## SECURITY ANALYSIS AND PORTFOLIO MANAGEMENT

**Solutions to Problem Set #1:**

**1)** r = .06 or r = .18

$$PVA[T = 10,\ r = .06] = \sum_{t=1}^{10} \frac{\$8000}{(1.06)^t} = \$8000\left(\frac{1}{.06} - \frac{1}{.06\ (1.06)^{10}}\right)$$

$$= \$8000(7.36009) = \$58{,}880.70 > \$50{,}000$$

$$PVA[T = 10,\ r = .18] = \$8000(4.49409) = \$35{,}952.70 < \$50{,}000$$

**2)** A bond is a security which typically offers a combination of two forms of payments:

i) A stream of fixed **coupon** payments paid at regular intervals up to a stated maturity date, referred to as the **term to maturity**.
ii) A "return of principal" at maturity which involves a payment on the maturity date equal to the stated **par value** of the bond.

If no coupons are offered the bond is said to be a **zero coupon** or **pure discount** bond.

If the price of the bond PB > M, the Par value, then the bond is a **premium** bond. This occurs when the annual coupon payment C satisfies: (C/M) > y, the yield to maturity on the bond.

If the price of the bond PB < M, then the bond is a **discount** bond and (C/M) < y.

If PB = M, the bond sells at its par value and (C/M) = y.

Coupon payment frequency differs across bonds. Many government bonds pay coupons semiannually. In this case, half of the stated annual coupon (C/2) is paid every 6 months. Similarly, for quarterly coupon payments, (C/4) is paid every three months.

Bonds are identified and quoted according to the amount of coupons which are paid in one year. For example, a $100 par value, 10% coupon bond which pays coupons quarterly would make a payment of $2.50 each quarter.

Let C = $8, M = $100, and, the term to maturity (T) be T = 8

$$PB^{A} = \$8 \sum_{t=1}^{8} \frac{1}{(1.1)^{t}} + \frac{\$100}{(1.1)^{8}} = \$8\ (5.335) + \$100\ (.467) = \$89.38$$

$$PB^{SA} = \$4 \sum_{t=1}^{16} \frac{1}{(1.05)^{t}} + \frac{100}{(1.05)^{16}} = \$4\ (10.84) + \$45.80 = \$89.16$$

**2b)** $PB^A$ = \$12 (5.335) + \$46.70 = \$110.72
$PB^{SA}$ = \$6 (10.84) + \$45.80 = \$110.84

**2c)** C = \$8 T = 10 y = .10

$PB^A$ = \$8 (6.145) + \$38.60 = \$87.76 $PB^{SA}$ = \$4 (12.46) + \$37.70 = \$87.54

C = \$12 T = 10 y = .10

$PB^A$ = \$12 (6.145) + \$38.60 = \$112.34 $PB^{SA}$ = \$6 (12.46) + \$37.70 = \$112.46

RESULT:

For discount bonds, with the same yield but different term to maturity, $PB^{10} < PB^8$
For premium bonds, with the same yield but different term to maturity, $PB^{10} > PB^8$

--Shorter term bonds have higher prices when the bonds sell at a discount and longer term bonds sell at higher prices when the bonds sell at a premium.

For discount bonds, with the same term to maturity **and yield to maturity**: $PB^{SA} < PB^A$
For premium bonds, with the same term to maturity and **yield to maturity**: $PB^{SA} > PB^A$

This result is somewhat misleading because with the same C and T a semi-annual coupon bond will always be preferred to an annual coupon bond, **because a portion of the cash flows are received sooner**. Hence, the semi-annual bond will sell for a higher price, and lower yield to maturity. (See the Additional Questions).

**3)** Current Yield = CY = (Annual Coupon Paid)/(Bond Price) = C/PB

The current yield is of interest because it is easy to calculate and was, at prior to the widespread use of calculators, quoted by dealers and in the financial press.

When PB = M, the bond sells at par, then CY = y

When PB > M, for premium bonds CY > y with the difference increasing as the bond has a greater premium.

When PB < M, for discount bonds CY < y with the difference increasing as the bond has a greater discount.  For a zero coupon bond, CY = 0.

In general:

$$CY = \frac{C}{\sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{M}{(1+y)^T}} \quad \Rightarrow \quad CY\{\sum_{t=0}^{T-1} (1+y)^t\} + \frac{M}{PB} = (1+y)^T$$

Some special cases:

For T = 1

$$CY + \frac{M}{PB} = (1+y) \quad \Rightarrow \quad y = CY + \{\frac{M}{PB} - 1\}$$

For T = 2:

$$CY(1+y) + \frac{M}{PB} = (1+y)^2 \quad \Rightarrow \quad y = CY + \{\frac{M}{PB(1+y)} - 1\}$$

As T increases, the size of the deviation shrinks.

For a perpetuity:

$$CY = \frac{C}{\frac{C}{y}} = y$$

When T goes to infinity, the current yield equals the yield to maturity.

**4)** Riding the yield curve is an investment strategy which involves purchasing bonds which have a longer term to maturity than the investment horizon.  In this case, instead of the bond maturing at the end of the investment horizon, the bond will be sold in the secondary market.

This strategy can be profitable due to the typically upward sloping shape of the yield curve.  This generates two **potential** sources of profit.  Firstly, the yield to maturity of the longer term security is higher, but the return is uncertain because of the need to sell the bond at the end of the investment horizon.  Secondly, when the bond is sold it will have a shorter maturity than when it is purchased. Hence, the increase in yields required to suffer a sufficiently large capital loss for riding the yield curve not to breakeven is more than indicated by the yield to maturity on the longer term bond.

Example: $z_1 = .05$ $z_2 = .06$ $z_3 = .07$ where **z** is the yield to maturity on a zero coupon bond and the investor has funds to invest for 2 years. Let M = $100

Strategy 1 involves buying a two year zero and earning 6% with certainty over the two year period. Observing $PZ_2$ = $89, if the investor buys this bond then at the end of the investment horizon, the investor will have $100.

Strategy 2 involves buying a three year zero, earning a promised 7% over the two year period, but having to sell the three year zero in two years, at which time it will be a one year zero. Because $PZ_3$ = $81.63, if the $89 is invested in this bond then $109.03 in par value can be purchased. **If yields are unchanged in two years time**, the zero can be sold at the present one year zero yield of 5% and $PZ_1$ = $95.24. Multiplying by the par value means that the investor will have $103.84, a significantly larger amount of money than investing in the two year.

What is the breakeven rate on the one year zero to be sold in two years time? 9.028%

## Section II: More on Time Value of Money

**1) Amortization** period: period over which the loan will be liquidated
**Term** of the loan: period over which the conditions of the loan, such as the applicable interest rate, are fixed.

For simplicity, assume an annual payment frequency. If the amount of the loan is PVL, and the stated (APR) interest rate on the loan is r, then:

$$PVL = \$A \sum_{t=1}^{T} \frac{1}{(1+r)^t} = \$A \left\{ \frac{1}{r} - \frac{1}{r\,(1+r)^T} \right\}$$

where $A is the annual loan payment and T is the amortization period. For any given amortization period and PVL, r can be changed, which will change the size of the loan payment.

To avoid the problem of interest variation at the end of the term, either assume that rates in 5 years will be equal to current rates, or assume that the maximum term currently available on mortgages is 5 years. In both cases the borrower wants the longest term possible.

There are two sources of saving on the concessionary financing: the difference in the loan payments over five years; and, the difference in the mortgage paydown.

Difference in mortgage payments:

$A_1$ = payment on 8% mortgage = \$200,000/9.81815 = \$20,370.4
$A_2$ = payment on 10% mortgage = \$200,000/8.514 = \$23,490.7

The present value of the loan payment savings, evaluated at the current market rate of 10%:

$$\sum_{t=1}^{5} \frac{A_2 - A_1}{(1+r)^t} = \sum_{t=1}^{5} \frac{(23490.7 - 20370.4)}{(1.1)^t} = \$11{,}829.1$$

Notice that this savings is only available for 5 years. At the end of five years, the loan must be refinanced at market rates.

To calculate the difference in the paydown on the mortgages, determine the amount owing at T=15 for both loans:

$$PVL_{15} = \sum_{t=1}^{15} \frac{\$20{,}370.4}{(1.08)^t} = \$174{,}350 \qquad PVL_{15} = \$23490.7 \sum_{t=1}^{15} \frac{\$23490.7}{(1.1)^t} = \$178{,}670$$

This difference occurs as a future value. The present value of the difference is:

PVD = (178,670 - 174,350)/{$(1.1)^5$} = \$2682.38

The total savings is the sum of these two values: \$11829.1 + \$2682.38 = \$14,511.50

**ADDITIONAL QUESTIONS:**

A1) A 10% coupon government bond (paid semi-annually) currently sells at par. At current market prices, what coupon rate would be required if the coupons were paid annually?

STEP 1: Equating future values for single payments

The future value of one payment received in T years, compounded m times per year at interest rate r produces:

$$FV^m = (1 + (r/m))^{Tm}$$

For annual compounding m = 1 at interest rate y this reduces to $FV^A = (1 + y)^T$

Equating the future values $FV^m = FV^A$ produces:

$(1 + y)^T = (1 + (r/m))^{Tm}$ ---> $y = (1 + (r/m))^m - 1$ ----> $r = (1 + y)^{1/m} - 1$

The interest rate which provides the annual equivalent for a stated interest rate which involves compounding at a greater than annual frequency is known as the **effective yield**. In the US, the effective yield is often referred to as the **Annual Percentage Rate** or **APR**.

STEP 2: The relationship between Future Value and Present Value

Present and future value represent the valuation of cash flows at different points in time. To translate a future value at time T to a present value involves discounting the value at the appropriate interest rate:

$PV^A = FV/(1 + y)^T$ $PV^{SA} = FV / (1 + (r/m))^{mT}$

When the effective yield is used, then equating the future values is the same as equating the present values.

STEP 3: Extending to Bonds

Work by example, consider a 1 year bond with an 10% coupon. **In all cases, it is assumed that the cash flows from the bond are reinvested at the stated yield to maturity.** A semi-annual bond has cash flows of C/2 = \$5 = (r/2)M which produces a time line of:

| | | |
|---|---|---|
| | \$5 | \$105 |
| t=0 | t=6 months | t=1 year |

The future value of this stream of cash flows is:

$$FV^S = (\frac{r}{2}M)(1 + \frac{r}{2}) + M(1 + \frac{r}{2}) = M(1 + 2\frac{r}{2} + \frac{r}{2}) = M(1 + \frac{r}{2})^2$$

This can be compared with the future value for annual payments of $FV^A = M(1 + y)$

It follows that for one year bonds, the effective yield formula holds:

$$y = (1 + (r/2))^2 - 1$$

For the annual coupon bond to be sold at par, the coupon on the bond would have to be 10.25%.

For a two year bond, the analysis is much the same. The time line for the semi-annual coupon bond is:

| | | | | |
|---|---|---|---|---|
| | \$5 | \$5 | \$5 | \$105 |
| t=0 | t=6 months | t=1 year | t=1.5 years | t=2 years |

To calculate the future value of the semi-annual cash flows:

$$FV^S = \frac{r}{2}M(1 + \frac{r}{2})^3 + \frac{r}{2}M(1 + \frac{r}{2})^2 + \frac{r}{2}M(1 + \frac{r}{2}) + M(1 + \frac{r}{2}) = M(1 + \frac{r}{2})^4$$

This can be compared with the future value of the annual coupons: $FV^A = M(1 + y)^2$

It follows that for two year bonds to have equal prices (from STEP 2):

$$(1 + y)^2 = (1 + (r/2))^4 \;\text{--->}\; y = (1 + (r/2))^2 - 1$$

The effective yield result also holds in this case, and the annual coupon bond would have a coupon of 10.25% when the semi-annual coupon bond has a coupon of 10%, for both bonds to sell at par.