BAYES RULE
For the Evidence of Testimony

The conditional probability (Prob[E | A]) in this case is that an event (E; a miracle) took place, given that the witness said it took place (A). Bayes formula in this case requires the prior probabilities, Prob[E] and Prob[~E], leading to the formula:

$$Prob[E \mid A] = \frac{Prob[E] \;\; Pr[A \mid E]}{(Prob[E] \;\; Prob[A \mid E]) + (Prob[\sim E] \;\; Prob[A \mid \sim E])}$$

Observe, Prob[A | E] -- that the witness said it took place and the event did take place -- is the probability that the witness is truthful; and Prob[A | ~E] = (1 - Prob[A | E]) is the probability the witness is not truthful.

MIRACLES Did the Christian miracles occur?

EXAMPLE 1

Prob[E] = Prob. of Miracle = .01 Prob[~E] = Prob. of no miracle = .99
Prob[A | E] = Prob. of truthful witness = .75 Prob[A | ~E] = .25

Prob[E | A] = {(.01)(.75)} / {(.01)(.75) + (.99)(.25)} = .0294

EXAMPLE 2

At one point, Price argues that Hume is incorrect to allow prior probabilities to impact the judgment of whether an event happened. In this case:

Prob[E] = Prob. of Miracle = Prob[~E] = Prob. of no miracle = .5
Prob[A | E] = Prob. of truthful witness = .99 Prob[A | ~E] = .01

Prob[E | A] = .99

EXAMPLE 3 Prob[E] = 1 - Prob[A | E]

Prob[E] = Prob. of Miracle = .01 Prob[~E] = Prob. of no miracle = .99
Prob[A | E] = Prob. of truthful witness = .99 Prob[A | ~E] = .01

Prob[E | A] = .5

GENERALIZING BAYES

Bayes Rule is more general than the miracle/no miracle two alternative case. When there are more than two mutually exclusive alternatives of which one must occur and none has zero probability, the denominator can be further expanded as:

$$Prob[A] = \sum_{i=1}^{k} Prob[B_i] \; prob[A \mid B_i]$$

This produces the following generalization:

$$Prob[B_j \mid A] = \frac{Prob[B_j] \; Prob[A \mid B_j]}{\sum_{i=1}^{k} Prob[B_i] \; Prob[A \mid B_i]} = \frac{Prob[B_j] \; Prob[A \mid B_j]}{Prob[A]}$$

PARTIES: Did Tom have a 'good time'?

Having discovered that Tom had a good time at the party (event F happened), what is the probability that his ex-wife did not attend the party (Prob[B1 | F])?

Prob[Tom's ex-wife goes to the Party] = Prob[Tom's ex-wife does not go to the Party] = .5
Prob[B2] = Prob[B1]

Note: This is .5 because there is no information about wife's attendance at the party, i.e., B2 = ~B1 and the probabilities will sum to one.

Prob.[Tom has a good time if wife goes to party] = .2 = Prob[F | B2]
Prob.[Tom has a good time if wife does not go to party] = .75 = Prob[F | B1]

Note: These two probabilities do not sum to one, i.e., it is Prob[F | B1] + Prob[~F | B1] = 1

Solution:
Prob[B1 | F] = {Prob[B1] Prob[F | B1]} / {(Prob[B1] Prob[F | B1]) + {Prob[B2] Prob[F | B2]}
= (.5)(.75) / {(.5)(.2) + (.5)(.75)} = 0.79

What if it is certain that Tom will not have a good time if ex-wife attends, i.e., Prob[F | B2] = 0? Observe that in this (trivial) case, it is also certain the ex-wife did not attend –> plug into Bayes theorem to verify.