{dt}[CH_4] = k_4[CH_3][CH_3CHO] = k_4(k_1/k_6)^{1/2}[CH_3CHO]^{3/2}$$

### **An Explosive Reaction**

**Overall:** 

 $2H_2 + O_2 \rightarrow 2H_2O$ 

 $\begin{array}{c} H_{2} \rightarrow 2H & \text{initiation} \\ H + O_{2} \rightarrow O + OH \\ O + H_{2} \rightarrow OH + H \\ OH + H_{2} \rightarrow H_{2}O + H \\ H \rightarrow \text{wall} \\ H \rightarrow \text{wall} \\ H + O_{2} + M \rightarrow HO_{2} + M \\ HO_{2} + H_{2} \rightarrow H + H_{2}O_{2} \\ HO_{2} + H_{2}O \rightarrow OH + H_{2}O_{2} \end{array} \right\} \begin{array}{c} \text{slow} \\ \text{propagation} \\ \text{propagation} \end{array}$ 

At 700 K and 0.1 bar  $O_2$ , each initial H atom  $\rightarrow 10^{13}$  H atoms in 0.3 s.



## Branching Chain Reactions

A, B  $\rightarrow$  R<sub>1</sub> Initiation 1  $R_1 + A \rightarrow R_2 + R_3$  [R] increases 2 **Branching**  $B + R_i \rightarrow R_i + \dots$ 3 Propagation  $R_i \rightarrow P_1$ 4 Termination (wall) [R] decreases

The steady-state approximation does not apply.

If *n* is the number of radicals at time *t*,

(combination)  $2R_i \rightarrow P_2$ 

$$\frac{dn}{dt} = \rho_{I} + k_{b}n - (k_{w} + k_{g})n$$

$$= \rho_{I} + \phi n \qquad \phi = k_{b} - (k_{w} + k_{g})$$

$$n = \frac{\rho_{I}}{\phi} (e^{\phi t} - 1)$$

$$k_{eff}$$
The exponential increase in *n* for  $\phi > 0$  leads to explosion.
$$k_{eff} = k_{b} - (k_{w} + k_{g})$$

 $\phi > 0$ 



At pressures below 1,

wall termination is dominant

the limit depends on surface composition and area

 the limit is altered by the size of the reaction vessel Between 1 and 2,

is the explosion peninsula

the limits change with temperature because

branching reactions are T dependent, diffusion less so Between 2 and 3,

gas phase termination reactions are dominant
 At pressures above 3,

reaction products are important

 $\succ$  heat from exothermic reactions  $\rightarrow$  thermal explosion

### **Autocatalysis**

 $A + \mathbf{P} \xrightarrow{k} 2\mathbf{P}$ 

$$rate = -\frac{da}{dt} = \frac{dx}{dt} \qquad [A] = a_0 - x, \ [P] = p_0 + x$$
$$\frac{dx}{dt} = k (a_0 - x) (p_0 + x)$$
$$kt = \int_0^x \left\{ \frac{dx}{(a_0 - x)(p_0 + x)} \right\}$$
$$= \frac{1}{(a_0 + p_0)} \int_0^x \left\{ \frac{1}{(a_0 - x)} + \frac{1}{(p_0 + x)} \right\} dx$$
$$= \frac{1}{(a_0 + p_0)} \left[ -\ln(a_0 - x) + \ln(p_0 + x) \right]_0^x$$
$$= \frac{1}{(a_0 + p_0)} \ln \left\{ \frac{a_0}{(a_0 - x)} \frac{(p_0 + x)}{p_0} \right\}$$

Substitute  $\alpha = (a_0 + p_0)k$ ,  $\beta = p_0/a_0$  $\alpha t = \ln \frac{(1 + x/p_0)}{(1 - \beta x/p_0)}$   $x/p_0 = \frac{e^{\alpha t} - 1}{1 + \beta e^{\alpha t}}$ exponential increase

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# **Oscillations in Gas Phase Kinetics**

Consider the concentration profile of an intermediate in the  $H_2 + O_2$  reaction.



#### Flaring of phosphorus in a loosely stoppered flask (Robert Boyle, 17th century)

Cool flames = limited combustion of hydrocarbons due to "long-lived" intermediates which damp the explosion.

Pre-ignition (autoignition) producing "knock" in auto engines.

## **Cool Flame Oscillations**

Hydrocarbon fuels spontaneously ignite in the presence of  $O_2$  at T > 400-500 K.

"True" ignition gives CO,  $CO_2$ ,  $H_2O$  and T increases ~ 1000 K.

"Cool" flames produce ROH, RCHO, RCOOH and  $\Delta T \sim 100$  K



Oscillations occur because of both chemical and thermal feedback.



## **Oscillating Reactions**

The Lotka-Volterra Mechanism

$$A + X \rightarrow 2X$$
$$X + Y \rightarrow 2Y$$
$$Y \rightarrow B$$

[A] is held constant (replenished). [X] and [Y] oscillate.



## **Oscillating Reaction Models**

Brusselator	$A \rightarrow X$
Prigogine <i>et al</i>	$2\mathbf{X} + \mathbf{Y} \rightarrow 3\mathbf{X}$
	$\mathbf{B} + \mathbf{X} \rightarrow \mathbf{Y} + \mathbf{C}$
	$X \rightarrow D$

[A] and [B] are held constant. [X] and [Y] settle down to a limit cycle:



Oregonator Noyes *et al* 

 $A + Y \rightarrow X$  $X + Y \rightarrow C$  $B + X \rightarrow 2X + Z$  $2X \rightarrow D$  $Z \rightarrow Y$ 

The B-Z reaction is of this general form, with  $X = HBrO_2$  $Y = Br^ Z = 2Ce^{4+}$ 18 steps, 21 species!