APPENDIX II

Money Market Calculations

Some basic background is required to interpret the interest rate futures quotes provided in the financial press. In order to understand the quoting mechanism, recall the profit functions for the short and long futures speculators: short positions benefit from falling futures prices while long positions benefit from futures price rises. However, for money market securities, the use of prices would be at variance with cash market practices which quote in yields. Because yields move inversely to prices, a method was devised for quoting money market futures which retained the notion that longs (shorts) benefit when the futures quote rises (falls) and was still consistent with cash market practices. The futures contract is quoted as 100 - Quote = (discount rate expressed with the first digit starting as a whole number). For example, the 6/16/92 Tbill for Sept 92 delivery closed at 96.21. This converts to 100 - 96.21 = 3.79. The 3 month US Tbill for Sept 92 delivery is being offered at a discount rate of 3.79%. A similar quoting convention is also used for Eurodollars and the Montreal Exchange's BA futures contract.

When comparing interest rates derived from money market futures quotes, it has to be recognized that most US money market securities, Tbills, BA's, commercial paper and term repos, are quoted on a ***discount rate*** and ***not*** a true yield basis. In addition to using a different pricing formula, the discount rate calculation also involves calculating the year as though it has 360 days. To see this, consider the US method for determining the purchase price of a 1 year, \$1 million par value Tbill sold at a discount rate of 8%:

$$Discount = (\$1{,}000{,}000)(.08)(364/360) = \$80{,}888.89$$

As a result, the price paid for this Tbill maturing in one year is the maturity (par) value minus the discount or \$919,111.11. The general formula for arriving at the discount is:

$$D = d\,F\,(tsm/360)$$

where D is the discount, F is the maturity value, d is the discount rate and tsm is the time from settlement to maturity. Similarly, for a Tbill with 3 days to maturity, par value of \$1 million and discount rate of 9%:

$$D = (.09)\ (\$1{,}000{,}000)\ (3/360) = \$750$$

The bill is purchased for \$999,250 and matures in 3 days for \$1 million, returning \$750 in interest.

The method for valuing an 01 or one basis point (.0001 expressed as a decimal) for the 3 month \$1 million par value Tbill futures contracts should now be apparent:

$$D = (.0001)(\$1{,}000{,}000)(90/360) = \$25$$

In the discussion of dollar value hedge ratios in Chap. 6, reference was made to the value of an 01 for a \$1 million, 6 month Tbill. In this case:

$$D = (.0001)(\$1{,}000{,}000)(180/360) = \$50$$

Hence, it is necessary to use two Tbill futures contracts to provide a dollar value equivalent movement for a 6 month, \$1 million dollar Tbill position.

It should be recognized that the discount rate understates the simple interest, true yield or bond equivalent yield formula which is used in Canada and other countries for money market securities. The formula for calculating the annualized true yield (r) is the familiar:

$$r = \frac{F - P}{P} \frac{365}{tsm} \qquad or \qquad P = \frac{F}{1 + (r \frac{tsm}{365})}$$

where P is the price paid or $D = F - P$. Substituting this result gives:

$$r = (D/(F-D)) (365/tsm)$$

This can be solved in terms of the discount rate by substituting the method for calculating D for discount securities to give:

$$r = \frac{(365)d}{360 - d\,(tsm)}$$

Table 2.1A **Treasury Bill Rates**

TREASURY BILLS

Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.
Aug 11 '94	1	4.11	4.01	+ 0.08	4.07
Aug 18 '94	8	4.11	4.01	+ 0.06	4.07
Aug 25 '94	15	4.13	4.03	+ 0.04	4.09
Sep 01 '94	22	4.16	4.06	+ 0.04	4.13
Sep 08 '94	29	4.17	4.07	+ 0.04	4.14
Sep 15 '94	36	4.16	4.12	+ 0.04	4.19
Sep 22 '94	43	4.34	4.30	+ 0.05	4.38
Sep 29 '94	50	4.17	4.13	+ 0.04	4.21
Oct 06 '94	57	4.27	4.23	+ 0.03	4.32
Oct 13 '94	64	4.32	4.30	....	4.39
Oct 20 '94	71	4.39	4.37	+ 0.02	4.47
Oct 27 '94	78	4.41	4.39	+ 0.01	4.49
Nov 03 '94	**85**	**4.47**	**4.45**	**+ 0.01**	**4.56**
Nov 10 '94	92	4.50	4.48	+ 0.02	4.59
Nov 17 '94	99	4.52	4.50	....	4.62
Nov 25 '94	107	4.56	4.54	....	4.67
Dec 01 '94	113	4.60	4.58	....	4.71
Dec 08 '94	120	4.65	4.63	....	4.77
Dec 15 '94	127	4.66	4.64	+ 0.01	4.78
Dec 22 '94	134	4.68	4.66	+ 0.01	4.81
Dec 29 '94	141	4.65	4.63	+ 0.01	4.78
Jan 05 '95	148	4.76	4.74	+ 0.02	4.90
Jan 12 '95	155	4.83	4.81	+ 0.02	4.98
Jan 19 '95	162	4.86	4.84	+ 0.01	5.02
Jan 26 '95	169	4.88	4.86	+ 0.01	5.04
Feb 02 '95	**176**	**4.91**	**4.89**	**+ 0.01**	**5.08**
Feb 09 '95	183	4.93	4.91	....	5.11
Mar 09 '95	211	4.99	4.97	+ 0.01	5.17
Apr 06 '95	239	5.06	5.04	– 0.01	5.25
May 04 '95	267	5.17	5.15	....	5.38
Jun 01 '95	295	5.20	5.18	– 0.02	5.43
Jun 29 '95	323	5.26	5.24	+ 0.01	5.51
Jul 27 '95	351	5.29	5.27	+ 0.01	5.56

This is the formula for converting a discount rate into a true or simple interest yield.

To see how this works, consider the covered interest arbitrage example of Sec. 5.2. The discount rate for the US BA was quoted at .0477 or 4.77%. Using the conversion formula:

$$r = [365\,(.0477)]/[360 - \{(.0477)90\}] = .0489462$$

Similarly for the CP rate:

$$r = [365\,(.0480)]/[360 - \{(.0480)90\}] = .0492577$$

The general formula can also be used to show that the quoted discount rate will always be below the true yield, $r > d$, as well as showing that the discrepancy between r and d will be greater the higher is the rate of discount and the longer is the term to maturity. Some manipulation gives the discount rate associated with a given true yield:

$$d = \frac{360\,r}{365 + r(tsm)}$$

In recent years, the relationship between d and r is reported in the financial press, as reflected in the US Treasury bill rates reported in Table 2.1A.

To verify the formula for deriving the discount rate associated with an observed yield, consider the Nov. 3, 1994 bill in Table 2.1A. This bill is indicated in bold because it is the most recently issued 3 month bill and, due to the focus of market trading on the most recently issued maturities, this bill will tend to be the most liquid. The *tsm* for this bill is 12 weeks plus 3 days or 87 days with a reported yield of 4.56. Evaluating for d gives:

$$d = [360\,(.0456)]/[365 + \{(.0456)(87)\}] = .0445$$

This is the value reported in Table 2.1A.

Fixed Income Calculations

The pricing conventions for Tbond and Tnote interest rate futures contracts do not follow the same quoting conventions as for money market securities. Long term interest rate futures prices are quoted much as in the cash market, with the proviso that the underlying bond or note being quoted is a theoretical instrument. For example, for many years the US Tbond contract traded a theoretical 8% coupon -- since 2000 the Tbond contract has featured a 6% theoretical coupon -- with 15 years to the call date and the Tnote is based on a theoretical 6 year, 8% coupon note. The method of quoting prices for these contracts is in keeping with market convention for trading non-money market fixed income securities on the basis of a $100 par value. In addition, the market convention of quoting prices in 1/8, 1/4, 1/2 and so on is also retained. For the Tbond and Tnote quotes, the fractional part of the quote refers to 32nds. Hence, a Tbond quote of 73-20 for June delivery is offering to provide "equivalent" cash Tbonds at 73 20/32's for the 8%, 15 year (to call) theoretical bond.

In the case of delivery, the short can deliver ***any*** available cash US Tbond or Tnote which meets the maturity restriction. Because the prices of available Tbonds and notes differ according to coupon and maturity date, the delivery process requires a pricing procedure for determining the value of any specific Tbond relative to the theoretical 8% bond (otherwise the lowest coupon bond would typically be delivered). For this purpose, the CBT devised a method of determining ***conversion factors*** for assessing the invoice value of potentially deliverable bonds. The conversion factors for the Tbond contract can be obtained from the CBT exchange website In practice, the conversion factors are not exact, giving rise to the presence of a "cheapest deliverable" Tbond or Tnote. In addition to multiple delivery grades associated with the wide range of deliverable cash instruments which differ with respect to maturity and coupon, the quotes for note and bond futures are further complicated by other considerations associated with specifics of contract design. These additional factors also need to be taken into account in determining the cheapest deliverable bond or note.

The translation of a quoted futures price to a yield follows the standard conventions for bond pricing applied to semi-annual coupon Treasury securities. Consider the problem of translating the 73-20 bond futures price to a yield for the old-style 8% theoretical coupon Tbond contract:

$$P_B = \sum_{t=1}^{2T} \frac{C/2}{[1 + \frac{r}{2}]^t} + \frac{B}{[1 + \frac{r}{2}]^{2T}}$$

$$73.625 = \sum_{t=1}^{40} \frac{4}{[1 + \frac{r}{2}]^t} + \frac{100}{[1 + \frac{r}{2}]^{2T}}$$

Solving the last equation for the yield (r) gives: $r = .11352$ or 11.352% Because Tbonds and Tnotes are quoted in terms of prices, correspondence with the long/short profitability conventions will automatically be observed.

The above methodology can be used to solve for the conversion factor used in delivery of Tbonds. For example, a 20 year, 14% coupon Tbond would have a conversion factor which is determined as though yields were 8%:

$$CF = \sum_{t=1}^{40} \frac{7}{[1.04]^t} + \frac{100}{[1.04]^T} = 159.38$$

The actual conversion factor used is derived by dividing CF by 100. Given this, the deliverable amount of a given bond is determined by taking the par value of the contracts involved and multiplying by the futures settlement price to get the market value in terms of the theoretical 8% deliverable. This value is then multiplied by the conversion factor provided by the exchange, using the formula described. While, for most traders, the complications of bond delivery are typically of little importance, significant movements in Tbond prices can occur when the cheapest deliverable bond changes. Given that the formula tends to favour certain features such as longer maturities, this can occur following a Treasury auction of 30 year Tbonds. As a final note, to get the value of a "tick", in this case

1/32, point movement in the price of the Tbond all that is required is: [1/32] $100,000 = $31.25.

Analysis of the cheapest deliverable bond has received considerable attention in the academic literature, e.g, Livingston (1984, 1987) and the references cited in Duffie (1989, p.324-332). Table 2.3A has an illustration of the cheapest delivery problem selection process. Some of the interesting issues arising in this area involve the various options embedded in the Tbond delivery process: quality to deliver, the option to deliver the cheapest bond; accrued interest arising from the option to make delivery on a arbitrary trading day during the delivery period; end-of-month option which involves delivering in the one week period after futures have finished trading the delivery contract; and, the wild-card option, which involves substituting bonds on the delivery day. The issue of deliverable specification has had implications for a number of contract failures, e.g., the GMNA, CDR futures contract (see Duffie 1989, p. 339-42).