DELTA NEUTRALITY FOR DIFFERENT OPTION STRATEGIES

BUY the Straddle:

$$V = C[X,T]+P[X,T] \qquad \frac{\partial V}{\partial S} = \Delta_V = \Delta_C + \Delta_P = \Delta_C + (\Delta_C - 1) = 0$$

$$\rightarrow \quad 2\Delta_C = 1 \quad \rightarrow \quad \Delta_C = .5$$

This happens when $N[0] = .5$ or $d_1 = 0$.

SELL the Straddle:

$$V = -(\ C[X,T]+P[X,T]\ ) \qquad \frac{\partial V}{\partial S} = \Delta_V = -(\ \Delta_C + \Delta_P\ ) = -(\Delta_C + (\Delta_C - 1) = 0$$

$$\rightarrow \quad 2\Delta_C = 1 \quad \rightarrow \quad \Delta_C = .5$$

BUY the Vertical Spread with Calls ($X_1 < X_2$):

$$V = C_1[X_1,T]-C_2[X_2,T] \qquad \frac{\partial V}{\partial S} = \Delta_V = \Delta_{C1} - \Delta_{C2} = 0 \quad \rightarrow \quad \Delta_{C1} = \Delta_{C2}$$

When will this happen?! (See Final 19-2 Q#4)

BUY the Butterfly ($X_1 < X_2 < X_3$):

$$V = C[X_1,T] - 2C[X_2,T] + C[X_3,T] \qquad \frac{\partial V}{\partial S} = \Delta_V = \Delta_{C1} - 2\Delta_{C2} + \Delta_{C3} = = 0$$

$$\rightarrow \quad \Delta_{C_1} + \Delta_{C_3} = 2\Delta_{C2}$$

Not necessary for the middle options to be at the money for the butterfly to be delta neutral.

BUY the Strap (2 Calls and a Put; same X):

$$V = 2C[X,T] + P[X,T] \qquad \frac{\partial V}{\partial S} = \Delta_V = 2\Delta_C + (\Delta_C - 1) = = 0$$

$$\rightarrow \quad 3\Delta_C = 1 \quad \rightarrow \quad \Delta_C = \frac{1}{3}$$

If the strap is constructed with $X_1 < X_2$, for the calls to be at the money (approx. $N[0] = .5$ or $d_1 = 0$), then delta neutrality requires $2\Delta_{C1} + (\Delta_{C2} - 1) = 0$ put has to be deep-deep-in the money ($\Delta_{C2} = 0 \quad \rightarrow \quad \Delta_{C2} - 1 = -1$).