Lecture 4
Risk Management and Hedging

- Risk Minimization or Speculation?
- Optimal Hedging
- Transactions Hedging
- Managing Interest Rate Risk

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Risk Minimization or Speculation?

- Risk management decisions involve determining how much to hedge, insure and diversify
  - “The viability of a given technique will depend on the risk management philosophy adopted by the firm and the empirical characteristics of the firm’s risk profile” (p.293).
  - The optimal solution to the risk management decision problem involves a combination of risk minimization and speculative components
- Developing a risk management philosophy
  - Theoretical guidance about appropriate risk attitude is provided by expected utility theory

Properties of Expected Utility Functions

- Expected Utility (EU) approach extends the certainty-based utility theory of neo-classical economics to the case of 'uncertain' outcomes
  - Initial contribution by J. von Neumann and O. Morgenstern, Theory of Games and Economic Behavior (1947)

- Expected utility theory can be used to provide theoretical specifications of risk attitudes, i.e., what is risk aversion? What is risk neutrality? (See handout)

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Cost of Risk

- The cost of risk is the difference between the expected value of a risky prospect and its certainty-equivalent income → See handout on determining cost of risk

- Absolute and relative risk aversion measures can be used to identify the properties of commonly used expected utility functions
  - Important functions include:

    quadratic utility $W - bW^2$ Log utility $ln\ W$

    power utility W^{α} / α

Optimal Hedging

- Transaction Hedging vs. Optimal Hedging

  Together with insurance and diversification, hedging is an essential feature of corporate risk management.

  There are a range of possible approaches that can be used – the selection of a particular approach depends on the firm's risk management philosophy.

Reading: RSD, p.293-6; Box 1, p.297-8.

Midterm Exam Question

- Assuming mean-variance agents, derive an expression for the optimal speculative position size. What happens to this position as the sensitivity of the agent to risk diminishes? Based on this, what can you conclude about the equilibrium in a market dominated by risk-neutral speculators?

- A solution to this question appears as a component of the solution to the **stylized** decision problem involving a hedge of a cash position (the **optimal hedge** solution).

The Optimal Speculative Position

- Reading: RSD, p.113

- What is the objective function to optimize?

$$EU[\pi] = E[\pi] - b \, var[\pi]$$

The mean-variance expected utility function where *b (> 0)* measures the sensitivity of expected utility to changes in risk.

The Optimal Speculative Position (cont'd)

- Recall that the profit function for a speculator is:

$$\pi(1) = Q\,\{F(1,T) - F(0,T)\}$$

Solving the optimization problem gives the solution:

$$\max_{Q} \; Q\,\{E[F(1,T)] - F(0,T)\} - b\,Q^2\,\sigma_f^2$$

$$\frac{\partial EU}{\partial Q} = \{E[F(1,T)] - F(0,T)\} - 2b\,Q\,\sigma_f^2 = 0$$

$$\Rightarrow \quad Q^* = \frac{E[F(1,T)] - F(0,T)}{2b\,\sigma_f^2}$$

The Risk Minimizing Solution

- Risk minimizing hedgers do not take expected return on the hedge into account when arriving at an optimal solution.

- The objective function for a risk minimizer can be specified as:

$$EU[\pi] = - var[\pi]$$

Reading: RSD, p.114-5.

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The Hedger Profit Function

- For a hedger long-the-spot and short-the-forward, the profit function is:

$$\pi(1) = Q_S \{S(1) - S(0)\} + Q_H \{F(0,T) - F(1,T)\}$$

It follows that the variance of the profit function is:

$$var[\pi] = Q_S^2\, \sigma_S^2 + Q_H^2\, \sigma_f^2 - 2\, Q_S Q_H\, \sigma_{Sf}$$

Reading, RSD, p.535-7.

The Risk Minimizing Solution

- Important Result: the risk minimizing solution can be estimated empirically using a regression of the (change in) spot price on the (change in) futures price.

Note: some presentation use a minus sign.

$$\frac{\partial EU}{\partial Q_H} = 2Q_H\,\sigma_f^2 - 2Q_S\,\sigma_{Sf} = 0 \quad \rightarrow \quad \left(\frac{Q_H}{Q_S}\right)^* = \frac{\sigma_{Sf}}{\sigma_f^2} = \frac{\sigma_S}{\sigma_f}\,\rho_{Sf}$$

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Optimal Hedging Solution

- The optimal hedge ratio associated with max $EU = E[\pi] - b\ var[\pi]$ using the short hedge expected profit, $E[\pi] = Q_S\ \{E[S(1)] - S(0)\} + Q_H\ \{F(0,T) - E[F(1,T)]\}$, has the solution:

$$\max_{Q_H} \quad EU[\pi] = E[\pi] - b\ var[\pi]$$

$$\frac{\partial EU}{\partial Q_H} = (F(0,T) - E[F(1,T)]) - b\ (2 Q_H\ \sigma_f^2 - 2 Q_S\ \sigma_{Sf}) = 0$$

$$\rightarrow \quad \frac{Q_H^*}{Q_S} = \frac{\sigma_{Sf}}{\sigma_f^2} + \frac{F(0,T) - E[F(1,T)]}{2\ b\ Q_S\ \sigma_f^2}$$

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The Optimal Hedge Ratio

- Examining the solution reveals that the optimal hedge ratio for the stylized hedger problem is a combination of the optimal speculative solution and the risk minimizing solution.

- Exercise: do sensitivity analysis on the solution as the parameters in the formula are allowed to vary.

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Hedging Techniques

- **Transactions Hedging**

The *transactions hedging* approach emphasizes the trading mechanics involved in *fully hedging* a specific transaction. A cash position is identified and the appropriate forward position is described and determined (e.g., see 03-3 midterm and sample midterm, Q#4b.i).

It is conventional to have spot and derivative positions that have little or no basis risk though this does not have to be the case.

Reading: RSD, p.299-309.

Perfect Transactions Hedge for a Commercial Paper Issuer

- *Figure 6.2:* ***Perfect Hedge for a Commercial Paper Issuer (circa 1981)***

DATE	Cash Position	Basis	Futures Position
t=4/13	Decide to issue $20 million in 3 month commercial paper on June 3 and want to lock in current rate of 15.25%	1.93%	Short $20 June (3 month) Tbill contracts at 13.32%
t=6/3	Issue is offered and sold at 17.50%	1.93%	Close out position with Long contract at 15.57%

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Actual Hedge Profit for a Commercial Paper Issuer

Figure 6.4 ***Actual Hedge Profit for a Commercial Paper Issuer***

DATE	***Cash Position***	***Basis***	***Futures Position***
t=4/13	Decide to issue $20 million in commercial paper on June 3 and want to lock in current rate of 15.25%	1.93%	Short $20 June Tbill contracts at 13.32%
t=6/3	Issue is offered and sold at 17.50%	2.12%	Close out position with Long contract at 15.38%
Change:	+2.25	-.19	-2.06

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Sources of Basis Variation

- Basis variation in a transactions hedge can originate from three sources:
  - ***Cross-hedging, where the cash instrument being hedged is an imperfect match to the deliverable commodity***
  - ***Dollar equivalency, where the implied market value of the futures position differs from that of the cash commodity***
  - ***Cost of carry considerations, which include the 'time decay' (to $F(T,T) = S(T)$) built into the cost of carry relationship which determines the futures price.***

Strip Hedge versus Stack Hedge

- Example considers a hedge of a fixed rate loan that will be paid in quarterly installments (see also 03-3 midterm, Q#4b.ii).

- Another example, a refinery is hedging future purchases of 120,000 bbls. of crude oil that will be made continuously over the next year.

  Possible to **stack hedge** this by starting with 120,000 bbls. in the nearby – rolling forward into the next nearby as contracts mature and sequentially reducing the size of the hedge as purchases are made

Strip vs. Stack Hedge (cont'd)

- Also possible to *strip hedge* by placing 10,000 bbl. (x 12) hedge positions in each of the future months.

Actual Example: MGRM, RSD p.58-60.

In theory, the strip hedge will be superior to the stack hedge. However, in practice, insufficient liquidity in deferred contract months might make it difficult to execute a strip hedge.

Managing Interest Rate Risk

- Fixed Income Portfolio Management
  - Numerous applications including: depository institutions, such as chartered banks; pension funds; insurance companies; and corporate treasurers managing a debt portfolio.
- Possible strategies for fixed income portfolio management:
  - ***dedication*** or ***cash flow matching***, where the cash flow from the fixed income assets is structured to match the requirements of a portfolio of predetermined liabilities;
  - ***immunization***, where the interest rate sensitivity of the cash flows from the fixed income assets and liabilities is matched;
  - ***horizon matching,*** which combines dedication and immunization methods by dividing the assets and liabilities being managed into two parts, one part being managed with dedication and the other with immunization.

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Classical Immunization Rules

- Redington (1952) posed the problem: What allocation of assets and liabilities would minimize a life insurance company's possibility of losses from unexpected changes in market rates of interest?

- Using a Taylor series expansion, ***Redington's rules***, known as the ***classical immunization conditions***, are derived as**:**
  - ***Duration matching***: Duration of cash inflows equals the duration of the outflows.
  - ***Higher Convexity of Assets***: When there is more than one planning period for the fund to satisfy, the value of the cash inflows should be more "dispersed" around the duration than the value of the cash outlfows.

The Investment Horizon

- A key parameter governing the immunization problem is the date at which an obligation is to be discharged, i.e., the ***planning*** or ***investment horizon.***
  - Funds that have ***multiple*** planning periods are difficult to administer using techniques that treat each individual liability separately by creating dedicated portfolios for each liability, i.e., using ***cash-flow matching***.
- The immunization approach handles the different funds associated with the different possible planning periods by managing these funds as a single investment fund with a single investment horizon.

Using Taylor Series Expansions

- Simplifications required to use the basic formulas to determine the bond price/yield:
  - Bond is valued on the issue date or coupon payment date
    - No need to take account of accrued interest.
    - It is possible to specify more complicated, exact formulas for price between payment dates.
  - The bond has no embedded options.
  - ‘Straight bonds’ with a bullet maturity are used.

Taylor Series Expansions

- The basic idea is a specific solution to the more general problem of approximation of functions, a topic that has occupied mathematicians for centuries.

- ***Taylor series expansion:*** for a function of one variable, *f[x],* the expansion takes the general form:

$$f[x] = f[a] + \frac{df[a]}{dx}(x-a) + \frac{1}{2!}\frac{d^2f[a]}{dx^2}(x-a)^2 + \frac{1}{3!}\frac{d^3f[a]}{dx^3}(x-a)^3 + \ldots\ldots$$

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Evaluating the Taylor Series

- The Taylor series expands the univariate function *f[x]* about the point fixed point *a*, where $b \leq a \leq c$. Each of the derivatives in the expansion are evaluated by setting $x = a$. For this expansion to be valid, the function *f[x]* must have derivatives of all orders over [*b,c*] (some of which can be zero).

- Term by term inspection of the Taylor reveals how the function *f[x]* is approximated.
  - The first term *f[a]* is a point, the value of the function evaluated at the point $x=a$.
  - The sum of the first term and the second term is the linear approximation to the function about the point *a*.

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More on Taylor Series

- The quadratic approximation is achieved by adding the squared term in the Taylor series to the linear approximation. (Will this ***necessarily*** improve the approximation?)
- See Figures 5.1 and 5.2 in SAIS
- Application to fixed income valuation: For default and option free bonds, the price function is convex in the yield to maturity, a condition that easily satisfies the conditions required for a Taylor series to be used.

Application to Fixed Income Valuation

- It is possible to define the bond price function in terms of yield and time, i.e., *P[y,t]*.
- The textbook explanation for the relationship between duration and convexity is to treat the bond price as a univariate function of yield, *P[y]*. Applying a Taylor series expansion to this function at y_0 gives:

$$P_B[y] = P_B[y_0] + \frac{dP}{dy}(y - y_0) + \frac{1}{2}\frac{d^2P}{dy^2}(y - y_0)^2 + \dots H.O.T.$$

$$\frac{P_B[y] - P_B[y_0]}{P_B[y_0]} \approx -DUR\,(y - y_0) + \frac{1}{2}\,CON\,(y - y_0)^2$$

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The Role of Convexity

- Consider two bond portfolios (*A* and *B*), with values P_A and P_B. These portfolios are constructed to have equal duration ($D_A = DUR_A = DUR_B = D_B$), equal initial yield to maturity (y_0) and $CON_A > CON_B$.
- The impact of an instantaneous interest rate change on these two portfolios would be approximately:

$$\% \Delta P_A - \% \Delta P_B \cong \frac{1}{2} (CON_A - CON_B)(y - y_0)^2$$

More on Convexity

- Observing that *CON > 0* because the bond pricing function(s) are convex, it follows that whether yields go up or down, the portfolio with the higher convexity will have a better percentage change in price.
  - See Figure 5-4, SAIS.
  - Consider also Table 5-4 (p.277) which evaluates the impact of interest rate changes varying from 1 to 300 basis points on the price of a 20 year, 10% coupon par bond.

An Example Illustrating Convexity

- SAIS, p.277-78.

- The bond trader example: the net investment in the position at $t=0$ is zero.
- Comparison of a (*barbell)* cash + twenty year zero coupon assets with five year zero liability.
- The example demonstrates that whether interest rates go up or down, the higher convexity position does better.