Some Expected Utility Basics

Typical to take the utility function to depend on the single variable: *terminal or end of period wealth* denoted as $W(T)$, $W(1)$ and so on with expectation taken at $t = 0$. It is also assumed that: the utility function $U[W]$ is strictly increasing in W; and, has continuous first and second-order derivatives ($U' > 0$ and U''). The sign of U'' determines the risk attitude.

Defining ***expected wealth***

$$E[W(1)] = \sum_{j=1}^{S} Prob_j \; W_j = \Omega \qquad where \qquad \sum_{j=1}^{S} Prob_j = 1$$

***Risk Attitude Properties*:**

Attitudes to risk: ***risk averse; risk neutral; risk lover***

Risk attitudes can be identified by comparing the utility of the expected value of terminal wealth with the expected value of the utility

Two State Example: State A will occur with *Prob.* and outcome W_A with State B occurring with *(1 - Prob.)* and outcome W_B

Utility of expected value:
$E[W] = Prob_A \; W_A + (1 - Prob_B) \; W_B \quad \rightarrow \quad U[\Omega] = U[Prob_A \; W_A + (1 - Prob_B) \; W_B]$

Expected value of utility: $E[U[W]] = Prob_A \; U[W_A] + Prob_B \; U[W_B]$

Let $Prob = ½$ and $W_A = 2$ and $W_B = 0$ then $E[W] = 1$

Utility of expected value: $U[1]$
Expected value of utility: $0.5 \; U[2] + 0.5 \; U[0]$

Risk Neutral $U[1] = 0.5 \; U[2] + 0.5 \; U[0] \; \rightarrow U'' = 0$
Risk Averse $U[1] < 0.5 \; U[2] + 0.5 \; U[0] \; \rightarrow U'' < 0$
Risk Lover $U[1] > 0.5 \; U[2] + 0.5 \; U[0] \; \rightarrow U'' > 0$