5. The Mechanics of Spread Trading

> Spreading as a futures technique is as old as the markets themselves, and is probably the single largest source of market liquidity, particularly in the forward months. Indeed, spread participants are the backbone of market liquidity, without which no viable futures market can exist.
>
> *Leo Melamed*, former chairman of CME and IMM

5.1 Butterflies, Tandems, Turtles and Stereos

Butterflies

A natural extension of the intra-commodity futures spread trades described in Chapter 3 is the ***butterfly***, e.g., Schwager (1984). Because the butterfly can be interpreted as an intra-commodity tandem trade, it also provides a useful introduction to inter-commodity trades. Recognizing that there are a number of possible variations on butterfly trades, consider the following generic version: *short (long) 1 nearby contract; long (short)* ***2*** *contracts of an intermediate delivery date contract; and, short (long) 1 distant delivery contract.* The interpretation of the trade can be captured as a "spread of spreads", a combination of a short (long) nearby spread and a long (short) deferred spread. The trades supporting the profit function are described in Figure 5.1.

Figure 5.1 **Profit Function for a Butterfly Spread**

DATE	***Nearby (N) Position***	***Intermediate (T)***	***Distant Position*** *(T*)*
t=0	Short *1* at $F(0,N)$	Long 1 at $F(0,T)$	
		Long 1 at $F(0,T)$	Short 1 at $F(0,T^*)$
t=1	Long *1* at $F(1,N)$	Short 1 at $F(1,T)$	
		Short 1 at $F(1,T)$	Long 1 at $F(1,T^*)$

The profit function for the short-long-short butterfly is:

$$\pi_b/Q = [\{F(1,T)-F(1,N)\} - \{F(0,T)-F(0,N)\}] + [\{F(1,T)-F(0,T^*)\} - \{F(0,T)-F(0,T^*)\}]$$
$$= \{F(0,N)-F(1,N)\} + 2\{F(1,T)-F(0,T)\} + \{F(0,T^*)-F(1,T^*)\} \qquad (5.1)$$

For this trade to be profitable, the nearby futures basis is expected to widen more than the deferred futures basis.

Analysis of the butterfly proceeds expediently by assuming that the trade has been "tailed"-- unlikely in practice, but an assumption typically only resulting in second order differences from (5.1). In this case the profit function can be approximated as:[1]

$$\pi_b/Q = F(1,N)\ \Delta\ ic(N,T) - F(1,T)\ \Delta\ ic(T,T^*)$$

In words, profitability of the butterfly depends on the behavior of the *term structure of futures prices*. For non-exchange members subject to higher transactions costs, this type of trade would usually not provide interesting opportunities because the associated price movements are small relative to the costs of trading as a non-exchange member.[2] On the other hand, floor traders can use this trade, for example, to flatten out the futures term structure. This could occur if the price of an intermediate contract became mispriced due, say, to a large position was being placed in a particular delivery month due to cash market considerations. As it turns out, the most interesting applications of trading spreads against spreads arise when the spreads are in different commodities. In this case, the trade is referred to as a *tandem*.

A variation of the butterfly trade, known as the ***condor***, is constructed by separating the spreads contained in the butterfly. Instead of short (long) 1 nearby contract, long (short) ***2*** contracts of an intermediate delivery date contract and, short (long) 1 distant delivery contract involving 3 different delivery dates, the condor requires four delivery dates. The two contracts in the intermediate delivery date required for the butterfly are established in two distinct, but still intermediate contract delivery dates. In this case, the tailed profit function would look like:

$$\pi_c/Q = F(1,N)\ \Delta\ ic(N,T) - F(1,T^*)\ \Delta\ ic(T^*,T^{**})$$

While, typically, $T^{**} > T^* > T > N$, it is also possible that $N < T^* < T < T^{**}$. Because the condor requires at least four distinct and actively traded delivery dates, this trade is not applicable to all commodities. As with the butterfly, because the potential profits from this trade will often be small, it is usually of more interest to exchange floor speculators than to off-exchange traders subject to higher transactions costs.

Tandems and Stereos

A ***tandem*** involves combining spreads in two different commodities. Interpreting the tandem as an inter-commodity butterfly, the profit function for a tandem follows immediately from the butterfly, allowing for differing position sizes in the two commodities. The profit function for the ***untailed short-the-nearby, long-the-deferred*** spread in the first commodity would be:

$$\pi_1 = Q_1\ [\{F(1,T)-F(1,N)\} - \{F(0,T)-F(0,N)\}]$$

And, for the second commodity, where the untailed spread is ***long-the-nearby, short-the-deferred***:

$$\pi_2 = Q_2\ [\{G(0,T)-G(0,N)\} - \{G(1,T)-G(1,N)\}]$$

Combining these two component spreads gives the general profit function for the tandem trade:

$$\begin{aligned}\pi_{tan} = &\{Q_1\ [F(1,T)-F(1,N)] - Q_2\ [G(1,T)-G(1,N)]\}\\ &-\{Q_1\ [F(0,T)-F(0,N)] - Q_2\ [G(0,T)-G(0,N)]\}\end{aligned} \qquad (5.2)$$

Determining the hedge ratio, the number of spreads in commodity 2 for each spread in commodity 1, involves dividing (5.2) through by Q_1. Substituting in the cash and carry equilibrium conditions gives the cash-and-carry arbitrage form of the profit function:

$$\frac{\pi_{\tan}}{Q_1} = [F(1,N)\ \Delta ic_F + ic_F(0)\ \Delta F(N)] - \frac{Q_2}{Q_1}\ [G(1,N)\ \Delta ic_G + ic_G(0)\ \Delta G(N)] \tag{5.3}$$

Later sections provide specific examples of calculating hedge ratios for inter-commodity trades.

Unfortunately, the presence of two commodities in the tandem trade means that interpretation of the profit function can be somewhat complicated, e.g., the TED tandem (Landau and Wolkowitz 1987, Kawaller and Koch 1992, Poitras 1989, 1995). Following the approach used for the butterfly, in order to simplify the cash-and-carry profit function for the tandem, analysis can proceed expeditiously by taking both sides of the trade to be tailed spreads. In this case, (5.3) becomes:

$$\frac{\pi_{tt}}{Q_1} = [F(1,N)\ \Delta ic_F\ ] - \frac{Q_2}{Q_1}\ [G(1,N)\ \Delta ic_G\ ]$$

$$\pi_{tt}^{\ *} \equiv \frac{\pi_{tt}}{Q_1\ F(1,N)} = \Delta ic_F - \frac{Q_2\ G(1,N)}{Q_1\ F(1,N)}\ \Delta ic_G$$

Choosing a dollar equivalence hedge ratio involves setting $\{Q_2\ G(1,N)\} = \{Q_1\ F(1,N)\}$, permitting the profit function to depend solely on the difference in the *ic* changes for the two commodities involved. In order to determine the hedge ratio in practice, *G(0,N)* and *F(0,N)* are used to approximate the unobserved prices, *G(1,N)* and *F(1,N)*. This approach to interpreting the profit function provides an immediate connection between tailed tandems and ***stereo*** trades.

A stereo trade has a profit function that depends on the difference in changes for the cost-of-carry interest rates implied in arbitrages for the selected futures contracts. A simple example of a stereo trade occurs with a tailed tandem involving gold and silver. For these commodities there is no significant return to holding the cash commodity and non-interest carrying charges involved in the cash-and carry arbitrage are small relative to interest charges. As a result, the profit function depends on the difference in the changes for the interest rates implied in gold and silver futures prices. More complicated forms of stereo trades occur for debt futures contracts, where the commodity has both an interest carry cost, the implied repo rate, and an interest carry return. Recognizing that the tailing procedure can be adjusted such that the profit function for the spread in each commodity depends only on the implied repo rate changes, the tailed tandem again becomes a ***stereo:***

$$\pi_s^{\ *} \equiv \frac{\pi_s}{F(1,N)\ Q_1} = \Delta irr_1 - \frac{G(1,N)\ Q_2}{F(1,N)\ Q_1}\ \Delta irr_2$$

where irr_i is the interest carrying cost implied by the cash-and-carry arbitrage for commodity *i*, e.g., the implied repo rate for financial futures. For a number of reasons, not all tailed tandem trades are stereos. In order for a stereo trade to occur, the tailing method must convert the profit function to depend only on the change in interest carrying charges. Except in special cases, untailed tandems will not be stereos.

The tailed tandem stereo trades are specific instances of "differential repo arbitrage" trades, a class of trades that also includes the turtle trades (Yano 1989). The profit functions for these inter-commodity trades depend either on the difference in the implied repo rates for two sets of financial futures contracts or on the difference in an implied repo rate and a surrogate for the cash market repo rate. Trading opportunities are identified when the *irr* for a given futures contract deviates significantly, either from the *irr* for other futures contracts, which generates a stereo trade, or from the cash market, which generates a turtle trade. Specific examples of these trades include the stereo NOB, which trades the *irr* from Tnote and Tbonds futures, and the stereo GUN, which involves the *irr's* from GNMA and Tnote contracts. To illustrate these trades consider the stereo NOB. This trade is constructed using tailed spreads in bonds and notes, where the tail is $(1 + irr)$ with resulting profit functions of the form $F(1,N)\ \Delta$ irr. The appropriate hedge ratios are calculated in the same fashion as for the naked NOB. Yano (1989) provides an elegant and slightly more precise method of arriving at the relevant position sizes.

Turtles and Stereos

Variations on tailed tandem trades occur where one of the positions is not an appropriately tailed spread but, rather, an open position. These types of trades are known generically as turtle trades. The basic idea of the turtle is to trade the difference between the ic or irr embedded in the futures price structure against some other variable, usually an interest rate. The simplest version of this trade is a metal turtle, which is discussed in more detail in later in this chapter. This trade involves, for example, combining a tailed gold spread with a Eurodollar futures position (Poitras 1987). The objective is to speculate on changes in the difference between the implied interest rate in gold futures and the Eurodollar rate. Among other reasons, this trade is of interest because there is a *one-sided* arbitrage relationship between gold futures prices and Eurodollar interest rates that can be used to fine-tune the spread trading decision. Because absence-of-arbitrage prevents the gold *ic* from being greater than the relevant Euro rate, the difference between the two rates can be used to identify trading opportunities. To calculate the "dollar value" hedge ratio for this trade, the technique of equalizing the value of a basis point movement for the naked Euro and the tailed gold spread is used.

In turtle trades involving debt futures, such as ***the*** turtle (Rentzler 1986, Easterwood and Senchack 1986) between Tbond spreads and Tbills, it is the *irr* and not the *ic* that is of interest. Much as in the stereo trades that speculate on changes in $(\Delta\ irr_1 - \Delta\ irr_2)$, the turtle is concerned with speculating on $\Delta\ irr - \Delta i$, where i is the interest rate on the appropriate open (naked) interest rate futures contract. This requires specification of the tail for the intra-commodity spread to be readjusted such that the resulting profit function is of the form: $\pi_{irr} = F(1,N)\ \Delta\ irr$. In order to identify the appropriate tail for this situation, observe that for debt futures $ic = irr - R$ or $irr = ic + R$. More precisely, taking the current yield to be a sufficient approximation to R:

$$irr(0,N,T) = \frac{F(0,T)-F(0,N)}{F(0,N)} + \frac{C}{F(0,N)}\frac{T-N}{365} = ic(0,T,N) + R(0) \qquad (5.4)$$

where C is the annual stated coupon on the underlying theoretical bond or note. Taking $\pi_{ts} = F(1,N)\ \Delta ic = F(1,N)(\Delta irr - \Delta R)$, to derive the appropriate tail observe that:

$$F(1,N)\Delta R = F(1,N)\{\frac{C^*}{F(1,N)} - \frac{C^*}{F(0,N)}\} = -\frac{C^*}{F(0,N)}\ \Delta F$$

where $C^* = C\ (T-N/365)$. Combining this with the result:

$$\pi_{ts} = \{1 + ic(0)\}\{F(0,N) - F(1,N))\} + \{F(1,T) - F(0,T)\}$$

$$= \{1 + ic(0)\}\{-\Delta F(N)\} + \{F(1,T) - F(0,T)\}$$

Substituting from the definition for π_{irr} gives:

$$\pi_{irr} = \pi_{ts} + F(1,N)\Delta R = \pi_{ts} - R(0)\ \Delta F(N)$$

$$= \{1 + ic(0) + R(0)\}\{F(0,N) - F(1,N)\} + \{F(1,T) - F(0,T)\}$$

Hence, for spreads involving profit functions using the implied repo rate, the appropriate tail is *{1 + irr(0)}* and not *{1 + ic(0)}* where *irr* is calculated using *(5.4).*

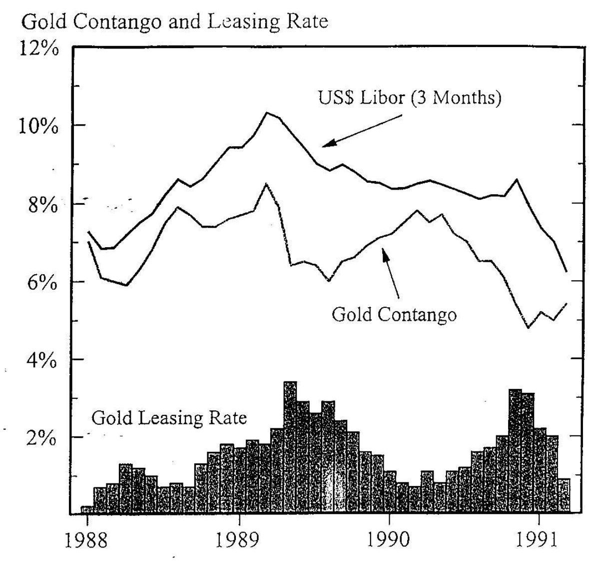
Compared to precious metal turtles, factors determining profitability of turtle trades involving debt futures are somewhat more complicated. As with the *{1 + ic}* tailed Tbond spread and the (naked) NOB, the turtle can be used to speculate on changes in yield curve shape, albeit only at the short end. More frequently, turtle trades involving debt futures are used to capture deviations of the *implied repo rate* from the actual or cash repo rate. These deviations emerge because the repurchase agreement used to finance cash transactions is primarily an overnight rate, with some term repo available in short maturities but, effectively, no terms to maturity that correspond to the deliveries of the relevant debt futures contract. Because the cash market does not provide a direct financing vehicle for doing arbitrages involving, say, Tbonds, it is possible for the *irr* associated with Tbonds to deviate substantially from the *irr* observed in the cash market (Allen and Thurston 1988). Turtles takes the form of a cash-and-carry quasi-arbitrage trade designed to exploit the observed deviation. This intuition for the turtle trade relies on the Tbill position being a surrogate for the cash repo rate.

Motivations for doing turtle trades with debt futures can be illustrated by considering a profit function that has been simplified by assuming the hedge ratio has been set appropriately:

$$\pi_{turt} \propto \{\Delta\ irr(N,T)\} - \Delta\ tbr(N,T)\}$$

where *tbr* is the interest rate reflected in the relevant Tbill futures and *irr* is the implied repo rate for the relevant debt futures contract. When the hedge ratio is set appropriately, this leaves the payoff on the turtle to be dependent on the difference in the implied repo and Tbill rate changes.[3] From this, turtle trades can be generalized to trades involving *{1 + irr}* tailed spreads and any other relevant money market futures contracts. Other possible configurations include *{1 + irr}* tailed Tnote spreads with Euros. Because the profit functions for the various possible turtle trades involve differencing two interest rates that are, invariably, determined by differing market forces, it is necessary to construct a behavioural foundation for explaining each specific trade's profitability. Yano (1989) recognizes this point: "The turtle trade is not riskless arbitrage. There seems to be a widespread fallacy that the [difference in the implied repo rates is] zero on average, but there is no necessary reason for this to be true." Referring to turtles derived from financial futures: "Different configurations will have their idiosyncrasies due to, but not limited to, heterogeneous expectations along the yield curve" (p.446).

5.2 Metal Turtles



Source: Consolidated Gold Fields, Gold 1991

In contrast to the currency tandem where the profit function was somewhat complicated to derive, the profit function for a metal turtle trade is straight-forward. Ignoring the hedge ratio, $\pi = \Delta\ ic - \Delta\ r$, where r is the interest rate on the interest rate futures contract selected for the specific turtle trade. One conceptual difficulty with a turtle occurs with specifying the time at which the trade is initiated. In the case of the precious metals, gold and silver, turtle profitability depends on the relationship between the *ic*, which is largely determined by interest charges, and the upper arbitrage boundary provided by the Eurodollar rate. For gold, this relationship is illustrated in Figure 5.2. Inspection of Figure 5.2 reveals that when the gold *ic* gets either "too close to" or "too far from" the Eurodollar boundary rate, a golden turtle trade can be established and held until the gold *ic* comes back to a more normal relationship with the boundary. At this time the position is closed out and profit on the trade calculated. This approach to defining a trading strategy differs from other studies (e.g., Monroe 1992, Monroe and Cohn 1986, Rentzler 1986) that use techniques such as moving averages and standard deviations to generate trading decisions.

Trade Triggers

In order to identify the appropriate number of basis points to use in determining when to initiate and close out the golden turtle trade, Poitras (1987) introduces an additional interest rate, the US Tbill rate, to serve as a lower boundary. The golden turtle trading strategy then involves evaluating the relationship between the three relevant rates, the gold *ic,* the Eurodollar rate and the Tbill rate. Poitras (1987) considered the following method of identifying golden turtle trading opportunities: for a period starting approximately fifteen months prior to the delivery date of the front gold contract and lasting until two months prior to the last delivery date on the front gold contract, if at any time during this period the annualized gold *ic* traded within a predetermined number of basis points of a boundary rate an appropriate trade was initiated. The trade was then examined daily to assess whether it should either be reversed or closed out. The reverse/close out decision was again made according to a whether the gold *ic* traded within a predetermined number of basis points of the other boundary rate. At this time, if there was more than six months left in the trading period the trade was reversed and the daily evaluation process was restarted. Otherwise, trading for that trading horizon was complete. If the trade had not been closed out by the end of last month of the trading period, the trade was closed out at market.[4]

An unanswered question in this trading strategy is the determination of the number of basis points from a boundary rate the gold *ic* must be in order for a trade to be initiated or closed out. The problem of specifying a strict rule is further complicated by occasional convergence of the boundaries, i.e., where the Eurodollar rate approaches the Tbill rate. With this in mind, Poitras (1987) selected a censored-percentage-trigger-rule. This rule works by determining the number of basis points from the boundary the gold *ic* must be to trigger a trade as a percentage of the size of the Tbill/Euro differential. For example, if the Eurodollar/Tbill differential is 80 basis points, a 10% rule would initiate or close a trade if the annualized gold *ic* came with 8 basis points of a boundary.[5] Using this trading technique, it was demonstrated that the golden turtle provided significant trading profits. The number of trades decreased with the number of basis points in the trigger rule while profits increased monotonically. The holding periods were typically several months, indicating that the golden turtle depends more on underlying fundamentals than pure noise trading.

Calculating the Tail and Hedge Ratio: Golden Turtle

In order to determine the number of gold and Euro contracts to be used in the trade it is necessary to specify the tailing procedure and the method of determining the 'hedge ratio' between gold spreads and Euro contracts. Recognizing that there are different methods of determining the tail, in the golden turtle the objective is to isolate *ic*. As discussed in Section 3.2, the tailed spread in this case can be specified such that for every long (short) deferred contract there will be $F(0,T)/F(0,N)$ short (long) nearby contracts. For every unit of the deferred contract, there will be $\{1 + ic(0)\}$ units of the nearby contract. The profit function for a short-the-nearby, long-the-deferred tailed gold spread is: $\pi_{TGS}/(100\ oz.) = G(1,N)\ \Delta\ ic(N,T)$. Assuming the trade is being initiated on Aug. 8, 1994 (see Fig. 4.3) and taking the gold price $G(1,N)$ to be equal to \$379, the $G(0,N)$ for the Oct 1994 delivery on Aug. 8/94, then the hedge ratio follows by observing that one basis point equals .0001, that the Comex gold contract is written for 100 ounces, and that the value of one basis point for a Eurodollar contract is \$25. Hence, the hedge ratio for the number of gold spreads per Euro contract is (\$25/[(\$379)(100)(.0001)] = 6.596. The value is combined with the value for the spread tail, to determine the appropriate number of the three contracts to use for the trade.

To see this more precisely, the basic problem is to derive the number of tailed gold spreads that, for a given basis point change, will (locally) have the same dollar value change as the corresponding dollar value change in the Euro contract. The profit function for a golden 'bear' turtle, long one Euro contract and short Q^* tailed gold spreads is:

$$\pi_{gt}(1) = \$2500\ (r_{EU}(0,T) - r_{EU}(1,T)) + 100\ Q^*\ G(1,N)\ (ic(1) - ic(0))$$
$$= \$2500\ (EU(1,T) - EU(0,T)) + 100\ Q^*\ G(1,N)\ (ic(1) - ic(0))$$

where: ***$r_{EU}(i,T)$ is a whole number, annual interest rate*** calculated as 100 minus $EU(i,T)$, the quoted Euro contract price at time i. When Q^* is selected to be consistent with the dollar equivalent hedge ratio, the golden 'bear' turtle will be profitable when the differential between the annualized gold *ic* and the Eurodollar rate narrows. The converse would hold for the golden 'bull' turtle, the trade will be profitable when the differential between the annualized gold ic and the Euro rate widens. Correct calculation of Q^* follows appropriately.

Given that the π for a Euro is \$25 per basis point, the dollar equivalency hedge ratio problem is to calculate the value of 1 basis point *(per $Q_T = 1$ spread)* for a tailed gold spread. Recalling that 1 basis point is .0001, what remains to be done is to set Δ *ic = .0001* and solve the tailed gold spread profit function. On a per contract basis this produces:

$$\pi_{TGS} = (100)\, G(1,N)\, (.0001) = (.01)\, G(1,N)$$

Because *G(1,N)* is not known at *t=0* when the trade is initiated, a proxy is required. In the absence of a better value, *G(0,N)* is appropriate. Recalling that this value for the Oct 1994 delivery on Aug. 8/94 is, \$379.30, then \$3.793 is the value of one basis point (***per contract***) in a tailed gold spread. Relating this basis point value to a Euro again provides the appropriate hedge ratio for the golden turtle:

$$Q^* = HR = \$25/\$3.793 = 6.591 = \#\ of\ tailed\ gold\ spreads\ per\ Euro$$

This number can now be used to construct the trade.

Because of the need to match the (whole) number of contracts in the tailed spread with the hedge ratio, the golden turtle is somewhat more complicated to implement than the tailed gold spread. Recall that in order to get a correct trade size for the tail, it was necessary to gross up *{1 + ic(N,T)}* until a "comfortable" integer relationship was established for the two legs of the spread. Using the one year, Oct/Oct spread values from 8/8/94 produced: *1 + ic(N,T)* = (399.5/379.3) = 1.0533. Observing that (3)(6.591) = 19.77 and (19)(1.0533) = 20.013 it follows that the trade can be roughly done using 19 deferred gold, 20 nearby gold and 3 naked Euro contracts. Slippage between trading profits and the theoretical profit function due to rounding involved in the calculations is reduced if a proportionately larger number of contracts are used.

What remains is to analyze various scenarios where the difference between the Euro rate and the annualized gold *ic* is expected to change. Consider the case where the difference between the Euro rate and the gold *ic* is expected to widen. This can happen a number of ways, e.g., the *ic* could stay constant and the Euro rate could increase. In this case, $r(1) - r(0) > 0$. Because the profit function for a long position is $\pi = \$2500\ \Delta\ EU = \$2500\ \{-\Delta\ r\}$, when r is expected to rise, a short position in Euros is profitable. Similarly, if the widening occurs because the Euro rate is unchanged $r(1) - r(0) = 0$, with the gold ic falling, $ic(1) - ic(0) < 0$, then it follows that a spread that is long-the-nearby and short-the-deferred is indicated. Further examples confirm: *when the Euro/gold ic interest rate spread is expected to widen the appropriate trade involves a short Euro combined with a tailed spread that is long-the-nearby and short-the-deferred.* Similarly, when the interest rate differential is expected to narrow then the appropriate trade is long the Euro combined with a tailed gold spread that is short-the-nearby and long-the-deferred. The appropriate combination of these positions involves calculation of the size of the tailed spread, adjusted for the appropriate hedge ratio.

Silver Turtle

Analysis for the silver turtle and other precious metal turtles follows in the same fashion as for the golden turtle trade. The hedge ratio for the silver turtle is developed from the profit function for the *{1 + ic}* tailed intra-commodity spread:

$$\frac{\pi_{TSS}}{5000\ oz.} = S(1,N)\ \Delta\ ic$$

Again, solve for one basis point in tailed silver spread profit per $(Q_T = 1)$ contract. For the Dec. 94 Comex silver price observed on 8/8/94, $S(0,N) = \$5.183$, for the 5000 oz. Comex futures contract:

$$\pi_{st} = (5000)\ (5.183)\ (.0001) = \$2.5915$$

Using the \$25 per basis point value for the Euro produces a *HR* of \$25/\$2.5915 = 9.647. The 8/8/94 Dec 95/Dec 94 tailed silver spread produces $\{1 + ic\}$ = (552/518.3) = 1.065. From the basic calculations (9.647)(3)=28.941 and (28)(1.065)= 29.82, it follows that on 8/8/94 a trade with some slippage can be constructed as 3 Dec 94 Euros combined with 28 deferred Dec 95 and 30 nearby Dec 94 Comex silver futures contracts.

Copper Turtle

The copper turtle differs from the precious metal turtles because *ic* is often dominated by factors other than interest charges. Following the approach used for the gold and silver turtles, the relevant per pound profit function for the $\{1 + ic\}$ tailed spread using Comex copper futures is:

$$\frac{\pi_{TCS}}{25{,}000} = C(1,N)\ \Delta ic_c$$

Using the 8/8/94 prices (see Fig. 4.3) and, again, taking the Euro basis point value to be \$25 with $C(0,N)$ = \$1.0895 as a proxy for the copper price $C(1,N)$, then calculating the value of $\Delta\ ic = .0001$, the per contract $(Q_T = 1)$ profit of one basis point for the tailed copper spread is:

$$\pi = \$1.0895\ \{25{,}000\}\ (.0001\} = \$2.724$$

This yields a *HR* of 9.178 = \$25/\$2.724. For the Dec 95/Dec 94 tailed copper spread, $(1 + ic)$ = (\$1.0515 /\$1.0895) = .965. Observing that (6)(9.178) = 55.07 and (56)(.965)= 54.04, it follows that on 8/8/94 a copper turtle could be constructed using 6 Dec 94 Eurodollar contracts combined with 56 deferred Dec 95 and 54 nearby Dec 94 contracts.

Spread behavior in copper futures can be considerably more volatile than for precious metals. For example, the 8/8/94 prices reflected a backwardation in futures prices, $F(0,T) < F(0,N)$ with $[F(0,T) - F(0,N)]$ = -\$.038 for the Dec 95/Dec 94 prices. This level of backwardation was not unusual during the decade of Hamanaka's activities in the copper market (see Sec. 2.3), though copper had largely been a contango market during the 1980s and, following the collapse of the scheme, the contango relationship in copper has re-emerged, e.g., as of 9/9/01 for the Dec 01 and Dec 02 Comex copper contracts, $F(0,T)$ = \$.7065 and $F(0,N)$ = \$.6775. Due the period of the scheme, there was variation in the degree of backwardon. For example, on 6/16/92 copper prices for a 1 year Dec 92-Dec 93 spread were $F(0,T)$ = \$1.1515 and $F(0,N)$ = \$1.0755, which is a wider backwardation than on 8/8/94 with $(1 + ic)$ = .934 and $[F(0,T) - F(0,N)]$ = -\$.076. Backwardation in Comex copper futures prices reached a peak during Sept. 1993 and continued until some time after the collapse of the Sumitomo scheme. To the astute spread trader, the collapse of the scheme presented a predictable reduction in the degree of backwardation that, in turn, presented a profitable trading opportunity for a copper turtle -- a tailed copper spread would also have been profitable..

To see the profit potential in the copper turtle, consider the 8/8/95 prices which reflect a severe backwardation in futures prices. Using the Dec 96/Dec 95 Comex copper futures prices: $[F(0,T) - F(0,N)]$ = [$1.1145 -$1.3155] = -$.201 with $(1 + ic)$ = .847. This yields a HR = $25/$3.29 = 7.60 and a copper turtle could be roughly established with 26 deferred, 22 nearby and 3 Euros. If the speculation at that time was that backwardation would disappear into contango within the following year, then a trade of long the Dec 95 Euro combined with short the Dec 95 nearby and long the Dec 96 deferred is appropriate. Because the nearby contracts will reach maturity before the end of the trade horizon is reached, to calculate the trade profit it is necessary to assume the contracts in the trade could be "rolled forward" with no cost. Given this, evaluating the approximate profit for the trade held to 8/8/96 produces: (3)(2500)(94.2 - 94.3) + 25,000 {26 (.889 - 1.1145) + 22 (1.3155 - .9085)} = -750 -146,575 + 223,850 = $76,525.

Treating a copper turtle to be similar to a precious metal turtle presents complications similar to using a $\{1 + ic\}$ Tbond spread in ***the*** turtle trade. Unlike the precious metals that are typically at or near full carry or full contango, copper has a term structure of futures prices that depends on convenience yield. In some periods, copper is at full carry while at other times there is backwardation where $F(0,N) > F(0,T)$. During the period of the Sumitomo copper corner, backwardation was often the case. Unlike the Tbond case where there is an observable price and coupon allowing the ic to be decomposed into two parts and a $\{1 + irr\}$ tail to be specified, convenience yield for copper is not so readily observable. To deal with this problem, assume the copper ic can be decomposed as $ic_c \cong irr_c - cy$, where irr_c is the implied interest charges associated with carrying copper and cy is the convenience yield that copper stocks provide. Taking $cy = irr_c - ic_c$ and using, say, the implied interest rate for gold, ic_G, as a surrogate for irr_c, it is possible to define a $\{1 + cy\}$ tailed copper spread that can be used to construct a $\{1 + cy\}$ tailed copper spread for the copper turtle. (See Question 4 at the end of the chapter).

5.3 TED Tandems and Currency Tandems

The TED Spread

To futures traders, a TED spread is an inter-commodity spread trade that combines a short (long) Eurodollar future with a long (short) US Treasury bill future. (The acronym TED comes from combining (T)reasury bill with (E)uro(D)ollar.) The trade is put on for a number of reasons. For example, the trade can be used to speculate on cash market credit spreads.[6] Combining TED spreads with different delivery dates produces the TED tandem. Both the TED spread and TED tandem have the desirable analytical feature that the position sizes $Q_1 = Q_2 = Q$, due to the equal maturity (3 month), par value ($1 million) and $25 basis point value of the contracts. Applying the general tandem profit function of Sec. 5.2 using a hedge ratio of one to correspond to the TED tandem produces:

$$\pi/Q = \{[F(1,T)\text{-}F(1,N)] - [G(1,T)\text{-}G(1,N)]\}$$
$$-\{[F(0,T)\text{-}F(0,N)] - [G(0,T)\text{-}G(0,N)]\}$$

where the Euro quote is $G(\cdot)$ and the Tbill quote is $F(\cdot)$. This profit function will be developed in more detail shortly.

To consider the intuition behind the TED tandem trade, assume the current date is Sept. 11, 1990, when the following set of futures prices was available:

	Euro	***Basis***	***Tbill***
Cash	8.06	.65	7.41
Nearby (Sept 90)	91.95	.69	92.64
Nearby (Dec 90)	92.02	.84	92.86
Deferred (Sept 91)	91.65	1.10	92.75
Deferred (Dec 91)	91.38	1.05	92.43

This pattern where the deferred basis is wider than the cash basis, combined with the requirement that the nearby basis must converge to the cash basis, is the motivation for tandem trading opportunities. Similarly for the June 16, 1992:

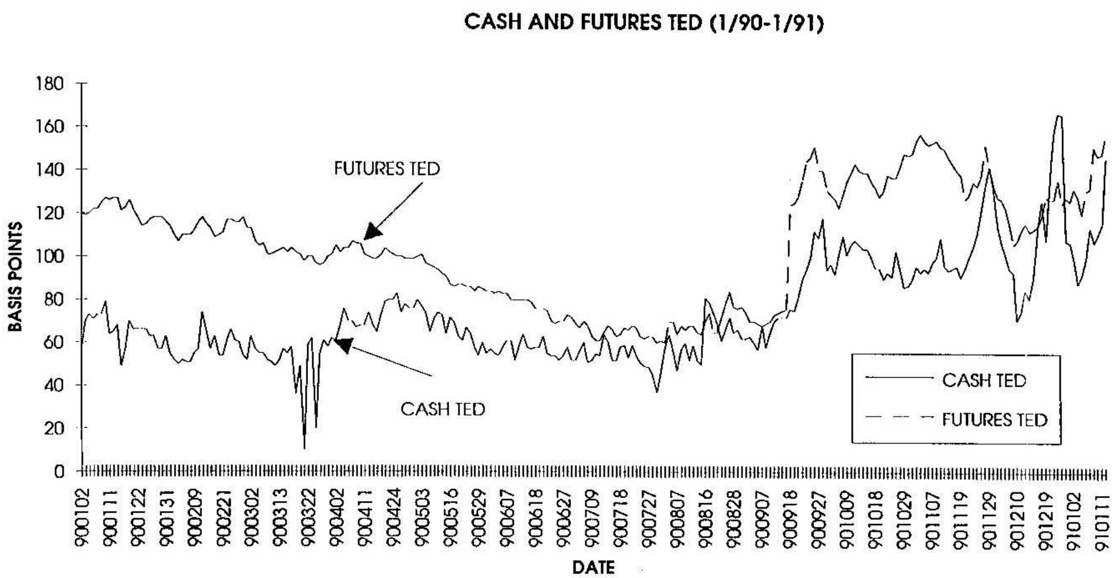
	Euro	***Basis***	***Tbill***
Cash	3 15/16		3.73
Nearby (June 92)	96.00	.31	96.31
Nearby (Sept 92)	95.83	.38	96.21
Deferred (June 93)	94.68	.58	95.26
Deferred (Sept 93)	94.24	--	---

Figure 5.3 provides a representative plot of the behavior of cash and futures TED spreads. It suffices to say that the TED is designed to benefit from futures to cash convergence. At delivery the futures TED must equal the cash TED. For a number of reasons, the distant contracts are less affected by the cash-and-carry arbitrage than the nearby contracts. As a result, futures to cash convergence will impact the nearby more than the deferred generating a potential profit opportunity.

The history of TED spread trading on futures markets begins with the introduction of Eurodollar futures contracts on the Chicago Mercantile Exchange's International Monetary Market (IMM) in December of

1981. Following an initially disappointing debut, the Eurodollar futures contract has grown to be arguably the most successful futures contract ever introduced. In addition, Eurodollar futures have been successfully introduced on other futures exchanges, most notably the London International Financial Futures Exchange (LIFFE) starting in September 1982. The other half of the TED spread, the US Tbill contract, has not performed as well. In the period following the introduction of IMM Tbill futures contracts in January 1976 to the start of trading in Eurodollar futures, the Tbill contract was a marked success. However, in the period since December 1981, volume and open interest in Tbills have declined to the point where TED spreaders are now a crucial component of distant contract liquidity in the Tbill pit. In other words, at a time when Eurodollar futures are adding more distant contracts at the IMM, Tbill futures are losing liquidity in the distant contracts.

Given this background, the history of TED trading can be roughly divided into two parts: an early period



(1982-3) where there was substantial divergence between the cash and futures TED spreads creating significant arbitrage opportunities; and, a later period (1983 to present) where deviations in the cash-futures TED from arbitrage equilibrium have substantially narrowed. The differences between the two periods can be attributed to market learning and the ensuing creation of trading operations to arbitrage significant divergence between cash and futures prices. Short-term TED trades based on cash-futures divergence that were profitable in the earlier period are, typically, no longer available. TED spreaders have had to adapt to changing conditions by either increasing position sizes on smaller anticipated moves or by basing trades on longer term fundamentals such as "flight to quality". In both cases, the nature of trading the TED spread has changed.

The Cash and Carry Arbitrages for US Money Market Futures

One aspect of the TED spread that has not changed is the arbitrage-based fundamentals for the relationship between the cash and futures TED spreads. Briefly, because of the mechanics of cash-futures arbitrage, there is an inherent bias causing the cash TED to differ from the nearby futures TED. In addition, the distant TED is generally undetermined because of the lack of a financing vehicle to execute the cash-futures Tbill arbitrage. Hence, there is some fundamentally-based potential to identify profitable TED trades. To see this, consider the cash-futures arbitrage trades for nearby Tbill futures contracts (e.g., Dym (1988), Hegde and Branch (1985)). This trade is similar to the cash and carry arbitrage for wi in Sec. 4.3. At time $t = 0\ (<N)$, the `long' cash arbitrage involves purchasing a Tbill deliverable on a futures contract maturing at $t=N$ at price $P[t,91+N]$, financing the purchase at the term repurchase agreement (repo) rate $(R[t,N](N/360))$, and covering the cash purchase by shorting a dollar equivalent amount of futures contacts at (invoice) price $TB[t,N]$. Observing that the cash tbill will also earn a carry return $r[t,N]$, the no arbitrage condition is:[7]

$$P[t,N]\ (1 + (R[t,N](N/360) - r[t,N])) \geq TB[t,N]$$

In other words, the net cost of purchasing the deliverable Tbill today and carrying it to delivery must be greater than the price received from simultaneously selling the same Tbill in the futures market. Violation of this condition will generate arbitrage opportunities.

The short cash arbitrage is similar. A Tbill deliverable at $t=N$ is acquired by doing a term *reverse* repo at the reverse repo rate $(RR[t,N])$ with the appropriately dated Tbill as the underlying collateral. This Tbill is then sold at $P[t,91+N]$ creating a short position. Simultaneously, the short is covered by taking a dollar equivalent number of long Tbill futures contracts. In this case no arbitrage dictates that:[8]

$$P[t,91+N]\ (1 + (RR[t,N](N/360) - r[t,N])) \leq TB[t,N]$$

It follows from combining these two conditions that $TB[t,N]$ is bounded:

$$\begin{aligned} &P[t,91+N](1+(RR[t,N](N/360) - r[t,N])) \leq \\ &\qquad TB[t,N] \leq P[t,91+N](1+(RR(t,N)(N/360) - r[t,N])) \end{aligned}$$

From this it can be shown that (see Poitras 1998):

$$\begin{aligned} RR[t,N](N/360) \leq\ & RTB[t,91+N](N/360) + (RTB[t,91+N] - r^t[N])(91/360) \\ &\leq R[t,N](N/360) \end{aligned}$$

where $RTB[t,91+N]$ and $r^t[N]$ are the appropriate interest rates for the $91+N$ day cash Tbill and the Tbill futures positions.

For expository purposes, assume that the equality holds in the above weak inequality relationships, i.e., that the repo (borrowing) and reverse (lending) rates are equal. In other words, $R = RR$. In this case, the conditions simplify to:

$$r^t[N] = RTB[t,91+N] + ((N/91)(RTB[t,91+N] - R[t,N]))$$

This result provides a direct relationship between the interest rate implied in a specific Tbill futures contract, with the cash rate for a Tbill that is deliverable on that contract and the financing rate applicable for the underlying arbitrage. However, because there is no deliverable cash Tbill for contracts more than 9 months to delivery, the cash and carry arbitrage can only hold for nearby Tbill futures contracts. In addition, there is virtually no market for term repo in that maturity range.[9] Hence, the fundamentals that drive the nearby Tbill futures rates tend to differ from the deferred futures.

The arbitrage for Eurodollar futures differs fundamentally from that for nearby Tbill futures. This follows because arbitrage financing is done in the repo market for Tbills while, for Euros, financing is done in the cash market. As a consequence, the financing rate for Euros is determined by the implied forward rate in the cash market. To see this, consider a `long' arbitrage for Euros. At time t, funds for the arbitrage are borrowed by issuing a Eurodollar deposit (at the bid rate, $RS[t,N]$) that matures on the delivery date for the Euro contract N days away. These funds are then invested for $91+N$ days (at the offer rate, $RL[t,91+N]$) and the resulting tail is covered by shorting the Euro contract at $EU(t,N)$ with implied interest rate $r^e[N]$. Unlike the Tbill case, the long Euro position is not deliverable on the futures contract, both because the Euro contract involves cash settlement and because the Euro deposit is non-negotiable. The position must be refinanced (at the bid rate) with the gain (or loss) on the futures position providing a mechanism to "lock in" the borrowing rate.

Ignoring the bid/offer difference for ease of notation, the long arbitrage implies:[10]

$$(1 + RL[t,91+N]((91+N)/360)) = (1 + RS[t,N](N/360))(1 + r^e[N](91/360))$$

Taking logs and ignoring second order terms gives:

$$r^e[N] = RL[t,91+N] + ((N/91)(RL[t,91+N] - RS[t,N]))$$

As shown in Poitras (1989, 1998), this indicates that the "arbitrage" equilibrium condition for Euros has the relevant implied forward rate in the cash market equivalent to the interest rate implied in the relevant Euro futures price.

These equations can now be used to determine the equilibrium value for the TED:

$$\begin{aligned} r^e[N] - r^t[N] &= (TED[t,91+N]) + ((N/91)((TED[t,91+N] - (RS[t,N] - R[t,N]))) \\ &= (1+(N/91))(TED[t,91+N]) - ((N/91)(RS[t,N]-R[t,N])) \end{aligned}$$

where $TED[t,91+N] = RL[t,91+N] - RTB[t,91+N]$. In other words, the futures TED spread is determined by the cash TED with an adjustment for the difference between the (short) Euro rate and the term repo rate. When $N=0$, this equation reduces to the arbitrage condition that cash and futures rates must be equal at maturity.

Given this, the formula for the term structure of the TED spread, essential to determining the profitability of the TED tandem, can now be calculated. More precisely, suppressing time dating for ease of notation:

$$\begin{aligned} (r^e[T]-r^t[T]) - (r^e[N]-r^t[N]) = \quad & (1+(N/91))(TED[91+T] - TED[91+T]) \\ & - (N/91)(EUROYC[T,N] - (R[T]-R[N])) \\ & + ((T-N)/91)(TED[91+T] - (RS[T] - R[T])) \end{aligned} \qquad (5.5)$$

where *EUROYC[i,j]* is the yield differences between the rates for Euros maturing in *i* and *j* respectively. For example, *EUROYC[T,N] = RS[T] - RS[N]*. Hence, the difference between the nearby and deferred TED's depends theoretically on the relative slopes of the Eurodollar and Tbill yield curves. However, at some point, the actual TED term structure will deviate from this condition due to restrictions on the underlying arbitrage. This divergence can provide a basis for designing profitable trading rules.

TED Spread Trading Strategies

As a spread trade, the TED has the desirable property of having equivalent basis point value for changes in the two contracts involved, $25 per basis point on both Euro and Tbill contracts. Hence, the dollar equivalence ratio of the number of Euro contracts to Tbill contracts in the TED is one-to-one. Given this, consider the payoff on a `long' TED trade (short the Euro and long the Tbill) that was established at *t* and closed out at *t+1*:

$$\pi_{TED} = (EU[t] - EU[t+1]) + (TB[t+1] - TB[t]) = (EU[t] - TB[t]) - (EU[t+1] - TB[t+1])$$

In terms of futures price quotes:

$$\begin{aligned}\pi_{TED} &= \{(100 - r^e[t,N]) - (100 - r^b[t,N])\} - \{(100 - r^e[t+1,N]) - (100 - r^b[t+1,N])\} \\ &= \{r^e[t+1,N] - r^b[t+1,N]\} - \{r^e[t,N] - r^b[t,N]\}\end{aligned}$$

Briefly, the *long TED* will be profitable when the price difference between EU and TB narrows or, put differently, $(r^e - r^t)$ widens. It follows that the *short TED* will be profitable when $(r^e - r^t)$ narrows. While futures to cash convergence may have some marginal impact, the payoff on this trade depends primarily on changes in the cash TED. Factors driving the cash TED include `flight to quality' and instrument supply considerations.

Based on analysis of the profit function for the TED tandem, there are arbitrage factors that cause the cash, nearby and deferred TED spreads to differ. However, on the delivery date for the futures contract, the cash and futures TEDs must be approximately equal. To exploit these discrepancies requires trading strategies that have payoffs based on spread convergence; in the case a tandem trade is indicated. For inter-market spreading, tandem trades have the practical advantage of combining spreads in different commodities. Because each side of the trade is a spread, this allows for substantially lower transactions costs and margin requirements.[11] Defining the `long' tandem as *short the deferred* Euro and *nearby* Tbill and *long* the *nearby* Euro and *deferred* Tbill, the associated payoff is:

$$\begin{aligned}\pi_{tan} = &\{(r^e[t+1,T]-r^b[t+1,T]) - (r^e[t+1,N]-r^b[t+1,N])\} \\ &- \{(r^e[t,T]-r^b[t,T]) - (r^e[t,N]-r^b[t,N])\}\end{aligned}$$

Hence, the long tandem profits when the difference between $(r^e[T]-r^b[T])$ *and* $(r^e[N]-r^b[N])$ widens over time. Similarly, the ***short*** tandem is profitable when the difference between $(r^e[T]-r^b[T])$ *and* $(r^e[N]-r^b[N])$ narrows over time. Similarly, the `short' tandem is profitable when the difference between $(r^e[T]-r^t[T])$ and $(r^e[N]-r^t[N])$ narrows over time. For example, assume at *t=0* that the nearby spread is 115 basis points and the deferred is 123. Further assume that at *t=1* the nearby spread falls to 95 basis points while the deferred spread stays relatively constant at 120 (due, say, to futures-cash convergence factors affecting the nearby spread). In this case, a long tandem would have generated a profit of 17 basis points.

This analysis demonstrates that arbitrage considerations require that the payoff on the tandem is primarily determined by relative yield curve slopes. Empirically, the slope of the Euro yield curve is typically much flatter than the Tbill curve. In addition, the $RTB[T]$ - $R[T]$ term is likely to be negative because repo financing rates are often above the return on the underlying Tbill. Observing that the three parts of equation (5.5) all have different weights, on balance as N gets small the last part will tend to dominate. This follows for a number of reasons: the difference in the maturities involved in the Euro yield difference is larger than in the first part, the net effect of RTB-R term is likely to be positive while the net effect of the TBYC term in the first part is likely to be negative (offsetting the positive EUROYC term); and, the $(T-N)/91$ weighting term will be comparable to $(1+(N/91))$ when N is small. This implies that, for small N, (5.5) will typically be positive. However, as N gets larger, the first part will tend to dominate and (5.5) will become less positive.

Based on this analysis of (5.5), it is possible to heuristically specify a trading rule based on the implied convergence behavior. To see this, assume that for large N the TED spread structure is flat. As N declines (the nearby contracts approach maturity) the front contracts become increasingly more affected by cash-futures arbitrage considerations. This drives $(r^e[N] - r^t[N])$ down relative to $(r^e[t] - r^t[T])$ creating a spread-trading profit opportunity. An example of this type of behavior is exhibited in Figure 5.3 which tracks the cash-futures TED spreads as the contract moves from creation to maturity. By trading spreads against spreads, both legs of the trade are protected against changes in the level of interest rates. A primary source of risk in the trade is the possibility that `perverse' yield curve shifts that distort the cash-futures convergence behavior could occur at the end of the trade horizon. In addition, the maturity dates for futures contracts selected should be far enough apart that the nearby contract can be affected by cash-futures arbitrage factors while the deferred contracts are not.

Because the payoff on the tandem depends on relative changes in interest rates, profits on this trade are dependent on small basis point moves. Given the underlying variability in the spread relationships, trade performance will be dependent on the selection of the trade's trigger and kill rules. Unfortunately, there is a sizable number of such possible rules with the practical selection criteria being empirical performance. The type of trigger rule considered in Poitras (1989, 1998) is naive: the trade is mechanically established a prespecified number of months prior to the delivery month for the nearby contracts. The kill rule can also be naive. More precisely, trades can be closed out a pre-specified number of days before the beginning of the delivery period for the nearby contracts. Naive rules are useful to provide a benchmark against which rules incorporating judgmental factors can be measured. Trades based on triggers that incorporate qualitative factors may outperform naive rules. For example, a more sophisticated rule could be based on fundamentally-motivated factors such as the "cheapness" of the cash yield curve relative to a Euro or Tbill futures strip.

Currency Tandem

Analytically, currency tandems are one of the most interesting of all spread trades. This background for this trade has already been considered in some detail in Sec. 3.2. From equation (3.3), it follows that:

$$\Delta\ [F(T)-F(N)] = \theta(0)\ \Delta\ F\ +\ F(1,N)\ \Delta\ \theta$$

By working directly with the CIP condition, this exact result can be used to derive a precise expression for the profit function.

Using the discrete time version of the differential in Sec. 4.2, evaluating the $\Delta\ \theta$ term gives:

$$\Delta\theta \ = \ \frac{\Delta\ (i\ -\ i*)}{(1\ +\ i*)} \ + \ (i - i*)\ \Delta \frac{1}{(1\ +\ i*)}$$

$$= \frac{(\Delta i\ -\ \Delta i*)}{(1\ +\ i*)} \ - \ (i\ -\ i*) \frac{\Delta i*}{(1\ +\ i*)^2}$$

Substituting this result into (3.3) and collecting terms gives:

$$\pi_{ct} \ = \ \Delta\{F(T)\ -\ F(N)\} \ = \ F(1,N)\left[\frac{\Delta i\ -\ \Delta i*}{(1\ +\ i*)}\right] \ + \ \frac{i\ -\ i*}{1\ +\ i*}\ \Delta F(N)$$

Observing that $(i - i^*)\ \Delta\ i^*$ is a product of differences in interest rates, it follows that this term on the lhs is, to a first approximation, second order and can be set equal to zero. If the spread is tailed, or tailing is unnecessary, the ΔF term is removed and this leaves only the first term on the lhs to determine the spread:

$$\pi_{ct} \ \cong \ \frac{F(1,N)}{1\ +\ i*}\ \{\Delta i\ -\ \Delta i*\}$$

The upshot is that *the profitability of an intra-commodity currency spread depends on the relative change in the appropriate interest rates for the US and the foreign country.* This result for a one-to-one currency spread extends naturally for a tandem, which can be used to speculate on changes in interest rates that are not US. The tandem permits speculation on relative foreign interest rate changes even when there is no liquid foreign currency futures contracts directly quoted in terms of the two foreign currencies.

Before proceeding to consider the currency tandem, one practical problem about the intra-commodity currency spread needs to be considered: under what conditions it is possible to simplify the profit function by ignoring the tail on the spread, e.g., Poitras (1997)? This question can be resolved by observing that the $\Delta\ F$ term in (3.3) is associated with the tail, with $\theta(0)$ representing the appropriate size of the tail. It follows that if foreign and domestic interest rates are approximately equal $(\theta(0) \cong 0)$, then it is not necessary to tail the spread. However, in cases where there is a significant difference, a tail may be required. To see this, assume that $F(0,N) = 1$, $i = .1$ and $i^* = .04$. If the exchange rate falls by 20%, then $\Delta F = .2$ and $\{\theta(0)\ \Delta F\}$ is around .012. If $\{\Delta i - \Delta i^*\}$ changes by .02 then $F(1,N)\ \Delta\ \theta$ is around .016. However, while there are definitely situations in which a tailing a currency spread is advisable, it is also possible to construct numbers for which a tail is not required for the currency spread. In general, because it is a product of two differences, the difference in foreign and domestic interest rate levels and the change in exchange rates, the $\theta(0)\ \Delta F$ term is likely to be of second order though this result does not apply when the difference between foreign and domestic interest rates is large.

Assuming, for simplicity, that it is not necessary to tail the two currency spreads comprising the tandem, calculation of a hedge ratio is required for the tandem. Reexpressing the currency tandem profit function gives:

$$\pi_{ct}* \ = \ (\Delta i\ -\ \Delta i_F) \ - \ \frac{Q_G\ G(1,N)\ (1\ +\ i_F)}{Q_F\ F(1,N)\ (1\ +\ i_G)}\ [\Delta i\ -\ \Delta i_G]$$

where * indicates that profit has been appropriately scaled. If the hedge ratio is chosen to be dollar equivalent $[Q_G\, G(1,N)(1 + i_F) = Q_F\, F(1,N)(1 + i_G)]$, then the US interest rate terms Δi will cancel and this profit function will depend on the difference in the two foreign interest rates:

$$\pi_{ct}^{\,*} \cong \Delta ic_G - \Delta ic_F$$

Hence, when the hedge ratio is set appropriately and tailing is done when necessary, the profitability of a currency tandem depends on the difference in the two foreign interest rate changes, with the US interest rate impact canceling out.

In practice, calculation of the hedge ratio involves solving the approximation $[Q_2\, G(0,N)] = [Q_1\, F(0,N)]$. This requires equalizing dollar value on both legs of the tandem at $t=0$. To see how this is accomplished for the currency tandem, consider a trade where the objective is to speculate on relative changes in Canadian and British interest rates using currency futures denominated in terms of US$. In this case the US$ value of the Canadian dollar contract is: *{US$/C$} $100,000 = F(0,N)* Q_1. And the US dollar value of the £ contract is: *{US$/£) (£62,500) = G(0,N)* Q_2. Hence: $(G(0,N)Q_2/ F(0,N)\, Q_1)$ = *[{US$/£) (£62,500)]/{US$/C$} $100,000 = (C$/£) (62,500/100,000)*. The hedge ratio is the product of the current Canadian to British exchange rate times .625.[12]

5.4 Synthesizing Foreign Interest Rates

The absence of a viable futures market for foreign money market securities could pose a legitimate problem for money market participants seeking to hedge foreign cash positions. In some circumstances hedging objectives can be achieved through some other means, e.g., in the Government of Canada treasury bill market through the use of when issued tbill positions. However, for some hedging situations, a potentially more practical alternative is to make use of instruments traded on US exchanges, specifically: foreign currency and Eurodollar futures contracts. Use of US money market futures by foreign hedgers will be complicated by basis risk considerations and by the hedge profit function being denominated in two different currencies. This section provides examples for hedging cash Canadian treasury bill positions, though the result generalizes in a straight forward fashion to the hedging of any admissible foreign money market security. The strategies examined exhibit differing position sizes for the US money market futures used in forming the hedge. Theoretically, it is demonstrated that the minimum variance hedge ratios can be interpreted and estimated as coefficients in an appropriately specified *multivariate* regression.

To accurately hedge a cash Canadian tbill position with US money market futures, hedge design should account for the interest and exchange rate relationships implied by covered interest parity (CIP). In Sec. 4.2 it was demonstrated that for arbitrage involving securities identical in all respects except currency denomination, the annualized CIP relationship is often restated as:

$$r = r^* + \frac{F - S}{S}\,(1 + r^*)$$

The CIP condition provides the framework for hedging Canadian tbill positions with US money market futures. The basics of the hedge strategy follow from the textbook covered interest arbitrage trade for 1 year securities where the covered Canadian rate exceeds the rate on a comparable US security (see Sec. 4.2). ***Converting the CIP trade to a synthetic interest rate futures trade involves substituting a short US money market futures position for the borrowed US funds and, to reflect the currency transactions, a tailed currency spread that is short the deferred and long the nearby.*** Given this, the hedge design problem is concerned with specifying appropriate hedge ratios for the futures positions.

Basis risk in the hedge arises from two factors: deviations from CIP for the relevant Canadian and US instruments; and, discrepancies arising from the use of futures positions as replacements for cash transactions. Of these sources, the replacement of cash transactions with futures positions introduces only limited difficulties. Basis risk arising from CIP deviations is another matter. CIP holds as an equality for instruments that are identical in all respects except for currency of denomination. This is not the case when Canadian tbills are compared with either Euros or US Tbills. Each of these instruments possesses differential risk characteristics. Due to a combination of risk and arbitrage considerations, either the Euro or US Tbill (or both) could be used in constructing the hedge. Both these rates can be considered to provide effective inequality bounds on the covered Canadian rate, i.e., the covered Canadian rate is bounded above by the Euro rate and below by the US Tbill rate. This implies that deviations from CIP are also bounded and, hence, variation of the hedge position is constrained. This provides some scope for designing hedging strategies that exploit the systematic portion of the basis risk to improve hedge performance. To understand how this is done it is necessary to examine the specific mechanics of how covered interest arbitrage applies to the relevant instruments under consideration.

As discussed in Chapter 4, for equality to hold in CIP the arbitrage must be 'two-sided'. In the Euro/Canadian treasury bill case, the arbitrage is one-sided. One of the arbitrage trades cannot be executed because only the Canadian government has the ability to issue liabilities in the Canadian tbill market. The Euro rate only provides an upper boundary on covered Canadian tbill rates (omitting time dating for convenience): $r^e \geq (r^* + ((F(0,T) - F(0,N))/F(0,N))(1 + r^*))$, where, for present purposes, r^* is the Canadian tbill rate and r^e is the Euro-US dollar rate. Considering the other direction for the arbitrage, a lower bound on the covered Canadian tbill rate is provided by a 'risk arbitrage' with the US Tbill rate, i.e., when r^u is the US Tbill rate and the inequality extends naturally: $r^u \leq (r^* + ((F(0,T) - F(0,N))/F(0,N))(1 + r^*))$. The lower bound is not driven by the "pure arbitrage" activity that drives the upper boundary. Given that the Euro rate will always lie above the US Tbill rate because of the differing risk characteristics of those two instruments, this implies that the covered Canadian tbill rate will fluctuate within boundaries provided by the Euro and US Tbill rates.

Given this, the key practical questions in implementing the hedge are which US money market futures contracts to use and how to determine the appropriate hedge ratios for the interest rate futures and currency futures spread positions. Selection of the appropriate interest rate futures contract is primarily an empirical question. Two hedge ratio specification procedures are examined: dollar value and minimum variance. The dollar value approach computes the hedge ratio directly from the value of the cash market transactions used in the textbook covered interest arbitrage trade. This approach produces accurate hedging outcomes when the price behavior of the futures contracts used closely matches that of the cash instruments. The greater the basis risk, the more likely it is that hedges based on the dollar value approach will be ineffective. The dollar value approach is used here primarily as a benchmark against which outcomes of the minimum variance hedge ratios can be compared.

To evaluate the dollar value hedge ratios, analysis of the CIP conditions developed in Sec. 4.2 is required. For ease of exposition, assume that the relevant variables satisfy the conventional calculus regularity conditions. (In practice, this assumption requires that the hedge positions be continuously adjusted.) Specifically, in Sec. 4.2 it was derived that:

$$dr = dr^* + (1+r^*)\frac{d(FPS)}{F(0,N)} - (r - r^*)\frac{dF(0,N)}{F(0,N)}$$

Examining this result, it can be seen that a dollar value hedge involves combining a US interest rate futures position and an appropriately tailed currency futures spread, with the appropriate hedge ratios given as the coefficients for the *dFPS* and *dF(0,N)* terms.

Following the discussion in 6.1, the accepted method for deriving minimum variance hedge ratios is to exploit the first order conditions for hedge variance minimization.[13] For the case at hand, this requires specification of the expected profit and variance of profit for the hedge position. For a hedge done with Euros:

$$E(\pi) = Q\,[r^*(0)' - E\,(r^*(1)'')] \quad + A\,[E[FPS(1)] - FPS(0)] \;+ B\,[eu(0) - E\,(eu(1))]$$
$$var(\pi) = Q^2\,\sigma^2_{*} + A^2\,\sigma^2_{f} + B^2\,\sigma^2_{e} \;+ 2(QA\sigma_{*f} - (QB\sigma_{*e} + AB\sigma_{fe}))$$

where: Q is the value of the cash position, A is the value of the legs of the currency futures spread *(FPS)*, B is the value of the interest rate futures position and σ is either a variance or a covariance defined by the subscripts f for futures spread, e for Euro and * for Canadian tbill. The ' and " superscripts allow for possibly differing times between settlement and maturity for the cash instrument. Given the variance function $var\,[\pi]$, generalized hedge ratios for both A and B unconstrained can be derived. This can be done by solving the two first order conditions *(foc)* for the minimum variance problem.

Specifically, differentiating $var\,(\pi)$ with respect to A and B gives 2 first order conditions that can be solved to get:

$$A^* = \frac{B\,\sigma_{fe} - Q\,\sigma_{*f}}{\sigma_f^2} \qquad\qquad B^* = \frac{Q\,\sigma_{*e} + A\,\sigma_{fe}}{\sigma_e^2}$$

Solving for A:

$$A^* = \frac{Q}{1 - R^2}\{\sigma_{*e}\,\frac{\sigma_{fe}}{\sigma_e^2\,\sigma_f^2} - \frac{\sigma_{*f}}{\sigma_f^2}\} \qquad where: \qquad R^2 = \frac{\sigma_{fe}^2}{\sigma_e^2\,\sigma_f^2}$$

In words, R^2 = the squared correlation between e and f. Rearranging A^* gives:

$$A^* = Q\,[(\sigma_{*e}\,\sigma_{fe} - \sigma_{*f}\,\sigma^2_{e})/\,(\sigma^2_{e}\,\sigma^2_{f} - \sigma^2_{ef})]$$

This is a special case of the multivariate hedge ratio discussed in Sec. 6.1, the minimum variance hedge ratio for the currency spread position is equivalent to the currency spread regression coefficient in a regression of Canadian tbill rates on Euro rates and currency spreads. Solving for B gives a similar result.

In order to transform the hedge ratio problem into its more familiar bivariate regression interpretation, one of the two hedge positions can be constrained-- thereby reducing the solution to a single first order condition. This can be done by setting the exchange rate adjusted value of the Euro position to be equal to the value of the Canadian tbill position or by setting the value of the currency futures positions equal to the implied CIP values, i.e., set $B = Q/F(0,T)$ or set $A = Q/F(0,T)$. If $B = Q/F(0,T)$ then differentiating $var\,(\pi)$ with respect to the single choice variable A gives:

$$A^{**} = Q\,\{\frac{1}{F(0,T)}\,\frac{\sigma_{fe}}{\sigma_f^2} - \frac{\sigma_{*f}}{\sigma_f^2}\}$$

A similar result holds for B^* if A is constrained. In this form, the hedge ratio expression involves bivariate regression coefficients. However, the variances and covariances are based on the *conditional* distribution and, as a result, imply a different regression specification than in the unconditional case.

Econometrically, use of regression to estimate minimum variance hedge ratios raises a number of interesting points, e.g., Kroner and Sultan (1993). One basic question involves the specification of the variables in the regression: should levels or first differences be used? Toevs and Jacob (1986) attempts to cast some doubt on the conventional wisdom (e.g., Hill and Schneweiss 1982) that first difference regressions are superior to regressions done in levels. However, regressions based on levels typically have undesirable statistical properties (e.g., non-stationarity as reflected in Durbin-Watson values significantly different from two). As a result, the minimum variance hedge ratio regressions should be based on *changes* in rates (or prices) and futures spreads. Because the ultimate objective is to minimize the variation in *prices*, regression results based on rates require some manipulation before being used as hedge ratios. Specifically, the following coefficient transformation is required: $\Delta P = \Delta r/(1 + r(0) + r(1) + r(0)r(1))$. In practice, rates must be adjusted to be comparable in maturity to the maturity of the currency spread. In addition, the coefficient on the interest rate futures position must be adjusted by the ($US/$C) exchange rate to get dollar equivalency. Poitras (1988b) provides more detailed discussion and various estimation results.

QUESTIONS

1. Derive the profit profile for a spread trade with equal position sizes. What factors determine the profitability of this trade? Derive the profit profile for a tailed spread and explain how this trade is different from one with one-to-one position sizes. Does your answer depend on the commodity under consideration?

2. What factors determine the profitability of: a copper turtle trade; an oil butterfly; a NOB tandem? What trading strategy is most applicable to trading the TED spread?

3. a) Assume you are convinced that the spread between long and short term interest rates is going to ***widen*** within the next few months but you do not know whether rates in general will be higher or lower than they are at present. As reflected in market prices, other investors appear to disagree with your prediction: they expect the spread to remain constant. How could you profit from your superior predictive ability by using: tailed Tbond futures spreads; or, a turtle trade combined US Tbond spreads and Tbill futures? Does your answer depend on whether the yield curve is inverted?
b) Assume that you are convinced that the spread between the implied carry return implied in gold futures will narrow relative to the implied return implied in silver futures. How would you design a trade to profit on your predictive ability in this case?

4. From Sec. 5.2 construct a $\{1 + cy\}$ spread for the 8/8/94 copper futures prices. Do the same calculations for crude oil futures. Explain the factors that would drive profitability for the two trades.

NOTES

1 The profit function is approximate because the exact profit function requires two tails to be specified for trade. The precise method of specifying the two tails is similar to setting a hedge ratio in the stereo trades.

2 Commodities where the butterfly may be a feasible trading strategy for traders with higher transactions costs are those with a significant seasonal factor in the term structure of futures prices, such as heating oil. For example, by forming the spread using fall-winter-summer contracts, changes in the butterfly could be used to speculate on, say, the appearance of an unexpectedly cold winter, without being concerned with changes in the level of heating oil prices.

3 Significantly, the *irr* depends fundamentally on the cheapest deliverable commodity. For Tbond spreads, it is possible for there to be numerous changes in the cheapest deliverable bond over longer trading horizons. This can give rise to variations in trade profitability.

4 The reason for avoiding the delivery month is that the gold *ic* drops well below the Tbill rate as the delivery period progresses. As there is no arbitrage relationship between gold futures and Tbills, this is not a violation of absence-of-arbitrage.

5 In devising this trigger strategy, two types of rules can be considered: censored percentage rules and censored fixed rate rules. Fixed rate rules establish the trade if the gold *ic* approaches the boundaries by an ad hoc number of basis points, say 20 for both the upper and lower boundary, but restricts (censors) trades if the Eurodollar/Tbill differential is less than an ad hoc number of basis points, say 80. The main difficulty with this approach is that it does not account for variations in the differentials due to changes in the level of rates and other related factors.

6. Further discussion of the naked TED spread can be found in Siegel and Siegel, p. 262-6. It is also possible to develop complex tandem trading strategies that combine TED's with spreads in other futures.

7. This arbitrage condition ignores variation margin costs as well as the related transactions costs including bid/offer differences.

8. In doing this arbitrage, it assumed that the funds received from doing the short are used to repay the funds used to do the originate the term reverse.

9. More precisely, as the financing term increases the difference between the repo and reverse rates widens significantly to the point where the reverse rate is almost zero. In other words, while it may be possible to do term repos (and reverses) for distant months, the quoted financing rates are sufficient to deter the arbitrage.

10. For Euros, dropping the bid/offer difference is much the same as assuming R and RR were equal for Tbills, i.e., the inequality conditions reduce to equality conditions. Hence, it is not necessary to consider the `short' arbitrage condition.

11. The TED spread proper is not treated as a spread in assessing transactions costs and margin requirements, i.e., both the Euro and Tbill positions are considered "open" positions. However, the tandem TED trader does qualify for spread treatment.

12 To be exact, the hedge ratios involve variables defined at $t=1$. These are, obviously, not known at $t=0$ and, as a result, must be approximated. In the absence of information that may improve the estimate, the ratio of current values can be used. In certain cases, hedge ratio adjustment during the life of the trade may be required and this will have to be incorporated into trade design. This practical substitution of current for future values occurs in virtually all the hedge ratio evaluation situations.

13. The hedge ratio calculation and estimation problem is examined in a number of sources, e.g., Toevs and Jacob (1986), Bell and Krasker (1986), Hill and Schneeweis (1981), Gemmill (1984), Cecchetti (1987). Benninga et.al. (1984) show that if futures prices are unbiased then the minimum variance hedge ratio is also an optimal hedge ratio for hedgers with quadratic utility functions.