Lecture 6
Natural Hedging, Diversification and Insurance

- Implementing Natural Hedges
- Diversification and Risk Management
- Various Forms of Portfolio Insurance
- Risk Management in Agriculture

What is a Natural Hedge?

- A natural hedge is the reduction in financial risk that can arise from an institution's normal operating procedures.
  - Example, a company that has a significant portion of its sales in one country will have a natural hedge to at least part of its currency risk if it also has operations in that country generating expenses in the currency. Firms may act to increase natural hedges by changing sourcing, funding, or operational decisions, but natural hedges are less flexible, and more difficult to reverse, than financial hedges.
  - The values of life insurance and annuity liabilities move in opposite directions in response to a change in the underlying mortality. Natural hedging utilizes this to stabilize aggregate liability cash flows.
- Natural hedges can be sources of sustainable corporate advantage.

Natural Hedges and Competitive Exposures

- Natural hedges involves making adjustments to marketing, production and capital structure to deal with competitive exposures
  - Associated with inherent differences in competitiveness across firms in a given business sector due to differing exposure to particular risks, e.g., FX, interest rate
    - Natural hedging is a long run, strategic process (vs. transactions hedging

- Financial institutions have inherent natural hedges, e.g., FX swaps, deposit/loan portfolios
  - Not immune to problems such as duration gap

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Accounting vs. Economic Exposure

- A key element in the development of a risk management philosophy is the method of measuring risk; Reading: RSD, p.151-6.

- **Accounting exposure** measures on a transaction by transaction basis. This approach is reflected in conventional textbook presentations of a risk management involving derivatives that assumes that there is only one transaction of interest.

- **Economic exposure** measures attempt to assess the impact of a specific financial or commodity price on the firm’s net cash flow. (Possible to run regressions.)

Hedging Corporate Economic FX Exposure

- See headline on next slide for example of economic exposure. See discussion in RSD, p.157-8

- How to measure the exposure? Optimal hedging theory provides a regression

- Is it necessary to hedge this risk? PPP arguments provide a theoretical rationale for not hedging

Reading: RSD, p.161-3

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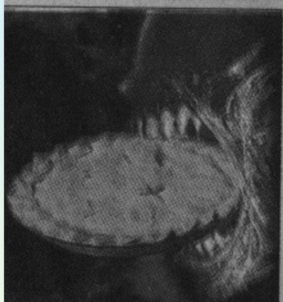

ACE'S FINAL FRONTIER

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n of CHUM Television, is get-
to the movie production busi-
with its first theatrical release,
s. Paul Gratton, VP and GM of
, said in a news release the
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describe *Decoys* as "*Ameri-
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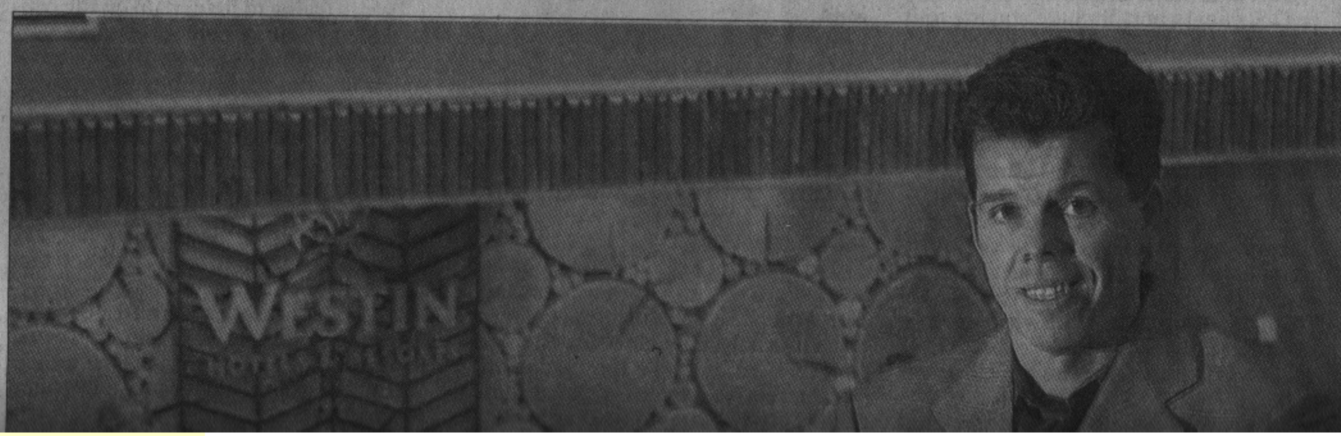
ONE-HOUR PHOTO FINISHED?

test advance in old-fash-
photography is coming soon:
service kiosk that can con-

Whistler visits drop with weak U.S. dollar



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Questions Relevant to Formulating Natural Hedging Strategies for FX Exposure

- 1. What is the foreign/domestic breakdown of sales?
- 2. Are the company's key competitors foreign or domestic?
- 3. What is the short and long run price elasticity of demand for firm output?
- 4. What is the foreign/domestic breakdown of production activities?
- 5. What is the foreign/domestic breakdown of input sources?
- 6. What currency is used to determine the firm's inputs and outputs?

Connection to Enterprise Wide Risk Mgmt. (from Lec. 5)

- What is Strategic Risk Management?

Strategic risk management is the process of identifying, implementing and monitoring systems for managing the range of risks confronting the firm.

In the consulting industry (CI), strategic risk management corresponds to **enterprise-wide risk management.** CI refits often emphasize decision support systems, IT methods.

→ **the construction of natural hedges is a fundamental, if unrecognized, part of ERM**

Diversification and Risk Management: Preliminary Concepts

Theorem: Moments of Linear Combinations of Random Variables

If $X(1), X(2), \ldots, X(N)$ are random variables and $a_1, a_2, \ldots, a_N$ are constants and $Y = a_1 X(1) + a_2 X(2) + \ldots + a_N X(N)$ then:

$$E[Y] = \sum_{i=1}^{N} a_i \, E[X(i)]$$

$$var[Y] = \sum_{i=1}^{N} a_i^2 \, var[X(i)] + 2 \sum_{i > j}\sum a_i \, a_j \, cov[X(i), X(j)]$$

where the double sum over $i > j$ extends over all values of i and j, from 1 to N, for which $i > j$.

Derivation of $var[Y]$ requires the observation that $cov[X(i), X(j)] = cov[X(j), X(i)]$.

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Methods of Managing Risk (from Lec. 5)

- The ***actuarial science approach***. Deals with insurance risk where the random variable of interest is a loss function taking a value of either *X(t)* (the size of the loss) or zero.
- Risk can be avoided; risk can be transferred; risk can be shared; risk can be reduced; and risk can be retained.

Reading: RSD, p.120-2.

The Insurance Principle:
Independent Random Variables and Zero Risk

If $cov[X(i), X(j)] = 0$ for all i and j where $i \neq j$, i.e., the random variables are all independent, and $a_1 = a_2 = ... = a_N = 1/N$ then $var[Y]$ has the property:

$$\lim_{N \to \infty} var[Y] = \lim_{N \to \infty} \left\{ \sum_{i=1}^{N} a_i^2 \; var[X(i)] \right\} = 0$$

This result has applications in insurance where the random variables are policy payouts and the a_i are the fraction of the portfolio of policies attributable to policy i.

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Some Useful Definitions

Definition: The expected return on the portfolio $E[R_p]$ is the value weighted sum of the expected returns on the individual securities, the $E[R_i]$:

$$E[R_p] = \sum_{i=1}^{k} w_i \; E[R_i]$$

where k is the number of securities in the portfolio. To calculate the value weights, w_i:

$$w_i = \frac{\$A_i}{\sum_{i=1}^{k} A_i} \qquad where \qquad \sum_{i=1}^{k} w_i = 1$$

with $\$A_i$ being the dollar value invested in security i and the sum over all $\$A_i$ being the total amount of money invested in the portfolio

Definitions continued

Definition: The standard deviation of portfolio returns, σ_p is the square root of the variance of portfolio returns $var[R_p] = \sigma_p^2$. Various ***equivalent*** forms for the portfolio variance formula are available:

$$var[R_p] = \sigma_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} w_i\, w_j\, \sigma_{ij} = \sum_{i=1}^{k} w_i^2\, \sigma_i^2 + 2 \sum_{i>j}^{k} w_i\, w_j\, \sigma_{ij}$$

$$= \sum_{i=1}^{k} w_i^2\, \sigma_i^2 + 2 \sum_{j=1}^{k} \sum_{i=1,\ i>j}^{k} w_i\, w_j\, \sigma_{ij} = \sum_{i=1}^{k} w_i^2\, \sigma_i^2 + 2 \sum\sum_{i>j} w_i\, w_j\, \sigma_{ij}$$

where $cov[R_i, R_j] = \sigma_{ij}$. In the double sum expression, when $i=j$ the covariance is a variance. These expressions can be further manipulated by making further substitutions using the definition for ρ_{ij}, the correlation between R_i and R_j, i.e., $cov[R_i, R_j] = \sigma_{ij} = \rho_{ij}\, \sigma_i\, \sigma_j$.

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Examples

Consider the simplest case where $k = 2$, when there are only two securities in the portfolio. In this case:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

$$= w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2 w_1 (1 - w_1) \rho_{12} \sigma_1 \sigma_2$$

Similarly for 3 assets in the portfolio:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2$$

$$+ 2 \{w_1 w_2 \sigma_{12} + w_1 w_3 \sigma_{13} + w_2 w_3 \sigma_{23}\}$$

When there are k securities in the portfolio, the resulting portfolio variance will contain k variance terms and $\{k(k - 1)\}/2$ covariance terms.

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The Minimum Variance Portfolio, 2 Assets

The minimum variance portfolio is an optimal portfolio. It is the portfolio that has the smallest risk in the set of all possible portfolios. This formula for this portfolio can be derived by minimizing $var[R_p]$ with respect to the choice variables, the value weights for each of the individual securities, subject to the restriction that the sum of the value weights be equal to one. In the simple case of the minimum variance portfolio for two securities, using the result that $w_1 + w_2 = 1$:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2 w_1 (1 - w_1) \sigma_{12}$$

$$\frac{d\sigma_p^2}{dw_1} = 2 w_1 \sigma_1^2 - 2 (1 - w_1) \sigma_2^2 + 2 (1 - 2w_1) \sigma_{12} = 0$$

$$w_1^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2 \sigma_{12}}$$

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Basics of Portfolio Insurance

- The basic idea behind portfolio insurance is to provide a rate of return on the portfolio which will not fall below a given floor. (Recall that a put is a type of insurance).

- The basic mechanics of portfolio insurance can be isolated from the put-call parity arbitrage condition for a non-dividend paying stock:

$$S + P = C + X\,e^{-rt^*}$$

For portfolio insurance, instead of an individual stock *S* now refers to a portfolio of stocks (dividends have been ignored for simplicity of exposition).

Basics of Portfolio Insurance (cont'd)

- Put-call parity provides two ***path independent*** insurance strategies.

- One strategy is $S + P$, buy puts against the portfolio. If S is an index portfolio, relevant exchange traded puts may be available.

- Another strategy is $C + X e^{-rt^*}$, buy calls and invest the remainder in appropriately dated bonds. Again, if the portfolio is an index portfolio, exchange traded calls may be available

Properties of Insured Portfolios

- ***Path Independence***
  - "A strategy that is not path independent gives an uncertain payoff, and therefore violates the very premise of portfolio insurance: giving a known payoff."
- ***Time invariance***
  - This requires that the insurance strategy does not depend on the time remaining in the program (or on the use of a fixed time horizon).
  - The primary practical difficulty arises because of the option price convexity with respect to time, e.g., an option with six months to expiration will cost less than twice what a three month option costs.

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Greeks for Insured Portfolios

- A delta neutral portfolio can be contrasted with portfolio insurance plans (which are delta positive).
- One possible method of insuring a stock position is to form a portfolio which combines purchased puts with a long stock position. In this case:

$$V = n_1 S + n_2 P \quad \rightarrow \quad \Delta_V = n_1 + n_2 \Delta_P$$

More on Dynamic Portfolio Insurance

- Dynamic portfolio insurance strategies can be illustrated by substituting the Black-Scholes formula into the put-call parity condition:

$$
\begin{aligned}
S + P &= S\,N[d_1] - X\,e^{-rt^*}\,N[d_2] + X\,e^{-rt^*} \\
&= S\,N[d_1] + X\,e^{-rt^*}\,(1 - N[d_2]) \\
&= w_1\,S + w_2\,X\,e^{-rt^*}
\end{aligned}
$$

where the weights w_1 and w_2 indicate the proportions of the portfolio held in stock and bonds in order to achieve insurance with an exercise price of *X* and time to maturity of t^*.

Different Types of Portfolio Insurance

Different Forms of
Equity Portfolio Insurance(a)

Strategy	Advantages	Disadvantages
Buying index puts against a portfolio	Insurance cost determined in advance. Investor captures portfolio nonmarket return.	Listed puts do not trade with expirations greater than 6 months. Must accept the pricing risk of subsequent options purchases.
Buying puts on individual stocks	Portfolio positions protected against decline on a stock-by-stock basis.	Premiums greater than for index puts. Not every stock has a listed put.
Buying index calls and money market securities	Can vary fixed income strategy around the call position; call performance tied to a diversified index.	Cannot capture nonmarket return on a portfolio of stocks. Must accept the pricing risk of subsequent options purchases.
Buying calls on individual stocks and money-market securities	Full participation in all gains from individual stock movement.	Premiums greater than for index calls. Not every stock has a listed call.
Selling stock index futures to create a synthetic put	Can create strike price and expiration date. Will capture portfolio alpha.	Actual cost cannot be predetermined. Must accept pricing risk of the futures contract.
Raising cash by selling stocks to create a synthetic put	No futures pricing risk.	Higher transaction costs and market impact costs in most instances.
Buying stock index futures to own a synthetic call	Can vary fixed income investment. Equity performance tied to a common index such as the S&P 500.	Cost cannot be predetermined. Position is exposed to index futures pricing risk.

(a) In addition to the insurance strategies using listed options and stock index futures, it is possible to create an over-the-counter European or American index option with a longer life than that available in the listed markets.