Lecture 3
Institutional and Theoretical Basics

- Definitions and Basic Profit Functions
- Types of Risk and Risk Calculation
- Value at Risk and Other Measures

Risk Management Decisions

- Traditional approach is to model specific situations → transactions hedging where the 'farmer' is deciding to lock in a harvest price at planting time
  - Determination of farmer risk measure can be specified by examining the hedger profit function
- Complex financial institutions require a different methodology to measure risk, e.g., Value at Risk

Risk Management Activities

- Hedging
  - Options, futures, forwards, swaps, other products
  - Natural hedges
  - FX strategies
- Insurance
  - Direct products and Exchange Traded Products
  - Self-insurance and Avoidance
- Diversification
  - Financial vs. Commodity Risk Management

Jump to first page

Basic Variable Definitions

In order to specify the profit function for the traditional farmer's hedging problem, some notation is required:

- *F(t,T)*: the futures or forward price observed at time *t* for delivery at time *T*

  Recognize that the dating convention for derivative securities requires at least two dates, the date the price is observed and the delivery or expiration date

  Usually, *t* will be either 0 (current date) or 1 and *T* will be either *T* for deferred delivery or *N* for nearby delivery.

More Definitions

- *S(t)*: the commodity spot price. The words "cash price" or "physical price" are used interchangeably with "spot price".
- *r(t,T):* when an interest rate has two time subscripts this refers to the **actual** interest rate paid over the time period, not the annualized rate *(r)*, e.g., for $t=0$ and $T = 3$ *months*, $r(0,3$ *months*$) = r / 4$.

Reading: RSD, p.6-7 and n.2, p.216.

Jump to first page

Basis Definitions

- **The Basis**: $F(t,T) - S(t)$
- **The Futures Basis**: $F(t,T) - F(t,N)$

Notice that as t changes, then the carrying charges embedded in the basis will decline due to the reduction in $(T-t)$ while the carrying charges in the futures basis will not decline because $(T-N)$ does not change

More on the Basis

- **Definitions Applicable to the Futures Basis and Basis**
  - Contango: $F(t,T) - F(t,N) > 0 < F(t,T) - S(t)$

Example: Gold

  - Backwardation: $F(t,T) - F(t,N) < 0 > F(t,T) - S(t)$

Example: Tbonds

Do not confuse this with the Normal Backwardation Hypothesis for the **Yield** curve

More Basis Definitions

- **The Future Basis**: *F(0,T) - S(T)*

Unlike the other types of basis, there is no agreed upon terminology for this basis.

This basis has to do with the accuracy of the forward/futures price as a predictor of the future spot price. (Reading RSD, p.165-171)

Margins

- Review the differences between futures and forward contracts (BUS 316 material)

  Reading: RSD, p.11-14, esp. Table 1.1

- Margins are ‘good faith deposits’ (performance bonds) that are required for futures contracts and may be part of a forward contract

- **Initial margin** vs. **Maintenance Margin**

Jump to first page

Profit Functions

- In this course, profit functions are algebraic representations of trading outcomes (trade payoff functions).

  Types of profit functions correspond to types of trades:

  --Speculative profit functions (no direct position in the commodity)

  --Hedger profit functions (contain current or future spot positions) → hedger profit function can be considerably more complex than the simple examples to be examined

  -- Arbitrage profit functions (associated with arbitrages)

Trading Schematic of Elevator Hedge

Figure 2.3 **Profit Function for a Grain Elevator Hedge using Futures Contracts**

DATE	Cash Position	Futures Position
$t=0$	Buy Q_A units of grain at $S(0)$ for storage in grain elevator	Short Q_H units at $F(0,T)$
$t=1$	Q_A units are sold at S(1) and loaded for shipment	Close out position with Long Q_H units at $F(1,T)$

If costs associated with carrying the commodity are ignored, the profit function for this type of hedge can be specified:

$$\eth(1,T) = \{S(1) - S(0)\}\, Q_A + \{F(0,T) - F(1,T)\}\, Q_H$$

Jump to first page

Variability of Hedger Profit (See Appendix 2)

Figure 2.4 **Stylized Short (Long) Hedge Profit Function**

DATE	***Cash Position***	***Futures Position***
t=0	Buy (Sell) Q_S at $S(0)$	Short (Long) Q_F at $F(0,T)$
$t=1$	Sell (Repurchase) Q_S at $S(1)$	Close out with Long (Short) at $F(1,T)$

This leads to the associated profit function for the short (long) hedger:

$$\pi(1) = Q_S \{S(1) - S(0)\} + Q_H \{F(0,T) - F(1,T)\}$$

$$(= Q_S \{S(0) - S(1)\} + Q_H \{F(1,T) - F(0,T)\})$$

The profit function can now be used to derive $var[\pi]$, that is the same for both the long and short profit functions:

$$var[\pi] = Q_S^2\, \sigma_S^2 + Q_H^2\, \sigma_f^2 - 2\, Q_S Q_H\, \sigma_{Sf}$$

where $\sigma_{Sf} = cov[S(1), F(1,T)]$, the conditional covariance of spot and futures prices with σ_f^2 and σ_S^2 being

Types of Hedges

- See Types of Hedges in chapter 3 .zip file
  - Stylized traditional hedge has various aspects
- Extensions to Financial Risk Management
  - Risk in a portfolio context
  - Risk and risk properties as mathematical variables, e.g., the delta of the option book
  - Applications of the Greeks

Jump to first page

Types of Risk

Market Risk, Credit Risk, Liquidity Risk, Operational Risk, Legal Risk

→ How does each of these arise in the farmer's hedging problem?

Risk as a distributional parameter, e.g., standard deviation of return

Risk as the possibility of loss without regard to profit → the insurance approach → only considers one tail of the distribution

Jump to first page

Measuring Market Risk

- Market risk is the risk of losses due to movements in financial prices
  - → market risk is determined using market prices, the standard deviation associated with the hedger profit function is a simplified market risk measure
- Common approaches to Measuring Market Risk
  - Sensitivity Analysis
  - Stress Testing
  - Scenario Testing
  - CAPM
  - Value at Risk

Sensitivity Analysis

- Sensitivity analysis measures the expected change in the value of a portfolio when there is a small change in a market risk factor
  - For bonds, the appropriate measure is $ duration
  - For equities, the beta of the stocks is needed
  - For options, the Greeks are used
- Sensitivity analysis is useful when changes in market risk factors are expected to be small
  - Not well suited to consider large changes in risk factors

Jump to first page

Example of Second Order Approximation

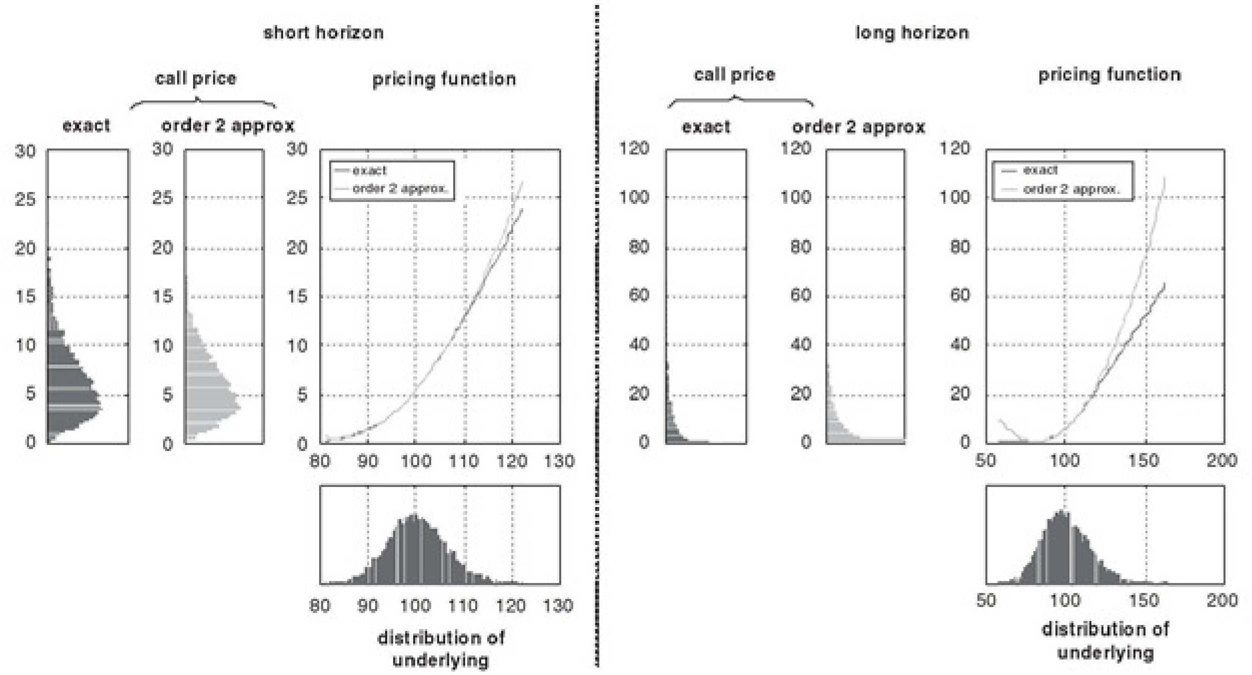


Figure 1: Goodness of delta–gamma approximation at different horizons.

Stress Testing: The Basic Method

Applicable to large (e.g., crisis) changes in market risk factors

1. Determine relevant market risk factors
2. Decide which factors to block together or move independently,
3. e.g., Latin American bank blocking together movements in Asian currencies which retained the US$ as a separate factor
4. Based on empirical data identify, say, what a six standard deviation move in each factor group would be
5. Apply these movements and revalue the portfolio/position/balance sheet
6. Determine the range of possible changes in value being sure to use accurate , possibly non-linear valuation models, e.g., use Black-Scholes rather than the delta-gamma approximation to price options.

Scenario Analysis

- Similar to stress testing
  - Instead of uniformly selecting six sigma movements in market factors (stress testing), scenario analysis uses tailored and subjectively chosen changes
    - Informed opinion is used to create a small number of ‘worst case’ scenarios → sources of opinion could include head traders, bank economists, risk management group

- Stress tests tend generate too many results with no method of determining the most likely result while scenario analysis generates too few and depends on the abilities of the ‘informed opinion’

A Stylized Example of Scenario Analysis

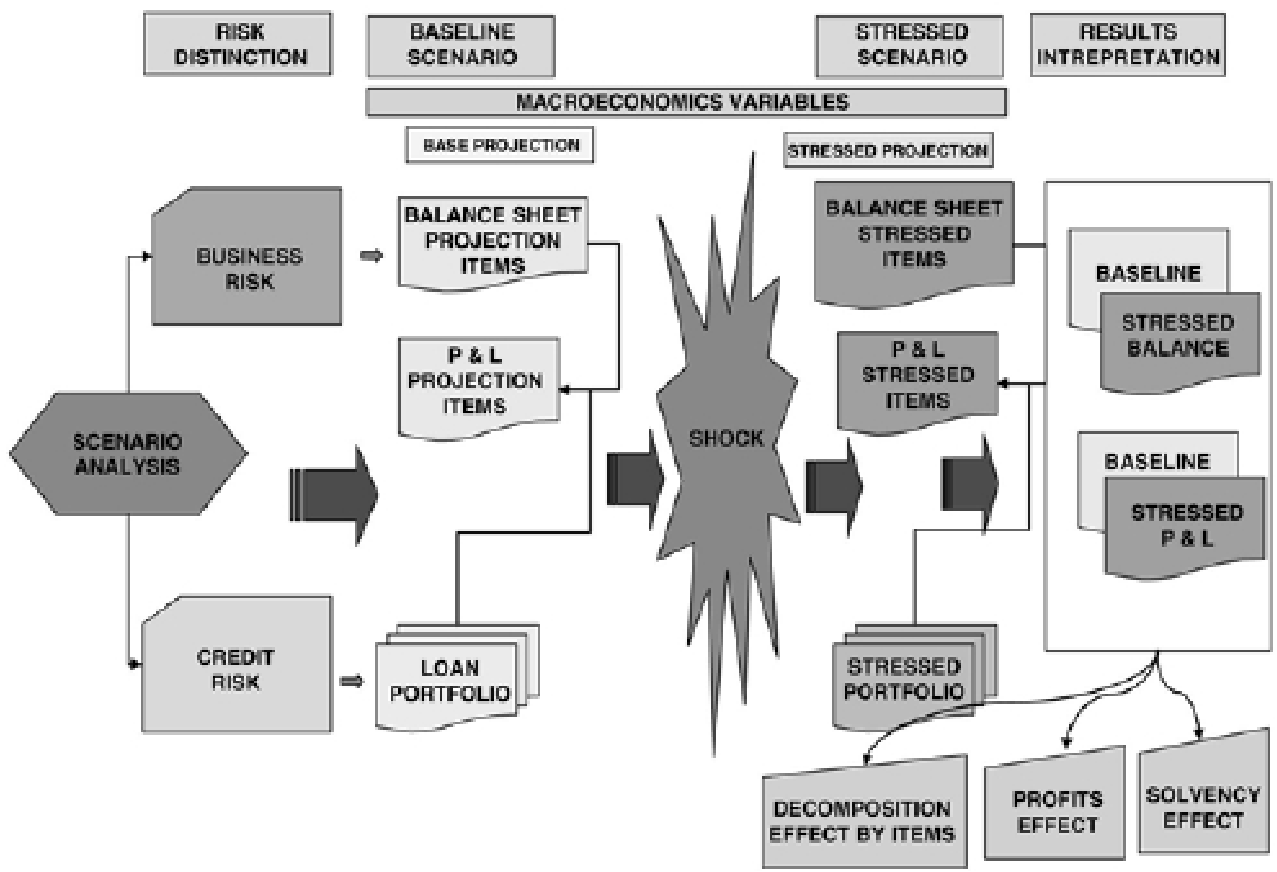


Figure 2 Scenario analysis diagram

Capital Asset Pricing Model (CAPM)

- Basic approach of financial risk management is to treat the decision problem in a portfolio context → CAPM is well suited to measuring portfolio performance
  - Sharpe ratio → $\{E[R_p] - R_f\} / \sigma_p$
  - Treynor ratio → $\{E[R_p] - R_f\} / \beta_p$
- This method of risk measurement does not yield a dollar value → better suited to performance measurement rather than managing risk of loss

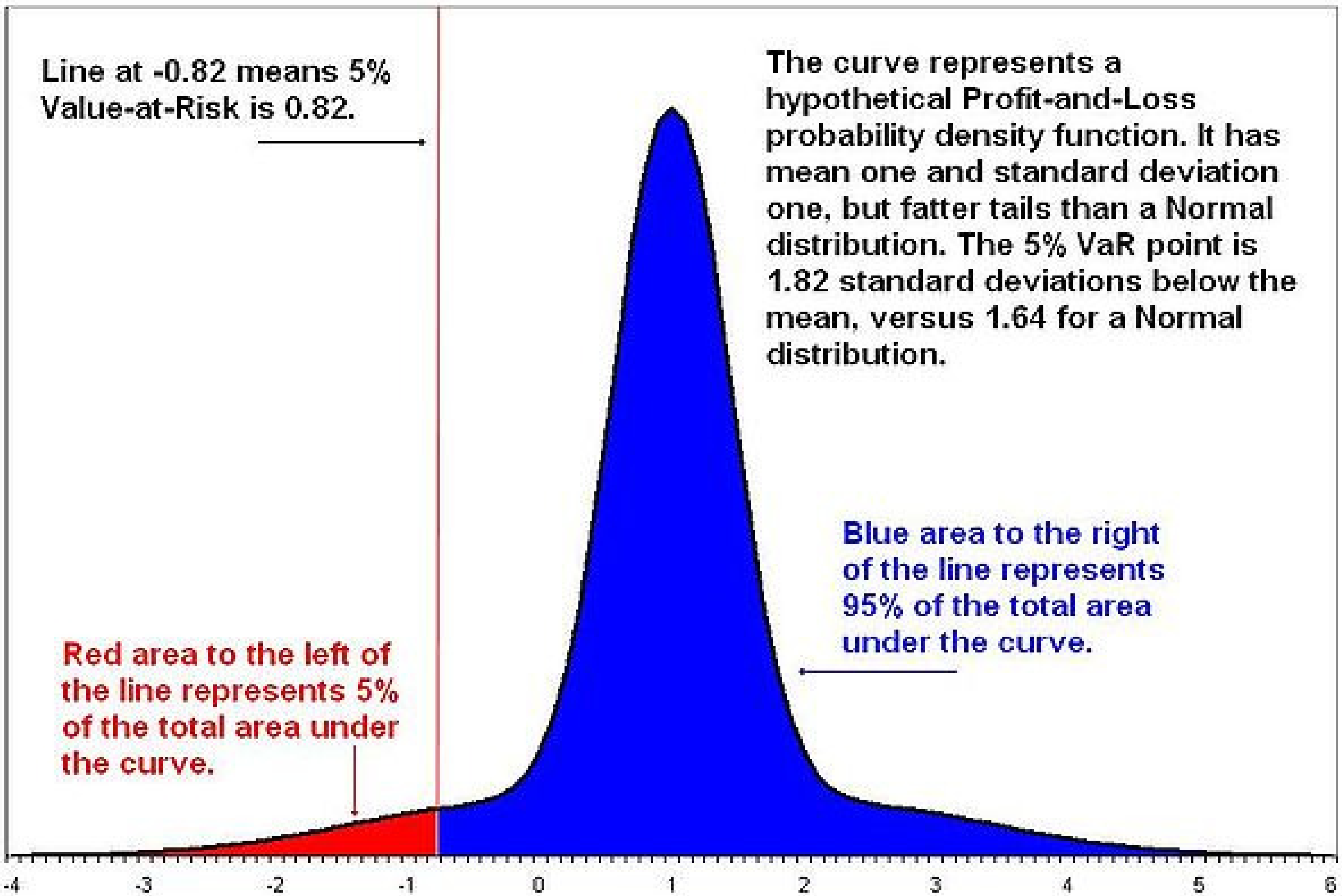
Value at Risk (VaR)

- VaR is the value of expected loss during adverse or severe fluctuations in market prices

→ Alternative definition: VaR is the maximum loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger

  - VaR is typically measured for daily intervals
  - Typical confidence intervals for VaR are 5% (5 observations out of 100) and 1% (1 observation out of 100)

VaR is the value at the cutoff point



Jump to first page

VaR Mathematics:

Φ[x] is usually the normal density (!)

$$confidence\ level = c = \int_{-VaR}^{\infty} \Phi[x]\ dx$$

$$probability\ level = 1 - c = \int_{-\infty}^{-VaR} \Phi[x]\ dx$$

Basel Rules and VaR

- VaR recommended by the Basel Committee as the standard for determining the minimum amount of capital to allocate for market risks
  - Use a horizon of 10 trading days (2 weeks)
    - Possible to create the 10-day VaR using an aggregation of daily Var's → $VaR(10,99\%) = \sqrt{10}\,(Var(1,99\%))$
    - Why $\sqrt{T}$?
  - Use a 99% confidence interval
  - Use at least one year of historical data with quarterly updating

VaR Confusions

- VaR does not describe the worst loss
- VaR does not adequately describe the losses in the left tail (there could be a huge loss far out in the tail)
- VaR depends on the sample used for estimation → changing the sampling period will change the VaR (how much?)

Example calculating weekly VaR for a bond

- For a single long bond or bond portfolio with a parallel yield curve shift, VaR can be calculated as:

  VaR = (Bond Price)(Modified Duration)(Worst yield increase at desired confidence level)

  E.g., Price = 103 Duration = 8.6

  Worst yield increase over a one week period estimated as 65 basis points

  → (103)(8.6)(.0065) = $5.76

Jump to first page

Problems with VaR

- Basic properties required of risk measures
  - Monotonicity
    - If the value of a portfolio is lowered, the risk measure will also be lowered
  - Translation invariance
    - Adding $\$k$ cash to a portfolio will only shift the Var by $\$k$
  - Homogeneity
    - Increasing portfolio size by a constant scaling will increase the risk measure by the same constant scaling
  - Subadditivity
    - The risk measure of the portfolio will be less or equal to the sum of the risk measures → VaR can violate sub-additivity

Alternative Risk Measures

- VaR only identifies a single point on the distribution → examine the whole distribution or calculate a measure for the whole distribution (esp., standard deviation)
- Semi-standard deviation → calculate the standard deviation only for data points which represent a loss
- Conditional VaR, also referred to a *expected shortfall, conditional loss, expected tail loss* → the expected value of loss for observations exceeding VaR

Calculation of conditional VaR → this measure does satisfy the subadditivity requirement

Let q be the -VaR value and $\Phi[x]$ the relevant distribution of returns (or profits). Then the conditional VaR can be calculated as the **negative** of:

$$E[\ X \mid X < q\ ] = \frac{\int_{-\infty}^{q} x\ \Phi[x]\ dx}{\int_{-\infty}^{q} \Phi[x]\ dx}$$

Observe that the denominator is the probability that the loss will exceed the VaR which adjusts the measure to satisfy the requirement that the probability sum to one. The numerator is the average of the losses that exceed the VaR.