

Bond Trader Example

To provide an illustration of the impact of convexity, consider the gains associated with the following self-financing portfolio:

- Purchase one pure discount (zero coupon) bond with a 20 year maturity, with par value of \$1 million.
- Sell short 1.92 pure discount bonds with a 5 year maturity and par value of \$1 million per unit. The full value of the short sale proceeds is immediately available.
- Invest \$1,127,490 in bonds with zero duration (cash has $D = 0$).

Without loss of generality, assume that the yield to maturity on all bonds is 5% initially. What is the initial investment in the portfolio? From an inspection of the balance sheet, it is apparent that no investment is required in the portfolio:

Assets:		Liabilities Plus Net Worth:	
20 Year bond	$\$376,890 = 1 \text{ million}/(1.05)^{20}$	Short sale	$\$1,504,370 = 1.92 \text{ million}/(1.05)^5$
Cash	<u>$\\$1,127,480$</u>	Net Worth	$= \$0$ (Self Financing)
Total:	$\$1,504,370$		

Computing the Macaulay duration of the assets and of the liability reveals that $D_A = (376,890/1,504,370)(20) + 0 = 5 = D_L$. Treating the liabilities as one portfolio and the assets as another, the position conforms to the requirement of equal duration and yield. Calculation of the convexity reveals that the duration of the assets is (21/6) times larger than the convexity of the liabilities. The portfolio is immunized according to the classical Redington conditions.

Consider what happens if the yield to maturity for all assets and liabilities immediately rises to 6%. In this case, the net value of the portfolio (value of assets less value of liabilities) changes to:

Assets:		Liabilities Plus Net Worth:	
20 Year bond	$\$311,800 = 1 \text{ million}/(1.06)^{20}$	Short sale	$\$1,434,740 = 1.92 \text{ million}/(1.06)^5$
Cash	<u>$\\$1,127,480$</u>	Net Worth	<u>$4,540$</u>
Total:	$\$1,439,280$		$\$1,439,280$

Now consider what happens if the yield to maturity of all assets and liabilities immediately falls to 4%:

Assets:		Liabilities plus Net Worth:	
20 Year bond	$\$456,390 = 1 \text{ million}/(1.04)^{20}$	Short sale	$\$1,573,100 = 1.92 \text{ million}/(1.04)^5$
Cash	<u>$\\$1,127,480$</u>	Net worth	<u>$10,770$</u>
Total:	$\$1,583,870$		$\$1,583,870$

This example illustrates that basic point: starting from a position of equal duration and yield for the assets and liabilities of the portfolio, whether yields increased or decreased the net worth of the portfolio increased.