

BAYES' THEOREM

Judge Kavanaugh vs. Prof. Blasey Ford

The issue is concerned with the conditional probability $\text{Prob}[G | B]$ where the event G , the sexual assault of Ford by Kavanaugh, took place, given that Ford said it took place.

It follows that $\text{Prob}[B | G]$ concerns the truthfulness of Ford, i.e., the probability that Ford gave testimony that an assault took place, given that an assault did take place. Similarly, $\text{Prob}[B | \sim G]$ is the probability that Ford was either mistaken or lying. Note: $\text{Prob}[B | G] = 1 - \text{Prob}[B | \sim G]$

Analysis

$\text{Prob}[G] = \text{probability of a sexual assault} = \text{Prob}[\sim G] = 0.5$ (Complete uncertainty about outcome)

$\text{Prob}[B | G] = .8$ (80% chance of Ford being truthful) $\text{Prob}[B | \sim G] = .2$ (Mistaken or lying)

$\text{Prob}[G | B] = \{(.5)(0.8)\} / [\{(.5)(0.8)\} + \{(.5)(.2)\}] = 0.8$

Observe when outcomes are equally uncertain – a “diffuse prior” – the probability of the outcome is fully determined by the “likelihood” of Ford being truthful.

Say there is some other “prior” information: 1) appearance that Kavanaugh is not being truthful in testimony; 2) claims by other individuals of similar behavior in other situations leading to:

$\text{Prob}[G] = .6$ and $\text{Prob}[\sim G] = .4$

$\text{Prob}[G | B] = \{(.6)(0.8)\} / [\{(.6)(0.8)\} + \{(.4)(.2)\}] = 86\%$

The Party Example Did Kavanaugh commit a sexual assault at the party?

Having discovered that Ford attended the party, what is the probability the Kavanaugh committed a sexual assault at the party?

$\text{Prob}[A] = \text{Prob}[\sim A] = \text{Probability that Kavanaugh attended the party} = .5$

$\text{Prob}[S | A] = .9$ Probability of a Ford sexual assault if Kavanaugh was at the party

$\text{Prob}[S | \sim A] = .5$ Probability of a Ford sexual assault if Kavanaugh not at the party (she was assaulted but mistaken about who it was)

$\text{Prob}[A | S] = \text{Probability that Kavanaugh attended the party and committed the assault}$
 $= \{(.5)(.9)\} / [\{(.5)(.9)\} + \{(.5)(.5)\}] = 64\%$