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ARMAX Modeling of Term Structure Supply Effects:
Theory and Some Evidence for Canada

by

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ABSTRACT

This study examines the term structure of interest rates in the market for Government of Canada bonds to determine whether changes in either the maturity composition of Canada debt or the US term structure have had a significant effect on the shape of the Canadian term structure. Methodologically, a reduced form time series model is estimated which incorporates exogenous information about both the level of domestic interest rates and the shape of the US term structure. The empirical results indicate that the null hypotheses of no debt management effects and no US term structure effects are rejected for monthly data.

Key words: Vector autoregression, reduced form estimation, causality testing.

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How changes in the maturity structure of aggregate debt affect the term structure of bond yields is a fundamental question in debt market analysis. Methodologically, the issue of "term structure supply effects" has been approached using either structural models of the bond market (e.g., Christofides (1975, 1976), Cook and Hendershott (1978), Friedman and Roley (1980), Roley (1981, 1982)) or reduced form/time series techniques (e.g., Capie, et.al. (1986), Plosser (1982, 1987), Poitras (1989), Boothe and Reid (1989)). The reduced form approach is appealing because the null hypothesis of no supply effects can be tested using reduced-form estimates with only minimal identifying restrictions imposed. In this regard, vector autoregressions (VAR's) have been the basis for specifying the reduced form in a number of studies. This study extends the VAR approach by including additional exogenous variables (other than the lagged endogenous) into the VAR-based reduced form, i.e., the estimated reduced forms are derived from a vector ARMAX model. Unfortunately, as in other reduced form studies, the lack of identification conditions inherent in reduced form estimation eliminates the possibility of determining the direction of contemporaneous causality amongst the endogenous variables.

In the following, Section I provides a brief outline of relevant previous studies on term structure behaviour as well as a brief institutional background on the Government of Canada debt market. Section II describes the estimation methodology, outlining the derivation of the autoregressive form of the ARMAX model. Section III provides some estimation results for Canada. Finally, Section IV outlines the important results contained in the paper. An Appendix describing the data set is also included.

1. PREVIOUS STUDIES AND INSTITUTIONAL BACKGROUND

Since the seminal work of Modigliani and Sutch (1967), reduced form procedures have often produced little evidence for term structure supply effect in either the US or Canada. Similar results have been presented for Canada (e.g., Pesando (1975), Boothe and Reid (1989)). In support of activist debt management, Capie, et.al. (1986) has recently found significant impact on yields from the 1932 Conversion Loan in the UK and Poitras (1989) has found evidence contemporaneous causality for Canada.

The issue of supply effects is intimately related to the distinction between the expectations hypothesis and the preferred habitat hypothesis in the term structure literature. Both the pure expectations and the liquidity premium-augmented expectations hypotheses implicitly maintain that debt management effects are insignificant. (Assuming that the term premia are not influenced by changing debt maturity composition.) See, for example, the discussion following Mankiw (1986). However, if investors do adhere to preferred habitats, then it is likely that debt management could make use of this behaviour through alterations in the maturity composition of the debt.

In examining variations in the term structure over a times series (1972-84), this study differs from many of the previous studies of supply effects which have concentrated on major debt management events (e.g., the Canadian Conversion Loan (1958), Operation Twist in the US, or the Stock Conversion Loan (1932) in the UK).

Much as with structural studies, reduced form results can be subjected to a number of qualifications and criticisms. Significantly, the problem of misspecifying the reduced form equation requires attention. Following Modigliani and Sutch, previous reduced form studies for Canada have modelled the behaviour of the Canadian long rates with a distributed lag on short rates. This approach presents at least two potential misspecification problems. Firstly, in a small open

economy the domestic term structure is dominated by the world (i.e., the U.S.) term structure (Beenstock and Longbottom (1981), Kool and Tatom (1988)). Hence, in order to examine the behaviour of the Canadian term structure, it may be necessary to introduce U.S. bond yields as exogenous variables. Secondly, if long rates are modelled as a distributed lag on short rates, the level of interest rates is left undetermined. Hence, a variable for interest rate levels may be necessary if the shape of the yield curve is not independent of the level of rates.

The current study exploits the ARMAX structure to address the potential problem of misspecifying the reduced form equation with respect to both the level of rates and the influence of the US term structure. Specifically, US term structure variables are included as exogenous variables in the reduced form estimations. To account for the level of interest rates, contemporaneous and lagged realizations of the Canadian TBill rate are also included as exogenous variables. This specification does not imply that the bond market and the TBill market are independent but, rather, that bond rates are recursively determined after the TBill yields. (This approach to short rate/long rate interaction is supported in a number of theoretical studies, e.g., McCafferty (1986)). In other words, bond market variables will, in general, be correlated with the TBill rate but the causality is assumed to run from the TBill market to the bond market so that the bond market can treat TBill yields as exogenous. This implies that TBill rates act as an "anchor" for the term structure, determining the yield curve's location at any point in time. As a result, the lag structure for TBills is restricted to be of the same order as that for Government of Canada bond yields. By introducing information about both the level of domestic interest rates and the shape of the foreign, U.S. yield curve, the reduced form estimates of the Government of Canada yield curve's shape should be more efficient.

This raises the issue of debt stock variability. If there is little variability in the debt stock over

time then statistical inference is complicated by the multicollinearity of the debt variables and the constant. Relative to the US data, one advantage of Canadian data is the greater amount of variation in the debt program over time. In the current sample, average maturity of Government of Canada domestic marketable debt outstanding (including TBills) increased from four years six months in 1972, to a low of three years nine months in 1975, increasing from there to seven years two months in 1979 finally ending up at five years seven months in 1984. Over the same period in the US, the average maturity of outstanding marketable Treasury debt went from three years, three months in 1972 to a low of two years seven months in 1976 then rising slowly to reach a high of four years, six months at the end of 1984.

A number of possible factors contributed to the variation in the maturity composition of Government of Canada marketable bonds over the period. For example, a portion of the variation in the maturity composition was a result of changes in the market's demand for bonds in the various maturity categories, i.e., due to "tailoring" of maturities to meet the demands of the market. Tailoring was due to technical features of the "primary distribution" method used to market domestic Government of Canada bonds. Restrictions were placed on the percentage of their allotment that dealers could drawdown. Early in the period when debt managers were attempting to develop the long end of the market, restrictions were in the 40-50% range. However, later in the period when rates were historically high and were generally expected to fall, long bonds restrictions were tightened to the 15-25% range.

2. ESTIMATION METHODOLOGY

Briefly, an ARMAX model is a vector ARMA model with exogenous variables included in the reduced form estimation procedure (Hannan, et.al. (1980), Poskitt and Tremayne (1984), Bierens (1987)). Exogenous variables are defined as predetermined (right hand side) variables other than

lagged endogenous variables (and the constant). Hence, the vector autoregressive form of an ARMAX model (the so-called VARX model) is the conventional VAR model with exogenous variables included as additional regressors. The VARX methodology begins with the assumption of a jointly covariance stationary set of time series x_t , where x_t is a column vector containing k variables, the x_{it} observations ($i = 1, \dots, k$), at time t . In modelling the behaviour of the term structure, the x_t vector contains sub-vectors of bond yields for various maturities, the associated bond stocks, treasury bill rates and, for the small open economy case, foreign bond yields for various maturities (i.e., $k = 3(\text{number of maturity classes}) + 1$). Assuming joint covariance stationarity, the multivariate version of Wold's decomposition theorem states that the x series can be represented (Hannan (1970)) as the sum of a deterministic component (which may vary through time), and a moving average of white noise errors. Without loss of generality, x_t can be centered about its (possibly time varying) mean and the moving average representation may be inverted to yield a vector autoregressive process in which x_t is a linear function of lagged realizations of x_t and a vector of contemporaneous v_t shocks. Contemporaneous correlation amongst the x_t 's is reflected in the off-diagonal elements of the variance-covariance matrix (V) of the v shocks. In conventional econometric terms, the VAR model is a reduced form model where the only predetermined variables are the lagged endogenous variables. Extending to the VARX framework involves incorporating additional information regarding the exogeneity of variables. If exogenous specification is correct, this will lead to more efficient reduced form estimates. In early specifications of the ARMAX model (Hannan, Dunsmuir and Deistler (1980)), the ARMAX model incorporates such information by decomposing the x_t vector into $x_t = (x_{1t}, x_{2t})$ where x_{1t} are the 'true' endogenous variables and the x_{2t} are the 'true' exogenous variables. With this in mind, the vector moving average specification given in (1) corresponds to the following structural system which underlies the ARMAX model:

$$(1) \quad \begin{vmatrix} x_{1t} \\ x_{2t} \end{vmatrix} = \begin{vmatrix} A_{11}(L) & A_{12}(L) \\ 0 & A_{22}(L) \end{vmatrix} \begin{vmatrix} e_{1t} \\ e_{2t} \end{vmatrix} = A(L) \begin{vmatrix} e_t \\ e_t \end{vmatrix}$$

$E[e_t] = 0$, $E[e_t e_{t-n}] = 0$ for all n not equal to zero
 $E[ee'] = Z$, where Z is diagonal and non-singular.

where (1) is the "orthogonalized moving-average representation" of the x process since the e innovations are orthogonal by construction. $A(L)$ is a polynomial in the lag operator L and, by convention, the A_0 matrix is assumed to be triangular to ensure identification. Contemporaneous and lagged correlations across variables are solely a function of $A(L)$. Using (1), the reduced form equation for the x_{1t} follows:

$$(2) \quad x_{1t} = D(L)x_{2t} + B(L)x_{1,t-1} + v_{1t}$$

By construction, the x_{2t} variables are independent of the residuals for the x_{1t} regression.

As in the conventional VAR model, contemporaneous (or "structural") relationships between the x_{1t} variables are imbedded in the estimate of the variance-covariance matrix V of the v_{1t} 's (equal to $Z_{11}A_{11,0}A_{11,0}'$) and are not identified. An important feature of the VAR representation, as with any reduced-form representation, is that while estimation of the model provides an estimate of the V matrix it does not (without the imposition of identifying restrictions) provide information regarding the ordering imposed contemporaneously (i.e., on the $A_{11,0}$ matrix). The system can be estimated using OLS which is consistent and efficient given the assumptions made in specifying the model.

Comparing the modified VAR modelling procedures with the more familiar Box-Jenkins approach (e.g., Tiao and Box (1981)), the identification stage of the Box-Jenkins based approach must be altered since, with exogenous variables in the model, it is no longer generally necessary (or possible) to test for the stationarity of the endogenous variables prior to model estimation. This is because stationarity of the endogenous variables is required to hold only after the influence of the

exogenous variables has been removed. Hence, if the VAR model includes exogenous variables, stationarity tests are generally conducted on the residuals of the fitted model.

For present purposes, the endogenous variables are assumed to be Government of Canada bond yields and Government of Canada bond stocks. The remaining variables, the x_{2t} , include the specified exogenous variables -- the Government of Canada TBill rate and U.S. bond yields.

The model (2) has an immediate connection to the Granger method of causality testing. A variable z is said to Granger (or Weiner-Granger) cause a variable y if the projection of y on an information set which is exclusive of past z has a prediction error which is significantly greater than the projection of y on the information set which includes past z . This definition has been operationalized in a number of different ways, e.g., Feige and Pierce (1980), Geweke, et. al. (1983), Cooley and LeRoy (1985) and Pierce and Haugh (1977) for further description of these causality tests. Much of the discussion of causality is conducted in a bivariate context. However, Skoog (1976) examines the tri-variate and higher order forms of causality testing and finds that the interpretation of causality test results is somewhat altered in these cases.

To illustrate how Granger's approach may be utilized to test the hypothesis of supply effects in the bond market, decompose the x_{1t} vector into $x_{1t}' = (r_t', b_t')$ where r_t is the vector of bond yields and b_t is the vector of bond stocks. Equation (2) can now be expressed as:

$$(2') \quad \begin{array}{l} |r_t| = B(L)|r_{t-1}| + D(L)x_{2t} + |v_t^r|, \quad E[v_t v_t'] = V \\ |b_t| \quad |b_{t-1}| \quad |v_t^b| \end{array}$$

$$B(L) = \begin{vmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{vmatrix} \quad V = \begin{vmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{vmatrix}$$

The Granger procedure for testing causality from stocks to rates is to perform an OLS regression of current r on a prespecified number of lagged r 's and lagged b 's and test for the significance of the

coefficients on the lagged b's, i.e., test the hypothesis that $B_{12}(L)=0$. If $B_{12}(L) = 0$, b is found not to Granger cause r. Similarly, the condition for r not Granger causing b is $B_{21}(L) = 0$.

An important drawback to causality tests such as Granger's arises if the lag length is not appropriately specified. If the lag structure is incorrect then the test statistics are not valid under the null hypothesis. In the VAR context, technical conditions for truncation of the lag polynomial need to be satisfied. Another drawback arises because Granger (and contemporaneous) causality provides a necessary but not a sufficient zero restriction on the "structural" equation coefficients (Jacobs, et.al. (1979), Cooley and LeRoy (1985)).

If exogenous variables are included in the regressions, Granger causality from b to r must be interpreted as b causing r controlling for the effect of the exogenous variables on r. In this case, the necessary (but not sufficient) condition for zero contemporaneous causality between r and b will be $V_{12} = 0$. In other words, the residuals from the VAR regressions are computed and examined for significant cross-equation relationships. If significant relationships are observed (V_{12} not equal 0) then the null hypothesis of no instantaneous causality between r and b can be rejected. However, the direction of causality will be undetermined. Where exogenous variables are included in the model, contemporaneous values of the exogenous variables must be included to avoid potential bias in the estimation of V.

3. ESTIMATION RESULTS

3.1 Identification

The order of the model (2') is determined using the sequential testing procedure suggested by Jenkins and Alavi (1981) and Poskitt and Tremayne (1984); the model's lag structure is overfitted and a systems version of the likelihood ratio test is used to determine the true lag specification. Specifically, to test for lag specification the model is estimated both with $k+1$ lags and k lags and the null hypothesis of k lags is tested by comparing the adjusted log ratio of the determinants of the variance/covariance matrices of the residuals using the likelihood ratio test proposed by Sims (1980). This procedure is also used to test for the significance of exogenous reduced form variables. The test statistic is defined as:

$$LRD = (T-k)[\log|V'| - \log|V|]$$

where $|V'|$ and $|V|$ are the determinants of the variance/covariance matrices of the restricted and unrestricted models respectively, T is the sample size and k is the number of estimated coefficients per equation. LRD is distributed asymptotically chi-squared with r degrees of freedom, where r is the number of total system restrictions, under the null hypothesis. Once the order of the model is established, the estimated coefficients are examined to test for supply effects. The residual variance-covariance matrix is also examined to determine whether or not the necessary condition for a lack of contemporaneous causality (zero cross correlations between the residuals of the debt supply and rate equations) is satisfied.

Following Sims (1980), the correction factor k has been included because the corrected statistic gives small sample results closer to the asymptotic distribution. While there are a multitude of proposed methods for testing the order of a VAR model (e.g., Lutkepohl (1986)), the selection of one method over another depends on the final purposes for the selected time series model. Because

the objective here is primarily to provide a parsimonious representation of the data in the sample, LRD has desirable properties compared to other methods. When two test procedures designed for forecasting purposes, Aikaike's final prediction error (FPE) criteria and Schwarz's method were tried in the current study, the general ordering of the models was unchanged, except that Model 2 was preferred for the full sample, i.e., the model with a greater number of parameters was selected.

A number of different definitions for the debt supply variables can be used. For example, the three bond maturity classes could be defined in terms of levels. However, because the nominal stock of debt has grown substantially over time due to increases in financing and inflation, the use of debt stock levels risks biasing the results towards acceptance of the null hypothesis of no supply effects. A more accurate definition is to use the share of the maturity class in the total of publicly held domestic marketable debt, i.e., total debt is defined as the sum of the stock of debt outstanding for the three bond maturity classes and TBills. As well as avoiding the problems associated with nominal values, use of shares has the added advantage of allowing the indirect introduction of the effect of TBill financing (through its affect on the total stock of debt outstanding). By construction, the shares will not sum to one because of the use of TBills in total debt. If there are supply effects, variations in the bond shares will impact bond yields as the share of debt between maturity classes changes.

Table 1 illustrates the results of the preliminary search for the optimal lag length and exogenous variable specification. The initial system estimated (Model 1) is a six equation model with a five month lag length for the Canadian bond yields and debt shares plus an additional lag at the twelfth month. In the mnemonics of Table 1, this lag structure for the endogenous variables is indicated 1-5,12 (0-5,12 for TBills because the contemporaneous value of TBills is also included). The lag length for the foreign rates was restricted to be two months or less. The six equation model was re-

estimated under a number of different restrictions on both the lag length of the endogenous variables and the specification of the exogenous variables. For example, to test for the significance of foreign rates, no US rates are included in Models 6 and 9. To test for an annual 'seasonal', the twelfth lag is omitted in Models 5 and 7.

Table 1 also presents the chi-squared statistics and marginal significance levels of the log-determinant-based likelihood ratio (LRD) tests. The chi-squared statistic tests the null hypothesis that the restrictions imposed are valid. As indicated in the Table 1, for the full sample, the hypothesis of valid restrictions on the fifth and fourth lags could not be rejected at the 90% confidence level while restricting the lag length to order two was rejected. Based on a lag length of order three (Model 3), tests for specification of the exogenous variables and seasonality were conducted. For the U.S. term structure variables, the null hypothesis of no US effects was strongly rejected by the data indicating that the world term structure of interest rates is an important determinant of the term structure in a small open economy. Regarding seasonality, the likelihood ratio test produced mixed results for the twelfth lag on the Canadian variables. The small seasonal effect observed is likely due to the shocks to shares produced by the annual CSB sales program.

Regarding residual diagnostics, because the model treats the Canadian TBill rate and the foreign bond yields as exogenous variables, it is not possible to test for stationarity prior to fitting the model. Stationarity is tested indirectly by examining the residuals of the fitted model. The portmanteau Q-statistics for the preferred models and a number of other models are given in the Table 2. For both preferred models, there was no evidence of non-stationarity in the residuals, i.e., the significance level of the Q-statistic was small enough to fail to reject the null hypothesis of stationary residuals. (The Q statistic tests significance of the sum of a prespecified number of residual auto-correlations. The tests reported here incorporate the changes recommended by Ljung and Box (1978) to correct

for small sample problems.) This was confirmed by simulating the response of the endogenous variables to exogenous shocks. In both the full and subsample cases the preferred model was stable. In addition, it was observed that when the TBill rate was omitted as an exogenous variable significant auto-correlations among the rate equations emerged, suggesting that while the premiums of bond yields over the TBill yield are stationary, the raw yields are not. This result confirms the specification of the yield curve model as being anchored by the TBill rate as opposed to a model that explains the term structure in isolation.

3.2 Granger and Contemporaneous Causality

Testing for supply effects in the Government of Canada bond market involves evaluating the conditions for contemporaneous and Granger causality between bond yields and the debt shares. Two types of supply effects are considered: the first is that a change in aggregate maturity composition of the debt will affect bond yields; the second is that a change in the share of debt within a specific maturity class will affect bond yields. In other words, one type of supply effect arises when the total maturity composition changes; the other supply effect arises when the share for a specific maturity class changes. In both cases, testing for contemporaneous causality involves examining a sub-matrix of the correlation matrix of the residuals for the six estimated equations of the preferred model. Regarding the effect of the total maturity composition, contemporaneous causality is examined by testing the hypothesis that the cross correlations between all shares and the chosen bond yield are zero. Contemporaneous causality between the share in a specific maturity class and specific bond yields is examined by testing whether specific cross-correlations are zero. Contemporaneous causality between the share for a specific maturity class and all rates is examined by testing the hypothesis that cross correlations between the chosen share and all rates are zero.

The test for total share change on a specific rate and for maturity specific share change on all

rates is based on Quenouille's (1947) chi-squared test. This test is based on the asymptotic properties of the correlation coefficient, i.e., $T^{1/2}$ times r is $N(0,1)$. As is well known, this statistic is not well-behaved in small samples. However, because the number of observations used in this study is well over one hundred, the large sample properties of the test statistic are assumed to hold.

As Table 3 indicates, for the full sample, the null hypothesis of no maturity composition effects (i.e., the row chi-squared tests) is rejected for all bond yields. In addition, the hypothesis of zero contemporaneous causality between bond yields and short and medium maturity shares also cannot be rejected at the 90% level of confidence. However, with regard to the individual cross correlations, the correlation between the errors of the long share equation and the short and medium yield equations indicate the hypothesis of a zero cross correlation is rejected. For the test of contemporaneous causality between the long share and the term structure (the column chi-squared test) the hypothesis of zero contemporaneous causality is also rejected. The implication of these results is that for the full sample there does appear to be some contemporaneous feedback between innovations to both the long maturity share of outstanding debt and the term structure of Government of Canada bond yields. Subject to the caveat that the tests performed cannot identify a specific direction of causality, the positive sign of the significant correlations indicates lengthening of the maturity composition of the debt by increasing the long maturity share corresponds with a contemporaneous flattening of the yield curve.

To test for Granger causality of the share in a specific maturity class on rates, F tests are constructed to test whether all the coefficients for the lag structure associated with a share variable are zero in the yield equations. The results of these tests are contained in Table 4. For the full sample, examination of the F tests indicates that the coefficient-lag-structure for bond shares does not have a significant impact on bond rates. Changes in the share of debt in a specific maturity class

do not appear to Granger cause any bond yields. The F tests indicate that changes in a specific share over time only Granger cause that specific share while bond yields tend only to Granger cause other bond yields. (The barely significant effect of the long share on the medium share is most likely an artifact of the maturity crossing problem.) While the yield results are not surprising, the share results are interesting because they indicate that there was little interaction between shares over time. This indicates that there were, at best, only minimal debt management effects on share behaviour-- in the Granger sense.

Conventionally, a supply effect arises because a relative decrease in the debt share in a specific maturity class should increase rates comparably for other maturity classes. Higher rates have to be offered in order to encourage investors to hold the increased supply in the other maturity classes. Hence, the predicted sign for supply effects is negative for shares and rates not for the same maturity class (positive for shares and rates in the same maturity class). Given the contemporaneous and Granger causality results, limited evidence was found for conventional (negative) supply effects based on the full sample behaviour. Similarly, if dealers were determining the maturity composition, the predicted contemporaneous causality sign would also be negative, i.e., when rates are 'high' and falling dealers would want to lengthen term and, if rates are 'low' and rising, dealers would want to shorten term. Hence, dealer determination of the maturity composition would also be consistent with a negative cross correlation between bond yields and shares.

Evidence was presented indicating positive contemporaneous causality of the long share with medium and short rates. This form of supply effect could originate from activist debt management motivated by interest cost minimization. Specifically, if rates are high and are expected to fall, interest cost minimization would dictate that funding requirements be shifted into the short end. Similarly, if rates are low and expected to rise, the tendency would be to shift funding requirements

into the long end. Given that there is a tendency towards yield curve inversion at the peak of an interest rate cycle and yield curve steepness at interest rate cycle troughs, part of the effect of changes in rate levels on debt shares would be picked up by bond yields and not by the exogenous TBill rate. The absence of any contemporaneous feedback between bond yields and short and medium shares can be attributed to other, extraneous (i.e., non-interest rate) factors affecting those shares. For example, because the long share was the primary "new money" financing vehicle, short and medium shares had to absorb the impact of shocks arising from the CSB program, shortfalls in tax receipts and foreign exchange losses. The insignificance of a contemporaneous relationship between long shares and rates could be due to the lesser amount of interest-rate-cycle-related volatility in long rates.

Examining the evolution of yields and shares over the sample reveals the nature of the impact of debt management. In the early part of the sample the yield curve evolved from steeply sloped in early 1975 to flat to slightly inverted at the end of 1979-- the level of rates rising from around 6% to 12% for short maturity bonds over the period. Debt share behaviour during this period was largely driven by a dramatic shift into long bond financing. This shift was encouraged by debt managers both through moral suasion and the stated 'restrictions' on long bonds which, in practice, acted as indicators of how much of an issue the Government expected the dealers to take in long bonds. Consequently, the maturity composition of the debt was lengthening considerably at a time when the yield curve was flattening. Similarly, in the latter half of the sample rates were falling and the Government reduced term by discouraging drawdowns of long bonds. If expectations are based on a distributed lag on past rates, the persistence of rate trends during the sample would be consistent with the observed contemporaneous causality results.

4. SUMMARY

Using time series methodology, this paper examined the hypothesis that changes in maturity structure of outstanding Government of Canada marketable domestic debt affected the term structure of Government of Canada bond yields. Based on monthly data for the 1972-84 period, the empirical results found some evidence to support the hypothesis that there was feedback between debt management actions and changes in the maturity composition of the debt. The feedback was confined to positive contemporaneous effects of long shares with short and medium rates. Given the distribution method used to market Government of Canada bonds, the positive effect was likely due to the level and direction of rates affecting share determination decisions rather than the other way around. Evidence was also presented that U.S. term structure was highly significant in determining the Canadian term structure, although the US term structure variables had a shorter lag structure than that of Canadian rates.

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APPENDIX: DATA BASE DESCRIPTION

Data for the Government of Canada interest rate and bond stock data were derived from the the data base described by Boothe and Reid (1986). The Government of Canada bond market is aggregated into three maturity classes, shorts (S, zero to six years to maturity), mediums (M, seven to twelve years to maturity) and longs (L, thirteen years and over to maturity). Estimates for the representative Government of Canada bond yields for these maturity classes were derived from an estimated par yield curve at the three year (S), ten year (M) and twenty year (L) maturity points. The Government of Canada TBill yield is the average yield at tender for the three month TBill. U.S. bond yield data are DRI's constant maturity Treasury yields at the five, ten and twenty years to maturity points. All data series are used in their log level form. Regarding data frequency, Hamburger and Silber (1971) have hypothesized that data frequency should be less than a quarter (e.g., monthly or weekly) in order to adequately identify supply effects in reduced form models. The current sample is monthly, from January 1972 to December 1984.

Theoretically, if debt stocks are discontinuous, correct aggregation of bond stock data into maturity groupings requires the solution to an errors in variables problem produced by the discrete crossing of issues between maturity classes (Boothe (1987), Friedman and Roley (1980), Roley (1982)). The method employed here to handle the maturity crossing problem is to utilize the distinction between active and passive stocks. The passive stock is equal to that stock which would have resulted in the absence of any Government or Bank of Canada purchases or sales (including new issues) between the current and last observation. The active change to bond stocks is equal to the change in publicly held bonds due to new issues or open market sales or purchases. The quantity of par value bonds in the i 'th maturity class equals the passive stock plus the active change to that maturity class. The total stock in each maturity class is proxied by the sum of the active change to the stock plus a three month moving average of the passive stock (where the moving average begins at the current month and ends two months prior). In this fashion the crossover from one maturity class to another is smoothed, but the variations in the total stock due to changes in the amounts held by the Bank of Canada and/or the government or due to new issues are unaffected.

APPENDIX B: PROOFS

This system is derived from the more general form:

$$\begin{vmatrix} W_{11}(L) & W_{12}(L) \\ 0 & W_{22}(L) \end{vmatrix} \begin{vmatrix} x_{1t} \\ x_{2t} \end{vmatrix} = \begin{vmatrix} W_{11}^*(L) & 0 \\ 0 & W_{22}^*(L) \end{vmatrix} \begin{vmatrix} e_{1t} \\ e_{2t} \end{vmatrix}$$

Inverting the matrix associated with the x 's gives (1). Exogeneity of x_{2t} is represented by the zero restrictions imposed on the left hand side matrix of polynomial lag operators.

The lag operator L^i is defined as implying a lag of i length. The lag process $A(L)$ is the polynomial sequence $A_0 - A_1L - A_2L^2 - \dots - A_qL^q$, where q is non-negative and A_i is a $k \times k$ matrix of coefficients. Further, the imposition of identification restrictions on $A(L)$ is not necessary to specifying the model. Rather, identification restrictions are necessary for interpreting the model's results, Hannan (1969).

Solving for x_{1t} and inverting the moving average process yields:

$$\begin{aligned} (2) \quad x_{1t} &= A_{12}(L)A_{22}(L)^{-1}x_{2t} + A_{11}(L)e_{1t} \\ &= A_{12}(L)A_{22}(L)^{-1}x_{2t} + (I - A^*(L))A_{11,0}e_{1t} \\ &= (I - A^*(L))^{-1}A_{12}(L)A_{22}(L)^{-1}x_{2t} + B(L)x_{1,t-1} + A_{11,0}e_{1t} \\ &= D(L)x_{2t} + B(L)x_{1,t-1} + v_{1t} \end{aligned}$$

where $A_{11,0}$ is the appropriately dimensioned part of A_0 corresponding to e_{1t} and $A^*(L)$ is defined as $(A_{11,1}L + A_{11,2}L^2 + \dots)A_{11,0}^{-1}$, and $B(L)$ is defined as $I - (I - A^*(L))^{-1}$ from the inversion in (2) above.

These exogenous variables must obey certain technical restrictions. Sufficient conditions for deriving the asymptotic properties of the ARMAX model require that (Hannan, Dunsmuir and Diestler (1980), Poskitt and Tremayne (1984)): i) the predetermined variables must be independent of the error process (the innovations); ii) the predetermined variables must have covariance functions which are well-defined and jointly stationary in the limit; and iii) the means of the exogenous variables are well-defined in the limit.

Table 1

Model Specification Tests

Model	Lag Length			
	r	b	r^{tb}	r^*
1)	1-5, 12	1-5, 12	0-5, 12	0-2
2)	1-4, 12	1-4, 12	0-4, 12	0-2
3)	1-3, 12	1-3, 12	0-3, 12	0-2
4)	1-2, 12	1-2, 12	0-2, 12	0-2
5)	1-3	1-3	0-3	0-2
6)	1-3, 12	1-3, 12	0-3, 12	-
7)	1-3, 12	1-3, 12	0-3, 12	0-1
8)	1-3, 12	1-3, 12	0-3, 12	0-3

Likelihood Ratio Tests

Sample: 1972-1 to 1984-12

Hypothesis	Chi-Squared Statistic **	Significance Level
2) preferred to 1)	31.71 (42)	0.88*
3) preferred to 2)	41.41 (42)	0.50*
4) preferred to 3)	64.57 (42)	0.01
5) preferred to 3)	58.00 (42)	0.05
6) preferred to 3)	141.96 (54)	0.00
7) preferred to 3)	41.47 (18)	0.00
3) preferred to 8)	24.19 (18)	0.15*

* Accepted at the 90% level of confidence. A significance level of p% indicates the probability that the test statistic is equal to or greater than the calculated value under the null hypothesis.

** Degrees of freedom, given in brackets, determined by the total number of system restrictions imposed.

r = bond yields b = debt shares r^{tb} = TBill yield r^* = US bond yields

Table 2

Q Statistics*

Sample: 1972-1 to 1984-12

<u>Model**</u>	<u>Equation</u>					
	<u>r^s</u>	<u>r^m</u>	<u>r^l</u>	<u>b^s</u>	<u>b^m</u>	<u>b^l</u>
3 (33)	41.3 (15.1)	24.3 (86.4)	33.5 (44.2)	30.1 (61.2)	31.6 (53.8)	39.9 (19.0)
4 (33)	41.4 (14.8)	26.7 (77.3)	33.1 (46.1)	41.9 (13.7)	34.1 (41.4)	43.0 (11.5)
2 (33)	44.9 (8.1)	34.8 (38.0)	44.4 (8.9)	31.3 (55.0)	33.9 (42.4)	37.2 (28.2)

*Q-statistics as defined Ljung and Box (1978). Marginal significance in brackets.

** Number in brackets is degrees of freedom. Degrees of freedom are determined by $\min\{T/2, 3(T^{1/2})\}$.

r^s = short bond yield r^m = medium yield r^l = long yield
 b^s = short debt share b^m = medium share b^l = long share

Table 3
Cross Correlations Between
Stock and Rate Equation Errors

Sample: 1972-1 to 1984-12 Model 3

<u>rates\shares</u>	<u>short</u>	<u>medium</u>	<u>long</u>	<u>$\text{Chi}^2(3)$</u>
<u>short</u>	-0.024	-0.026	0.206*	6.16
<u>medium</u>	-0.073	0.018	0.170*	4.87
<u>long</u>	-0.082	0.076	0.062	2.30
<u>$\text{Chi}^2(3)$</u>	1.03	0.91	10.60*	

* Significantly different from zero at the 95% level of confidence for the individual correlations and the 90% level for the row and column sums of the squared correlations (the chi-squared tests).

Table 4

F Tests for the Significance of
the Lagged Coefficients

Sample: 1972-1 to 1984-12 Model 3

<u>Equation</u>	<u>Variable</u>					
	<u>r^s</u>	<u>r^m</u>	<u>r^l</u>	<u>b^s</u>	<u>b^m</u>	<u>b^l</u>
r^s	9.84**	1.98**	1.39	1.37	1.26	0.47
r^m	5.11**	3.80**	2.68**	0.47	0.61	1.02
r^l	2.25**	3.63**	3.70**	0.27	0.31	1.07
b^s	1.12	0.65	0.67	170.80**	0.25	0.80
b^m	1.08	0.72	0.76	0.92	46.84**	0.47
b^l	1.58	1.18	0.63	0.71	2.70**	54.35**

** Significant at the 90% confidence level.

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