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Hedging and Crop Insurance

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Traditional treatments of the optimal hedging problem, e.g., Rolfo (1988), Cecchetti, et.al. (1988), Myers (1991), ignore the possibility that the optimizing trader can use other types of hedging instruments than futures. In cases where the precise amount to be hedged is uncertain when the hedge is initiated, this distinction is particularly important. In practice, this can occur in the "farmer's hedging problem" where the quantity to be hedged at planting time must be determined based on the (uncertain) potential crop at harvest time. Hence, the farmer must hedge against both price and quantity uncertainty.[1] To do this, farmers have a number of practical alternatives. In addition to being able to hedge with derivative securities such as futures contracts, farmer's also have access to (possibly subsidized) crop insurance plans. Given this background, the primary objective of this paper is to provide solutions to various specifications of the farmer's optimal hedging problem when both futures and crop insurance are available to hedge sources of uncertainty.

In the following, Section I provides an overview of previous approaches to the farmer's hedging problem. A brief discussion of crop insurance is also provided. Section II examines the specification and solution of the farmer's optimal hedging problem without crop insurance. Starting from these initial results, Section III introduces crop insurance into the problem specification. This involves reformulating the terminal wealth function associated with the underlying optimization. Section IV evaluates the impact of crop insurance on farmer hedging behaviour. Among other things, it is demonstrated that with both price and quantity uncertainty, futures hedging activity depends fundamentally on the type of crop insurance provided. In certain cases, the fraction of total planted acres insured will be independent of the scale of the investment in crop production. Finally, Section V summarizes the contents of the paper.

Section I: <u>Background</u>

Despite the obvious practical connection, analysis of the farmer's hedging problem has traditionally omitted the possibility of using crop insurance.[2] This practical omission is often compounded by assuming the size of the position to be hedged is nonstochastic, i.e., the only

source of uncertainty is price variability (e.g., Danthine 1978, Holthausen 1979, Feder et.al. 1980, Lapan, et.al. 1991). Even in cases where both price and output uncertainty are admitted (e.g., Rolfo 1980, Grant 1985, Karp 1987), optimal hedge solutions are limited in a number of ways, e.g., there is often an implied market incompleteness: while two sources of risk are present, only one derivative security is provided to hedge the risk. In this vein, Myers (1988) demonstrates that market completeness will affect the real supply response of farmers, i.e., with two sources of uncertainty both futures contracts and crop insurance are required to complete the market. By implication, by adding crop insurance to the hedging problem, the resulting market completeness will change the associated hedging optimality conditions.

In addition to market completeness, various practical and analytical issues must be addressed in specifying an adequately descriptive solution of the farmer's risk management problem. An analytical example arises with the amount of crop to plant which can either be a choice variable or set exogenously. In turn, when planting is a choice variable, the extent to which the production and hedging decisions are "separable" is often of interest. While in certain cases, uncertainty only affects the level of hedging and not the production decision, e.g., Danthine (1978), Holthausen (1979), this result will not hold in general, e.g., Paroush and Wolf (1989), Lapan et.al. (1991). Another analytical concern is the conditions under which the optimal hedge ratio is independent of risk preferences. While independence can be established with one source of uncertainty and unbiased expectations, e.g., Heaney and Poitras (1991), it is not known whether this result extends to, say, the case where two sources of uncertainty are present. On balance, the complexity of the problem defies a general solution.

Given that it is necessary to abstract from some features of the farmer's optimal risk management decision, Wolf (1987) uses a mean-variance approach to consider the optimal hedging problem using options, ignoring the production decision. Lapan et.al. (1991) maintain that, because options truncate the probability distribution for prices (violating normality and monotonicity for profit or wealth), mean-variance will not necessarily provide an accurate representation of expected utility. Using a general expected utility function, Lapan et.al. go on to model the farmer's production decision allowing both futures and put options to hedge a single

source of diversifiable uncertainty, i.e., price risk. Assuming both that options are "fairly priced" and that futures prices are unbiased and linearly related to cash prices, Lapan, et.al. demonstrate that when both options and futures are available to hedge price risk, only futures will be used to hedge. In effect, futures make options a redundant hedging instrument.[3]

In the present context, the Lapan et.al. result is relevant because put options represent one potential form of crop insurance. In general, three kinds of insurance schemes are possible:

   A. Quantity Insurance: where the physical yield is restricted from falling below some minimum amount, usually set as some percentage of historical yields. This case is consistent with many traditional crop insurance plans.

   B. Price Insurance: where the crop delivery price is restricted from falling below a minimum amount. This type of insurance can be accomplished using put options.

   C. Mixed or Revenue Insurance: where the total revenue is restricted from falling below a minimum amount; this case is consistent with farm income stabilization and, to a lesser extent, disaster relief programs.[4]

In practice, federally administered crop insurance in the US is effectively quantity-based (A) insurance. Disaster relief programs are also available. In Canada, all three types of insurance have been offered as alternatives under the recently introduced GRIP program. Various schemes are offered in other countries (e.g., Hazell, et. al. 1986).

   While there are presently no studies dealing directly with solving the farmer's optimal risk management decision in the presence of both crop insurance and derivative securities, there has been some related work. For example, the issue of how crop insurance premiums should be priced is discussed in an insurance context in Ramaswami and Roe (1989) and in an option pricing context in Turvey and Amanor-Boadu (1989). In addition to numerous practical government publications on the types of insurance and their uses (e.g., FCIC 1989), Nelson and Loehman (1987) examine the economic role of crop insurance and assess the rationales for subsidization.[5] While there is some debate about the most appropriate method of implementation, (subsidized) crop insurance is found to be desirable where increasing food supplies is the objective, e.g., in the developing countries. Finally, King and Oamek (1983)

assess the impact of a specific type of crop insurance on farm risk management while, as noted previously, Myers (1988) evaluates the effect that completing the market with crop insurance and futures hedging has on farmer supply response.

Section II: <u>The Basic Model</u>

The basic model is discrete. Farmers have access to a variety of possible risk management instruments to hedge production decisions. The representative farmer plants a crop at time t and harvests it at time t+1. Both the price at harvest and the quantity harvested are unknown at time t, the date the relevant risk management and planting decisions are initiated. As conceived here, in addition to choosing the usage of hedging instrument(s), the farmer's optimization problem also involves choosing the amount of initial wealth to invest in crop production, i.e., the production decision is treated in a portfolio context. As a result, the costs associated with planting the given acreage are also determined. Starting from a given initial level of wealth, the farmer's objective is to maximize the value of terminal wealth assuming that the balance (possibly negative) of initial wealth which is not allocated to planting costs will earn (pay) the riskfree rate of interest.

Given this basic structure, it will initially be assumed that the only hedging instrument available is futures contracts. In this case, the underlying wealth dynamics can be specified:

$$W_{t+1} = A\, Y_{t+1}P_{t+1} + [W_t - C(A)]\,(1+r) + Q_f\,(f_{t+1} - f_t) \qquad (1)$$

where: W_{t+1} is wealth at time t+1 and W_t is the known level of initial wealth; A is the number of acres planted; Y_{t+1} is the random yield per acre observed when the crop is harvested at t+1; P_{t+1} is the random spot price at t+1; C(A) is the known cost function associated with planting the A acres; r is the riskfree interest rate; Q_f is the quantity of futures contracts sold (-) or bought (+); and f_{t+1} and f_t are the futures prices observed at t+1 and t respectively. Manipulation of (1) gives:

$$W_{t+1} = W_t\,(x(1+R) + (1-x)(1+r) + HR_f) \qquad (2)$$

$$= W_t\,((1+r) + x(R-r) + HR_f)$$

$$= W_t + \pi_{t+1}$$

where: π_{t+1} is the profit defined by (1) realised at time t+1, x is $(C(A)/W_t)$ the fraction of initial

wealth invested in the crop production, H is the value (f_t times Q_f) of the hedge position divided by initial wealth (<u>not the value of the spot position</u>), R_f is $(f_{t+1} - f_t)/f_t$ and $(1+R)$ is $[(A\ Y_{t+1}P_{t+1})/C(A)]$ one plus the rate of return on planting.

  Given this, the farmer's optimal risk management decision problem is to choose x and H such that the expected utility of terminal wealth is maximized. The decision problem is modelled with a general expected utility function. However, in order to achieve analytically concise results, joint normality of R and R_f is invoked. This leads to the following:

<u>Proposition I</u>: The Crop Investment and Hedging Decision

  Assuming that the returns R and R_f are jointly normal random variables, and that the farmer chooses x and H so as to maximize the expected utility of terminal wealth given by (2) then:

$$H* = \{-\frac{E[U']}{(E[U''] \ W_t)(1-\rho)^2}\}\{\frac{E[R_f]}{\sigma^2_f} - \rho\frac{E[R]-r}{\sigma_R\sigma_f}\} \qquad (3)$$

$$x* = \{-\frac{E[U']}{(E[U''] \ W_t)(1-\rho)^2}\}\{\frac{E[R]-r}{\sigma^2_R} - \rho\frac{E[R_f]}{\sigma_R\sigma_f}\} \qquad (4)$$

where:

$$\rho = \frac{\sigma_{Rf}}{\sigma_R\sigma_f}$$

$$\sigma_{Rf} = Cov(R, R_f), \ \sigma^2_f = Var(R_f)$$

$$\sigma^2_R = Var(R)$$

and U is the farmer's utility function for wealth (U'>0, U"<0).

Significantly, while Proposition I reveals that the individual optimal solutions (denoted by *) to the farmer's risk management problem (x*, H*) depend on preferences, the ratio (H*/x*) <u>only involves statistical parameters</u>.

  The portfolio-theoretic intuition behind the Proposition is as follows: the farmer faces two problems, one involving hedging, the other involving the scale of production. To determine the fraction of the crop to hedge, the farmer must solve a portfolio problem involving two risky "assets" with returns (R - r) and R_f. From mean-variance portfolio theory, it is well known that if asset returns are jointly normal and riskless borrowing and lending is permitted then all investors, regardless of preferences, hold the same portfolio of risky assets. In addition, the ratio

of any two assets in an optimal portfolio will be independent of risk preferences. Since the farmer's choice of the fraction of initial wealth to invest in crop production (x) is unconstrained, as long as returns are independent of the scale of production-- and the other assumptions relevant to Proposition I are satisfied-- (H*/x*) will not involve preferences.

When used to analyze (H*/x*), the practical implication of Proposition I is that the fraction of the investment in crop production to be hedged ($Q_f f_t / C(A)$) is independent of the size of the crop. Further, when the futures price is unbiased ($E[R_f] = 0$), only joint normality is required to motivate ordinary least squares as the optimal hedge ratio estimation technique. Though similar types of conditions have been derived for related problems, e.g., Benninga, et.al. (1984), Heaney and Poitras (1991), this result has not been recognized as applying to the farmer's hedging problem with two sources of uncertainty. (While a similar conclusion can be reached by manipulating the results in Grant (1985), this was not done). On balance, Proposition I is significant because it establishes the connection between the results of portfolio theory and the farmer's hedging problem.

Section III. Introducing Crop Insurance[6]

In the absence of crop insurance, the farmer's terminal wealth function with hedging is given in (2). To see the implications of admitting insurance, it is necessary to derive the terminal wealth functions for the price, yield and revenue forms of crop insurance. For example, if in addition to hedging with futures the farmer is assumed to buy revenue insurance against the full value of the crop ($AP_{t+1}Y_{t+1}$), the terminal wealth function can be specified:

$$W_{t+1}^i = W_t \{(1+r) + x[\max\{\underline{R}, R\} - s - r] - HR_f\}$$

$$= W_t \{(1+r) + x[(R - r) + \max\{0, \underline{R}-R\} - s] - HR_f\}$$

$$= W_t + \pi_{t+1} + W_t \{x[\max\{0, \underline{R}-R\} - s]\} \qquad (5)$$

where: s equals (S A)/C(A) with S being the price (insurance premium) per acre for the revenue insurance and $\underline{R}$ is the income floor specified in the insurance plan. It can be seen from (5) that the effect of adding revenue insurance to the risk management problem is to increase terminal wealth by x times the purchase price adjusted payout on a "put option" written on the return R,

with exercise "price" R.

While the terminal wealth functions for the other forms of crop insurance (price and yield) follow appropriately, some motivation is required. In particular, in the absence of crop insurance and hedging, there is a natural minimum on R. Either a complete crop loss where $Y_{t+1}=0$, or a spot price of zero at time t+1 corresponds to the case (1+R)=0. Significantly, unlike revenue insurance, neither yield insurance nor price insurance by itself can guarantee a higher minimum return when (1+R) equals zero. For example, price insurance guaranteeing $K a bushel ($P_{t+1} >$ K) cannot prevent a 100% crop loss due to drought, nor can quantity insurance providing for, say, $\underline{Y}$ bushels an acre ($Y_{t+1} > \underline{Y}$) prevent the future spot price falling to zero. However, both price and quantity insurance do reduce the probability of the total return attaining low values and, as a result, alter the distribution for terminal wealth. As it turns out, there are substantive differences in how price and yield insurance accomplish this result.

Examining the price insurance case involves introducing put options (written on the futures price). This leads to:[7]

$$
\begin{aligned}
W_{t+1}^{z} &= AY_{t+1}P_{t+1} + (W_t - C(A))(1+r) \\
&\quad + Q_f(f_{t+1} - f_t) + Q_z(\max[0, K-f_{t+1}] - z) \\[1em]
&= W_t\{x(1+R) + (1-x)(1+r) \\
&\quad + HR_f + \frac{Q_z f_t}{W_t}[\max[0, \frac{K-f_{t+1}}{f_t}] - \frac{z}{f_t}]\} \qquad (6) \\[1em]
&= W_t\{x(1+R) + (1-x)(1+r) \\
&\quad + H R_f + \gamma(\max[0, -R_f] - \frac{z}{f_t})\} \qquad (7)
\end{aligned}
$$

where K is exercise price on the put option which in going from (6) to (7) is assumed to be "at the money" (i.e., $K = P_t$), z is the price per unit of output of the put, Q_z is the number (in output units) of puts purchased, with the ratio γ being the value of the option position divided by initial wealth.[8] This specification can be contrasted with that for physical yield insurance where, instead of the number of options to purchase, it is the number of acres to insure which is the decision variable:

$$W^Y_{t+1} = AY_{t+1}P_{t+1} + (W_t - C(A))(1+r)$$

$$+ Q_f(f_{t+1} - f_t) + Q_y(P_{t+1}\max[0,\underline{Y}-Y_{t+1}] - L) \qquad (8)$$

where L is the price (insurance premium) per acre for the crop insurance, Q_y is the number of planted acres covered by physical yield insurance and $\underline{Y}$ is the yield floor provided by the insurance plan.

In practice, $\underline{Y}$ is set based on a percentage (<100%) of relevant historical physical yield averages. While the price used would actually depend on a specific method of price election selected by the farmer, taking the price elected to be the harvest period cash price (P_{t+1}) is not unrealistic (e.g., FCIC 1989). Assuming that $Q_y = A$ in (8), i.e., all planted acres are insured, leads to the following:

$$W^Y_{t+1} = W_t\{(1+r) + x[(R-r)$$

$$+ \max[0, \frac{P_{t+1}\overline{Y}A - P_{t+1}Y_{t+1}A}{C(A)}] - l] + HR_f\} \qquad (9)$$

where l equals (LA/C(A)). Observing that the expression inside the max function involves the difference between two random variables illustrates the primary analytical difference between (9) and (5). This distinction depends crucially on assuming that both price and quantity are uncertain. Observing that it may be unrealistic to assume that all acreage is insured, allowing the farmer to choose the acreage insured in (5) and (9) leads to:

$$W^i_{t+1} = W_t\{(1+r) + x[R-r] + HR_f + x\theta\{\max[0,\underline{R}-R] - s\}\} \qquad (5')$$

$$W^Y_{t+1} = W_t\{(1+r) + x[R-r] + HR_f + x\lambda[\max[0,\underline{RR}-R] - l]\} \qquad (9')$$

where θ and λ are the fractions of the total planted acreage insured under the revenue and physical yield crop insurance schemes and $\underline{RR} = \{P_{t+1}\underline{Y}A\}/C(A)$.

Given these terminal wealth functions for the three general types of crop insurance schemes, it remains to derive the associated optimality conditions and compare these results with those in Proposition I to assess the impact that introducing crop insurance has on both on x and H. Unfortunately, the random variables induced by the presence of crop insurance bring into question the normality assumption upon which the derivation of the Proposition I results

depended. In general, this prevents the Stein-Rubinstein lemma from being applied. However, similar solutions can still be obtained by assuming a mean-variance expected utility function. This approach is used with the caveat that this framework suppresses consideration of certain features associated with the distributional shape of the relevant random variables.[9]

To see why a specific functional form is required, consider the price insurance case. Given that this risk management instrument is consistent with a market-traded put option, it is feasible to assume that the option is fairly priced ($z = E\{max[0, K - f_{t+1}]\}$). It is also possible to further simplify by assuming that the futures price is an unbiased predictor, i.e., $E(R_f) = 0$. With a general expected utility function defined over terminal wealth as given in (7), the following first order conditions apply:

$$E[U'(R-r)] = E[U'] \ E[R-r] + cov[U',R] = 0$$

$$E[U'R_f] = E[U']E[R_f] + cov[U',R_f] = 0$$

$$E[U'max[0,-R_f] - \frac{z}{f_t}] = E[U']E[max[0,-R_f] - \frac{z}{f_t}]$$
$$+ cov[U',max[0,-R_f] = 0$$

Examining these results, it is apparent that even though the assumptions of fair pricing and unbiasedness can be used to simplify the last two first order conditions, it is still not possible to derive a closed form solution. In effect, given that the distribution of max[·] is decidedly non-normal, unless trivial or decidedly unrealistic assumptions are made there is no available method for solving the covariance terms.

In a similar problem with only price and (nondiversifiable) futures basis risk uncertainty, Lapan, et.al. 1991 are able to solve the covariance terms by assuming futures and cash prices are linearly related and invoking the global second order condition. The upshot of their results is to show $\gamma^* = 0$. The intuition underlying this result is that, with only one (diversifiable) source of uncertainty affecting the production activity, futures contracts are able to hedge away this risk more effectively than options. In effect, once the market has been completed with the introduction of futures, there is no residual risk to left to hedge with options. However, in the present case, the addition of yield uncertainty prevents market completeness from being obtained.

Hence, it is necessary to resort to specific form for the expected utility function to obtain sharper

results.

Section IV. <u>The Effect of Crop Insurance on Hedging</u>

The results in the section require assuming that: the farmer optimizes an expected utility

function of the form: $EU = E[W_{t+1}] - b \, var[W_{t+1}]$, where b is a measure of the sensitivity of EU

to changes in $var[\cdot]$; that all options and insurance premiums are "fairly priced"; that futures (not

cash) prices are used to specify the price insurance option as in (7); and that futures prices are

unbiased predictors. Given this, then the following applies:[10]

<u>Proposition II</u>: Hedging with Price Insurance

Assuming that the farmer optimizes an expected utility function defined over the mean and
variance of terminal wealth as given by (7), then the optimal values for the futures and options
positions are:

$$H* = \frac{x*}{1-\rho^2_{fz}} \left\{ \frac{\sigma_{Rz}}{\sigma_{fz}} \rho^2_{fz} - \frac{\sigma_{Rf}}{\sigma^2_f} \right\} \qquad (10)$$

$$Y* = \frac{x*}{1-\rho^2_{fz}} \left\{ \frac{\sigma_{Rf}}{\sigma_{fz}} \rho^2_{fz} - \frac{\sigma_{Rz}}{\sigma^2_z} \right\} \qquad (11)$$

where:

$$\rho_{fz} = \frac{cov\{R_f, \max[0, -R_f]\}}{\sigma_f \, \sigma_z} \qquad \sigma_{Rz} = cov\{R, \max[0, -R_f]\}$$

$$\sigma^2_z = var\{\max[0, -R_f]\} \qquad \sigma_{fz} = cov\{R_f, \max[0, -R_f]\}$$

Using (10) and (11), it is now possible to analyze the use of derivatives by evaluating the sign

and size of the relevant parameters. Unfortunately, given the nature of the random variables

involved, it is not practical to do this in a general fashion. However, it is possible to illustrate

the implications of Proposition II with examples.

To see this, assume that $f_t = \$2 = C(A)/A$ and that there are four, equally likely future states

of the world with the following associated outcomes:

STATE

| 1 | 2 | 3 | 4 |

Y_{t+1}	1.0	1.2	1.6	1.9
P_{t+1}	2.4	2.2	1.8	1.6

These assumptions are roughly consistent with the view that when there are bumper crops prices tend to be lower. In addition, when (partial) crop failures do occur (Y = 1.0) the impact on prices is not dramatic. Conversely, it is possible to have higher yields without substantial price deterioration. While other potential scenarios are possible, these assumptions are not unrealistic in an aggregate sense for certain types of crops, e.g., soybeans. In any event, these assumptions make for the most interesting analytical case.

These values for Y and P lead to the following outcomes for the random variables appearing in Proposition 2:[11]

STATE

	1	2	3	4
(1+R)	1.2	1.32	1.44	1.52
R_f	.2	.1	-.1	-.2
max[·]	0	0	.1	.2

Given these numbers, evaluating the parameters in Proposition II produces the counter-intuitive result that H* is positive (long futures) and γ* negative (sell puts).[12] With these specific parameters, H* would also be positive in Proposition I. Heuristically, this is a consequence of the assumed negative relationship between yields and prices. Instead of providing downside R protection, the conventional short futures hedge makes matters worse by requiring payouts at times when R is low. On the other hand, the put provides premium income when R is low at the expense of losses when R is high.[13]

By examining the effect of using yield (Y) instead of price insurance, it is possible to evaluate whether these unconventional results are changed by admitting market completeness, i.e., by providing one risk management instrument appropriate to handle each source of uncertainty. In this vein, there are two cases to consider: where the amount of yield insurance is constrained to be equal to total acres planted (9); and, where the fraction of acres to insure is a choice variable (9'). Using the same assumptions as those invoked to derive Proposition II, in the constrained

case:

<u>Proposition III</u>: Constrained Yield Insurance and Hedging

The mean-variance solution to the hedging and physical yield insurance case where the farmer is constrained to insure all planted acreage is:

$$H^* = -x^* \frac{\sigma_{if}}{\sigma_f^2}$$

$$x^* = \frac{E[R] - r}{2b\sigma_i^2} - H^* \frac{\sigma_{if}}{\sigma_i^2}$$

where $\sigma_{if} = \text{cov}\{R_f, \max[\underline{RR}, R]\}$ and $\sigma_i^2 = \text{var}\{\max[\underline{RR}, R]\}$. Comparing this case to Proposition I with $E[R_f] = 0$, the introduction of physical yield insurance has two offsetting effects on H: a direct reduction in the hedge ratio due to a reduction in covariance of crop and futures price returns and an indirect increase due to a higher amount of initial wealth invested in crop production arising from the reduction in the variance of the crop return.

More significantly, constraining the amount of physical yield insurance to cover the number of acres planted produces a change in the sign of H* using the values from the scenario considered previously. Assuming that the $\underline{Y}$ is set at the average Y over the states: $\underline{Y} = 1.4$, then the following values apply:

STATE

	1	2	3	4
max[<u>RR</u>, R]	.68	.54	.44	.52
R_f	.2	.1	-.1	-.2

This tableau is consistent with the notion that "farmers pray for hail", i.e., the best outcomes occur when crops fail due to substantial crop insurance receipts. Given this max function, the introduction of insurance is sufficient to change the sign of H* because σ_{if} is now positive (short futures hedge). However, given that, in practice, $\underline{Y}$ is set at some percentage of historical Y (< 100% of the average), this outcome will not always occur. In other words, unless the yield floor is set high enough, insurance payouts may not be large enough to prevent the lowest returns occurring when there is increasing prices and, hence, σ_{if} negative and H* positive.

With this in mind, consider the solution to the yield insurance problem, invoking the same

assumptions as in Propositions II and III, where both the fraction of total acreage to insure and

the amount to hedge have to be determined:

Proposition IV: Physical Yield Insurance and Hedging

Assuming that the farmer optimizes an expected utility function defined over the mean and variance of terminal wealth as given by (9'), then the optimal values for the futures and physical yield insurance positions are:

$$H^* = -\frac{x^*}{1 - \rho_{RRf}}\{\frac{\sigma_{Rf}}{\sigma_f^2} - \rho_{RRf}^2 \frac{\sigma_{RRR}}{\sigma_{RRf}}\}$$

$$\lambda^* = -\frac{1}{1 - \rho_{RRf}}\{\frac{\sigma_{RRR}}{\sigma_{RR}^2} - \rho_{RRf}^2 \frac{\sigma_{Rf}}{\sigma_{RRf}}\}$$

where: $\sigma_{RRR} = \text{cov}\{R, \max[0, \underline{RR}]\}$, $\sigma_{RRf} = \text{cov}\{R_f, \max[0, \underline{RR}]$

Under the condition that $\underline{Y}$ is the average over future states, evaluation of optimality conditions

in Proposition IV using parameters associated with the previous scenario reveals that the sign of

H* is now positive, in contrast to the (constrained) case where the all planted acreage is insured

and H* is negative. However, λ^* does have the appropriate (positive) sign and is independent

of x*. In other words, the optimal fraction of planted acres to insure does not depend on the

fraction of initial wealth invested in crop production.

Examining the specific values from the assumed scenario ($\underline{Y} = 1.4$), the reason H* is positive

when λ is a choice variable follows immediately. In this case, the insurance pays off in state 1

with amount (1.68) - 1.08 and state 2 with amount (1.54) - 1.32:

STATE

	1	2	3	4
(1+R)	1.2	1.32	1.44	1.52
R_f	.2	.1	-.1	-.2
max[·]	.6	.18	0	0

In effect, the max[·] function in the constrained case has been "unbundled". Given these values,

$\sigma_{RRR} = -.02605$, $\sigma_{RRf} = .0345$, $\sigma_{Rf} = -.019$, $\sigma_f^2 = .025$ and $\sigma_{RR}^2 = .06$. Evaluating λ^* and H*

reveals that virtually none of the crop would be insured and that a long futures position is

indicated. Insofar as the tableau numbers are realistic, these results are consistent with a

reluctance by farmers both to acquire crop (physical yield) insurance (even when partially subsidized) and to engage in futures hedging.

The constrained and unconstrained (H, λ) outcomes associated with the scenario featuring an inverse relationship between yields and prices raises a number of questions. Of interest in the present case is the extent to which the results are due to: the symmetry of the R and R_f distributions; the inverse relationship between yields and prices; and, the level of $\underline{Y}$. Consider the implications of dropping symmetry, e.g., by assuming that crop failure occurrences are particularly severe:

STATE

	1	2	3	4
Y_{t+1}	.5	1.2	1.6	1.9
P_{t+1}	2.4	2.2	1.8	1.6

Assuming that $\underline{Y} = 1.15$, the associated values of direct interest are:

STATE

	1	2	3	4
(1+R)	.6	1.32	1.44	1.52
R_f	.2	.1	-.1	-.2
max[·]	.89	0	0	0

Observing that the R distribution is definitely asymmetric, calculation of the relevant parameters gives: $\sigma_{\underline{R}RR} = -.13684$, $\sigma_{\underline{R}Rf} = .0445$, $\sigma_{Rf} = -.0485$, $\sigma_f^2 = .025$ and $\sigma_{\underline{R}R}^2 = .1485$. With these values, while the sign of H* is still positive, the fraction of total acres insured (λ*) has risen to .73.

Given the implications of dropping the symmetric distribution construction, consider a case where prices and yields have a slight positive relationship (while still retaining approximate symmetry of the relevant random variable distributions):

STATE

	1	2	3	4
Y_{t+1}	.8	1.2	1.6	1.9
P_{t+1}	1.8	2.4	1.6	2.2
1+R	.72	1.44	1.28	2.09

R_f	-.1	.2	-.2	.1

This scenario is consistent with farmers producing for an international market, e.g., wheat, where the domestic harvest conditions are not strongly related to pricing behaviour. Setting $\underline{Y}$ = 1.375, the average yield over the four states, leads to max[0, $\underline{RR}$ - R] = [.5175, .21, 0, 0]. Evaluation of parameters for this scenario reveals: $\sigma_{\underline{RR}R}$ = -.0827, $\sigma_{\underline{RR}f}$ = -.0115, σ_{Rf} = .04225, σ_f^2 = .025 and $\sigma_{\underline{RR}}^2$ = .04489. Observing that some of the parameters have changed sign, it follows that in this case H* is the negative (short hedge) while λ^* > 1 (over insurance).

Working through numbers appropriate for other scenarios reveals that: as $\underline{Y}$ (> Y_{min}) falls the amount of insurance increases. This is intuitive since a greater amount of insurance is required to protect against the unchanged risks. In addition, when prices and yields are unrelated (e.g., consider the last tableau with prices in states 1 and 4 equal to 2.0) then H* will be negative and λ^* will definitely be greater than zero. Consideration of the Proposition IV conditions reveals that $\sigma_{\underline{RR}f}$ will be zero (or near zero) in this case. Hence, the size of the hedge and insurance positions will be "decoupled" and depend on the direct variances and covariances. If there is limited price variability, fluctuation in R will originate largely from yield variation and crop insurance will be the most important risk management instrument. Conversely, if there is limited yield variation, then variation in R will originate from price fluctuation and hedging will be most important.

The final issue to consider is the change to the optimality conditions arising from the use of revenue insurance. Comparing the relevant profit functions reveals that the revenue and physical yield insurance cases differ only in how the max[·] function is specified. In physical yield insurance, the max function depends on random price behaviour while in revenue insurance the direct behaviour of R is the determinant. It follows that the mean-variance optimality conditions for the revenue insurance case will be virtually identical to those given in Proposition IV, allowing for the difference in how max function is specified (together with the resulting covariances and variances). Given this, insofar as yields as opposed to prices drive R, in the revenue insurance case cov[max{0, $\underline{R}$ - R}, R] will be higher than cov[max{0, $\underline{RR}$ - R}, R] with cov[max{0, $\underline{R}$ - R}, R_f] lower than cov[max{0, $\underline{RR}$ - R}, R_f]. However, while there will be a

general tendency toward higher fractions of the crop insured and lower levels of hedging with revenue insurance, specific outcomes will depend on the type of situation under consideration.

Section V: <u>Summary</u>

This paper has examined the implications of admitting both crop insurance and futures hedging into the farmer's risk management problem under the condition that both prices and yields are allowed to vary. It is demonstrated that making farm income the product of two random variables significantly changes the nature of the solutions to the (traditional) optimal hedging problem. Specifically, when the farmer is assumed to have access only to futures hedging, i.e., no crop insurance is available, it is demonstrated that joint normality of the rates of return to planting and futures prices is sufficient for the fraction of the crop position hedged to be independent of the scale of investment in crop production. In other words, the optimal amount of hedging can be determined on the basis of statistical parameters. If futures prices are assumed to be unbiased predictors, then further simplifications can be achieved.

For the case where farmers have access to both crop insurance and futures hedging, optimality conditions were derived for a farmer possessing a mean-variance expected utility function, assuming that insurance is fairly priced and that futures prices are unbiased. Specific examples were used to examine the implications of various types of yield and price outcomes for the optimal crop insurance and hedging positions. In the case where yields and prices are assumed to be positively related (with the R and R_f distributions symmetric), then farmers would go long futures and, in the case of physical yield insurance, may not insure. Various other cases were examined where, as expected, short futures positions were optimal and varying degrees of over and under insurance were identified. In general, the availability of crop insurance was found to have a significant impact on the nature of the optimal futures hedging solutions.

Appendix

Proof of Proposition I:

The farmer's problem is to maximize the expected utility of terminal wealth, $E[U[W_t + \pi_{t+1}]]$. Using equation (2) for the terminal wealth function this involves solving for H and x in the problem:

$$\max_{H,x} E\{W_{t+1}\} = \max_{H,x} E\{W_t[x(1+R) + (1-x)r + HR_f]$$

$$= \max_{H,x} E\{U[W_t + W_t((1+r) + x(R-r) + HR_f)]\}$$

where W_t is assumed constant. The first order conditions are:

$$H: \quad E[U'[\cdot]R_f] = 0$$

$$x: \quad E[U'[\cdot](R-r)] = 0$$

Using the definition of the covariance the first order conditions can be rewritten as:

$$E[U']E[R_f] + Cov[U',R_f] = 0$$

$$\quad (A1)$$

$$E[U']E[(R-r)] + Cov[U',R-r] = 0$$

The assumption of bivariate normality permits the use of the Stein-Rubinstein Lemma (Rubinstein 1976), i.e., if X and Y are bivariate Normal and f(X) is a function of X then under mild regularity conditions on f:

$$Cov[f(X),Y] = E[f'(X)]Cov[X,Y]$$

Applying this result to (A1) we obtain:

$$E[U']E[R_f] + E[U'']W_t Cov[((1+r)+x(R-r)+HR_f),R_f] = 0$$

$$\quad (A2)$$

$$E[U']E[R-r] + E[U'']W_t Cov[((1+r)+x(R-r)+HR_f),R-r] = 0$$

Solving the equations (A2) and using the definitions given in Proposition 1 gives:

$$cov[U',R_f] = E[U'']W_t (x\sigma_{Rf} + H\sigma_f^2)$$

$$cov[U',(R-r)] = E[U'']W_t (x\sigma_R^2 + H\sigma_{Rf})$$

Substituting in the covariances and manipulating gives:

$$x^* = -\{H^* \frac{\sigma_{Rf}}{\sigma_R^2} + \frac{E[U']}{E[U'']} \frac{E[R-r]}{W_t \ \sigma_R^2}\}$$

$$H^* = -\{x^* \frac{\sigma_{Rf}}{\sigma_f^2} + \frac{E[U']}{E[U'']} \frac{E[R_f]}{W_t \ \sigma_f^2}\}$$

Substituting x* and H* where appropriate and using the definition for ρ gives the solutions stated in the Proposition.

Proof of Proposition II:

Given the objective is to maximize the mean-variance expected utility of terminal wealth as specified in (7), using the assumed "fair pricing" of options and the unbiased prediction property of futures prices, this leads to the following mean and variance functions:

$$E[W_{t+1}^z] = W_t \ \{(1+r) \ + \ x \ E[R-r]\}$$

$$var[W_{t+1}^z] = x^2 \sigma_R^2 + H^2 \sigma_f^2 + \gamma^2 \sigma_M^2 + 2\{X\gamma\sigma_{RM} + Hx\sigma_{Rf} + H\gamma\sigma_{fM}\}$$

Using these functions, deriving the first order conditions for γ and H using the mean-variance expected utility function (EU = E[W$_{t+1}$^{z}] - b var[W$_{t+1}$^{z}]) and assuming the risk parameter b is not zero, gives the following two conditions:

$$\gamma^* = -\{x^* \frac{\sigma_{RM}}{\sigma_M^2} + H^* \frac{\sigma_{fM}}{\sigma_M^2}\}$$

$$H^* = -\{x^* \frac{\sigma_{RM}}{\sigma_f^2} + \gamma^* \frac{\sigma_{fM}}{\sigma_f^2}\}$$

Substituting in H* and γ* where appropriate, manipulating and using the definition for ρ provided in the Proposition gives the required results.

Proof of Proposition III:

As in the proof of Proposition II, the objective is to maximize the mean-variance expected utility of terminal wealth in this case as specified in (5), using the assumed "fair pricing" of

options and the unbiased prediction property of futures prices. This leads to the following mean and variance functions:

$$E[W_{t+1}^i] = W_t \{(1+r) + x\ E[R-r]\}$$

$$var[W_{t+1}^i] = x^2\sigma_i^2 + H^2\sigma_f^2 + 2Hx\sigma_{if}$$

where the variances and covariances are as defined in the Proposition. Evaluating the first order conditions for x and H using EU = $E[W_{t+1}^i]$ - b var$[W_{t+1}^i]$ and solving gives the results stated in the Proposition.

Proof of Proposition IV:

The proof is similar to that for Proposition II. Invoking the fair pricing and unbiased expectations assumptions, using (9') the expected value and variance of terminal wealth are evaluated as:

$$E[W_{t+1}^Y] = W_t \{(1+r) + xE[R-r]\}$$

$$var[W_{t+1}^Y] = x^2\sigma_R^2 + H^2\sigma_f^2 + \lambda^2 x^2\sigma_{\underline{RR}}^2 + 2\{HX\sigma_{\underline{Rf}} + H\lambda x\sigma_{\underline{RRf}} + \lambda x^2\sigma_{\underline{RRR}}\}$$

Evaluating the first order conditions for λ and H using EU = $E[W_{t+1}^y]$ - b var$[W_{t+1}^y]$, the first order conditions for H and λ are derived and solved for H* in terms of λ*x* and x* and for λ*x* in terms of H* and x*. Substituting for λ*x* and H* where appropriate, the optimality conditions in terms of x* alone are derived. The independence of λ* follows because when λ*x* is expressed solely as a function of x* alone, the x* cancels out.

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1. Given this, the "stylized" farmer being modelled is subject to the whims of weather, pests and plant disease. These problems are less typical of farming situations where the production functions are "controllable", e.g., feeder cattle.

2.   In a more practical context, Turvey (1989) and Turvey and Baker (1990) demonstrate that higher farm debt/equity ratios will increase the (optimal) use of derivatives. Hence, subsidized government farm loan programs and other initiatives which (indirectly) increase the farmers ability to borrow, e.g., marketing boards, may also impact the optimal risk management decision.

3.   In using a linear cash-futures price relationship, Lapan, et.al. are subject to much the same criticisms that they levelled at Wolf (1987).

4. Total revenue is the realized income from planting a given crop. Given that various types of farm income stabilization programs are possible, e.g., where payments are made prior to planting on the condition that farmers do not plant certain crops, the revenue insurance schemes being examined here are not fully descriptive of all possible plans.

5.   Hazell, et.al. provides a comprehensive international bibliography of government studies.

6.   In the following discussion, the problem of setting the deductible has been ignored. To account for this, the floor set by the insurance scheme can be viewed as net of the deductible, where applicable.

7. Given the ability to replicate call positions by combining puts and futures, little is gained by introducing a term for a call option. Similarly, while a straddle position would have an interesting random variable distribution, this would add little when futures are present.

8. It is also possible to specify the put option using cash prices. However, this significantly complicates the analysis. In addition, exchanged traded options are typically written using futures prices, so the present construction is potentially more realistic.

9.   Over time, there has been considerable debate on the appropriateness of mean-variance modelling. On balance, while mean-variance may suffer from specific analytical defects, there is considerable evidence that the approach can provide a good approximation to more theoretically acceptable results in many cases, e.g., Kroll, et.al. (1984).

10. In the following, if the option is not fairly priced it will probably be underpriced, due to government subsidies. In the present context, underpriced insurance will produce an increase in x^*. This result is well-known and does not add substantively to the following analysis. Similarly, if either futures prices

are biased predictors or price options are written on cash instead of futures prices, this introduces additional terms into the optimality conditions which only serve to qualify the results in ways which are not significantly revealing.

11. In keeping with the assumptions underlying Proposition I, the distribution for R_f is symmetric while the distribution for R is approximately symmetric. However, because R is a product of two random variables, changing the (P,Y) pairing will require adjustment of either Y or P in order to keep R symmetric.

12. The relevant parameters are σ_{Rz} = .00925, σ_{Rf} = -.019, σ_{fz} = -.010625, σ_f^2 = .025 and σ_z^2 = .006875. Observing that the ρ^2 term is .66 leads to H* = .5x* and γ* = .45x*.

13. Given the replication properties of options, the implied strategy of long futures and sell puts is similar to a strategy of buying calls.