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Mortgage Contract Design and Systemic Risk Immunization *

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ABSTRACT

This paper provides theoretical results for the design of contracts used in the market for residential household mortgages and mortgage securities. Critical elements in the problem of immunizing systemic risk through efficient contract design are identified. Using an extension of classical immunization theory, this paper demonstrates that systemic risk of long amortization mortgage contracts is reduced when term to maturity of the contract at origination is significantly less than the amortization period. In addition, incorporating prepayment and no recourse default options into the mortgage contract increases systemic risk when compared with full recourse mortgage contracts having yield maintenance prepayment penalties. The theoretical results are used to evaluate the systemic risk management problems that have plagued the US mortgage funding system.

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1. Introduction

The primary objective of this paper is to illustrate the implications of mortgage contract design for immunization of systemic interest rate and house price risks inherent in the residential mortgage funding system. The classical fixed income portfolio immunization model is extended to assess the implications of systemic interest rate and house price risk, e.g., Redington (1952), Reitano (1991a,b), Poitras (2007, 2013). It is demonstrated that shortening mortgage term to maturity and having a 'yield maintenance' prepayment penalty reduces the systemic risk inherent in the origination of long amortization period, single-family residential mortgages.[1] In addition to mitigating the difficulty of determining an actuarially sound fair market value at origination, shortening mortgage term to maturity strengthens adherence to underwriting standards by requiring borrowers (mortgagors) to periodically reaffirm both the equity value in the underlying asset and the source of household income required to service the mortgage. Even if the mortgage contract has no prepayment penalty and includes a no recourse default option, reducing mortgage term to maturity still significantly reduces the market value of these options when the mortgage is priced at origination, thereby reducing the systemic risk associated with the exercise of options that are unpriced or incorrectly priced.[2]

2. Mortgage Contract Design

The history of the mortgage contract stretches back to antiquity. Cuneiform tablets from the second millennium BC record debt-bondage contracts for consumption loans in ancient Mesopotamia that were structured with landed property as security. Much of mortgage contract history is concerned with: evolving legal interpretations of the contract, such as the remedies available to mortgagee and mortgagor in the event of default; and, how mortgage contract language can be structured to achieve a particular objective, such as including a power of sale clause to avoid costs of foreclosure for the mortgagee.[3] In the modern era, mortgage contract design varies substantively across countries and over time. These differences are the result of the unique evolution of the mortgage contract in each country. In particular, Campbell (2013, Fig.2) demonstrates that mortgage contract design in the US is anomalous compared to other countries in having a long amortization period with a fixed interest rate. This contract design feature is combined with other anomalous features: no prepayment penalties; and, in many state jurisdictions, limited recourse for deficiency claims. Consistent with fundamental support for individual freedom and distrust of concentrated financial power, the modern US mortgage contract is decidedly in favour of the mortgagor, though this has not always been the case.

The collapse of the mortgage funding system in the US brought on by the Great Depression was the result of a combination of factors, e.g., Rose (2011); VLR (1937); Fahey (1934). Whatever the causes, a fair estimate of the general collapse of residential houses prices was around a 50% decline. At the time, mortgage contract design called for short term to maturity mortgages, usually 3-10 years depending on the security of the borrower with loan-to-value ratios of 60% or less. Because the amortization period was usually much longer, 25 years or interest only being common, the unpaid principle was due on maturity (Jaffee and Quigley 2008, p.123).[4] Typically, this would be accomplished by taking another mortgage with the same lender. However, the collapse of house prices prevented this from happening as the value decrease of the underlying assets was so severe that the lenders could not fund such roll-over loans, even at much higher loan-to-value ratios. The consolidation of mortgage loans starting in 1933 under the Home Owners' Loan Corporation (part of the Federal Home Loan Bank System) combined with the home mortgage insurance under the National Housing Act (1934) led to the introduction of the conventional 30 year fixed rate, no prepayment penaltyUS mortgage contract that, more or less, has survived to the present.

Any practical discussion of issues associated with mortgage contract design would be incomplete without consideration of the seemingly incongruent regulatory and legal framework governing US mortgage origination. From the National Housing Act (1934) to the Community Reinvestment Act (1977) to the American Dream Downpayment Act (2003) to the Helping Families Save Their Homes Act (2009), the US federal government has actively promoted home ownership. Yet, various legislative initiatives aimed at achieving this end have often conflicted with the regulatory goal of maximizing economic efficiency. For example, consider the efficiency losses associated with Regulation Q. From the Banking Act (1933, Sec. 11) and the Glass-Steagall Act (1933) until passage of the Monetary Control Act (1980), Regulation Q was the cornerstone of a mortgage funding system that depended fundamentally on the thrift and S&L industry for origination of long term, fixed rate mortgages.[5] By prohibiting the payment of interest on demand deposits, Regulation Q provided a significant implicit subsidy to mortgage borrowers. The *de facto* collapse of Regulation Q in the face of inflation fuelled interest rate increases of the 1970's exposed the underlying systemic risk associated with the duration gap inherent in the balance sheets of the S&L's and other mortgage lenders.

While the originate-to-distribute, government sponsored enterprise (GSE)-based residential mortgage funding system has much earlier historical origins, passage of the FDICIA (1990) can be used to demarcate the transition from funding mortgages through the balance sheets of specialized depository institutions to funding through capital markets, e.g., Bentson and Kaufman (1997). Recognizing the underlying duration gap and credit risk problems confronting depository institutions funding mortgages, this transition sustained the development and growth of a range of primarily OTC cash and derivative products designed to "slice up" the cash flows from pools of conventional US mortgages underlying the GSE mortgage securities that were being traded, thereby mitigating risks and providing more efficient pricing.[6] A partial list of the 'innovative' financial engineering products include: credit default swaps and related synthetic derivatives; various exotic interest rate derivatives; and, rebundled mortgage pass-throughs producing tranche and Z-class CMO's. There were also variations on the conventional mortgage:sub-prime and Alt-A mortgage pools; and, alternative mortgage products such as interest-only and payment-option ARM's (GAO 2006). Following Hirtle (2009), Shin (2008, 2009) and others, it is now claimed that dispersion of default risk and duration gap risk through securitization does not necessarily enhance financial stability as financial engineering advocates have claimed. Instead, it is argued that these products directly contributed to a system wide increase in leverage that fuelled an expansion of bank balance sheets sustaining an overall reduction in mortgage lending standards.

Starting around mid-2005, the survival performance of single-family residential mortgages in the US deteriorated sharply, e.g., Sanders (2008); Mayer et al. (2009). For proponents of securitization such as Hayre et al. (2008) and Chen et al. (2010), this exposed weaknesses in methods for predicting mortgage defaults and estimating default loss severities used in the market valuation of collateralized mortgage products. However, while improvements in the data and models used to assess single-family mortgage default risk could partially mitigate problems for trading difficult-to-price securities, this paper examines the impact of conventional US mortgage contract design on the on-going failure of US mortgage markets to adequately originate long amortization period mortgages. Do specific US mortgage contract features combine in a systemically perverse fashion with 'originate-to-distribute' underwriting procedures used in the creation of collateralised mortgage obligations and related financing vehicles, e.g., Wilmarth (2009)?[7] Will improvement in pricing models permit accurate enough pricing of the interest rate and default options inherent in collateralized mortgage products to prevent future mortgage funding system failures, e.g., Kau et al. (1995); Deng and Gabriel (2006); Longstaff and Rajan (2008); Ozeki et al. (2009)? Or, alternatively, is a substantive change in mortgage contract design required?

3. What is Systemic Risk?

Despite numerous studies of systemic risk over many years, the situation described by Greenspan (1995) remains: "the very definition [of systemic risk] is still somewhat unsettled". More recently, FRBNY (2007, p.7) also observes: "Systemic risk in the financial system is difficult to define precisely". Kane (2010, p.251) accurately recognizes that "one must define [systemic risk] comprehensively and fashion from this definition one or more verifiable metrics for monitoring the target". Kane (2010, p.7-8) also finds "official definitions of system risk fail" to satisfy either of these requirements. "Official definitions focus on a perceived potential for substantial spillovers of institutional defaults across important firms in the financial sector and from this sector to the real economy." Bullard et al. (2009) provide an example of an 'official' definition: "Systemic risk refers to the possibility of a triggering event, such as the failure of an individual firm, will seriously impair other firms or markets and harm the broader economy." Similarly, Kupiec and Nickerson (2004) define systemic risk as: "the potential for a modest economic shock to induce substantial volatility in asset prices, significant reductions in corporate liquidity, potential bankruptcies and efficiency losses."

In contrast to general definitions, Saunders and Walter (2012, p39) provide a more targeted definition that identifies "financial firms that are 'systemic' in nature ... firms judged too big or too interconnected to be allowed to fail". In this case, systemic risk is associated with "what is gained and what is lost as a result of the available policy options" to deal with "the dominant role of systemically important financial institutions in the financial architecture." Similarly, Kane (2010, p.253) identifies: "The primary characteristic of systemic risk is the emergence of widespread concerns about the potential for substantial 'spillovers' of contagious defaults across counterparties in the financial sector and from these defaults to breakdowns in the real economy." Observing that actual 'spillovers' are limited due to government policy reactions, Kane (2010, p.253) proposes a precise metric for measuring systemic risk: "the capitalized value of the safety-net subsidies that financial firms capture represents a cogent way to measure what authorities ought to mean by 'systemic risk' and that regulation-induced innovation is the vehicle through which subsidies to systemic risk-taking are conveyed."

This paper adopts disparate elements from different definitions of systemic risk. Following Bullard et al. (2009, p.403) and many others, it is recognized that: "The financial crisis of 2008-09—the most severe since the 1930s—had its origins in the housing market." In contrast to most studies which focus on the financial system as a whole, the 'system' in this paper focuses only on the entities involved in the 'mortgage funding system'. No distinction is made between 'difficult to fail and unwind' (DFU) institutions and other mortgage system participants. Rather than assuming that the 'system' is inherently stable and that market failure is due to "defective risk management at DFU firms" and the 'undersupervision' of "innovative forms of risk taking at DFU firms" (Kane 2010, p.252), this paper takes the view that certain contractual features of the mortgage securities being issued and traded in capital markets can produce instability in the mortgage funding system that induces periodic crises in the greater financial system. From this perspective, reform proposals aiming to "toughen capital requirements, reconfigure the boundaries of regulation, and extend new powers to regulators" are underdeveloped and potentially counter-productive.

Many of the confusions about systemic risk in the mortgage funding system can be traced to inadequate attention to the definition of 'risk'. Bullard et al. (2009, p.403) hint at the issue in the US: "After several years of rapid growth and profitability, banks and other financial firms began to realize significant losses on their investments in home mortgages and related securities in the second half of 2007." The conventional view is to take an insurance perspective on 'risk' where outcomes involve either loss or no loss. Yet, **risk in financial markets is two-sided**. In general, accurate pricing of long term to maturity securities with embedded contingencies is complicated. In an 'originate to distribute' model of securitized mortgage origination, there is a compelling market incentive for issuers to underprice the contingencies at origination. The market discipline needed to prevent mortgagees from purchasing overpriced securitized loans is undermined by pricing conventions that provide false confidence the embedded contingencies have been accurately priced. In this context, significant reduction of 'systemic risk' can be achieved by altering mortgage contract terms to reduce the negative systemic impact of interest rate and house price changes and facilitate more accurate and efficient market pricing of the embedded contingencies.

Systemic risk from the mortgage funding system originates when the transfer of mortgage risks between households and mortgage lenders results in equity losses that exceed system equity available. To cover such losses, the mortgage funding system then needs to draw capital from elsewhere in the financial system creating 'systemic risk', i.e., the risk that equity losses from the mortgage funding system will spill over into the greater financial system with potentially catastrophic consequences. The ongoing debate over the causes and remedies of recent mortgage-driven US financial crises features a widespread perception that the risks inherent in the residential mortgage funding system can be mitigated, if not eliminated, by transferring the risks to capital markets, e.g., Green and Wachter (2005). Various potential gains from trade available from funding mortgages through capital markets are identified: enhanced diversification; correcting mismatches in the maturity composition of assets and liabilities; increasing liquidity and pricing accuracy through speculative participation; and, improved alignment of asset and liability composition with risk preferences, expectations and time horizon of individual traders. Though the value of such risk reduction benefits is indeterminate, the extent and severity of the recent US financial crisis suggests that inter-temporal gains to date have been small, at best, and illusory, at worst, e.g., Hancock and Passmore (2010).

4. Basics of Systemic Interest Rate Risk Immunization

The classical fixed income interest rate immunization model uses a univariate Taylor series expansion to derive two rules for immunizing a zero surplus fixed income portfolio against a change in the level of interest rates: 1) match the duration of cash inflows (assets) and outflows (liabilities); and, 2) set the asset cash flows to have more dispersion (convexity) than the liability cash flows around that duration, e.g., Redington (1952), Shiu (1987, 1990); Reitano (1991a, 1991b, 1992, 1996); Poitras (2007, 2013).[8] In the classical case of a fixed income portfolio associated with a life insurance or pension company, surplus immunization is structured around the balance sheet of an individual fund. In contrast, the framework for systemic risk immunization involves aggregating the balance sheets of households and mortgage lenders. In classical immunization the fund surplus is set equal to zero – with the actual surplus being segmented and treated separately – in order to obtain the well known solutions to that optimization problem, i.e., the duration matching and higher convexity of assets than liabilities conditions. In contrast, systemic risk immunization conditions need to specifically identify connections between different balance sheets of mortgage lenders and household mortgage borrowers. This requires the implications of having positive balance sheet surplus to be recognized.

Initially, the impacts from default and prepayment options will be ignored and simplified immunization conditions derived for the aggregated balance sheets of households and mortgage lenders. In this case, systemic interest rate risk immunization requires percentage changes in the (positive) market value of equity supporting the residential mortgage funding system to be non-negative after interest rates change. When this does not occur, the mortgage funding system will require net equity transfers from elsewhere in the financial system. When this occurs, the 'risk' in the mortgage funding system spills over into the broader 'financial system' producing 'systemic risk' where one part of the financial system -- the mortgage funding system -- has a net negative impact on the greater financial system, somehow defined. The so-called'sub-prime' financial crisis in the US revealed that the extent of the greater financial system is global. Financial institutions in all parts of the world were significant players in providing the equity transfers required to cover US mortgage funding system equity losses generated by the 'sub-prime' mortgage crisis, e.g., Harrington (2009, p.797).

Immunization of the market value of mortgage funding system equity involves explicit recognition of the household and mortgage lender balance sheet relationships:

$$A = L + S \rightarrow S[y,h] = A[y,h] - L[y,h]$$

where: A, L and S are assets, liabilities and surplus (equity); y is the yield to maturity on the annuity component of the mortgage; h is the growth rate of residential house prices; and, $\rightarrow$ indicates the next step in the derivation. For households: A_H is the market value of residential household assets associated with the mortgage; L_H is the market value of household mortgage liabilities; S_H is the household equity position. For lenders: A_L is the market value of the lender mortgage assets; L_L is the market value of mortgage lender liabilities used to fund mortgage assets; S_L is the lender equity position.

Upon aggregation, $L_H = A_L$ acts to transfer mortgage value changes between households and lenders. This balance sheet transfer is an essential avenue for mortgage funding risk transmission. In the simple case where only interest rate risk is considered and house price risk is ignored, a univariate Taylor series expansion is applied to the aggregate surplus function $S = S_H + S_L = A_H - L_H + A_L - L_L$ (where $S[y]$) to give:

$$S[y] = S[y_0] + \frac{dS}{dy}(y - y_0) + \frac{1}{2}\frac{d^2S}{dy^2}(y - y_0)^2 + \ldots\ H.O.T.$$

$$\frac{S[y] - S[y_0]}{S[y_0]} \cong -DUR_S\,(y - y_0) + \frac{1}{2}\,CON_S\,(y - y_0)^2$$

Because the duration of system surplus is the value weighted sum of the durations of surplus for the mortgage lender and household, it follows:

Definition: Duration of System Surplus

$$DUR_S = (S_H / S)\,DUR_{SH} + (S_L / S)\,DUR_{SL}$$

$$= \left(\frac{S_H}{S}\right)\left(-\frac{1}{S_H}\frac{dS_H}{dy}\right) + \left(\frac{S_L}{S}\right)\left(-\frac{1}{S_L}\frac{dS_L}{dy}\right)$$

where subscripts indicate households (*SH*) and mortgage lenders (*SL*). Similarly, $CON_S = (S_H / S)$ $CON_{SH} + (S_L / S)\ CON_{SL}$ is the modified convexity of system surplus.. The systemic interest rate immunization conditions follow: 1) set the duration of aggregate (system) surplus equal to zero; and, 2) have a positive convexity of aggregate (system) surplus.

While, in general, y_0 is the initial value of the interest rate which can be any plausible current value, a 'measure' of systemic risk can be obtained by setting y_0 equal to the expected value of the random variable, ($y_0 = E[y \mid \Omega_0]$). Taking conditional expectations, a measure of long run equilibrium systemic risk follows:

$$E\left[\frac{S[y] \ - \ S[y_0]}{S[y_0]}\right] \cong \frac{1}{2}\ CON_S\ \sigma^2{}_y \ = \ \frac{1}{2}\left(\ \frac{S_H}{S}\ CON_{SH}\ +\ \frac{S_L}{S}\ CON_{SL}\right)\ \sigma^2{}_y$$

where $\sigma^2{}_y = E[(y - E[y \mid \Omega_0])^2]$ is the conditional variance of interest rates. In the interest rate immunizing equilibrium, where the duration of system surplus is set equal to zero, having a positive convexity of system surplus is sufficient to ensure aggregate immunization is achieved, i.e., the expected change in system surplus from changing interest rates is positive. In addition, increase in the conditional expected volatility of interest rates is an essential component of increase in systemic risk. This implies that using interest rate changes as a tool of monetary policy has to be counterbalanced with the need to maintain interest rate stability to control systemic risk from the mortgage funding system.

Observing the subscripts *H* and *L* denote households and mortgage lenders and the superscripts *A* and *L* denote assets and liabilities, respectively, exploring the zero duration of system surplus condition – $DUR_S = (S_H / S)\ DUR_{SH} + (S_L / S)\ DUR_{SL} = 0$ – reveals that systemic interest rate risk immunization for a mortgage with term to maturity equal to the amortization period cannot be achieved unless interest bearing assets are included on the household balance sheet. More precisely:

Proposition 1: **The zero duration of systemic surplus (*S*[*y*]) condition**

$$DUR_S = ((S_H / S)\ DUR_{SH}) + ((S_L / S)\ DUR_{SL}) = 0$$

is equivalent to:

$$\left(\frac{A_H}{S}\ DUR_H{}^A - \frac{L_H}{S}\ DUR_H{}^L \right) + \left(\frac{A_L}{S}\ DUR_L{}^A - \frac{L_L}{S}\ DUR_L{}^L \right) = 0$$

Recalling $L_H = A_L$, it follows that zero duration of system surplus requires:

$$DUR_H{}^A = \frac{L_L}{A_H}\ DUR_L{}^L$$

When this condition holds, the aggregate surplus of the mortgage funding system, combining households and mortgage lenders, will be unchanged and the possibility of net equity draw from the greater financial system will be zero when interest rates change. Given that house price changes are not incorporated into this solution, for a no recourse mortgage where the only household asset is the residential property, $DUR_H{}^A = 0$ and zero duration of systemic surplus cannot be achieved.

The idealized theoretical systemic risk minimum when mortgage term to maturity equals amortization period is a 'complete recourse' mortgage where households hold all mortgage lender liabilities as security against mortgage default. In this idealized case, market value gains (losses) on the household mortgage liability from interest rate changes would be offset by losses (gains) on household interest bearing assets pledged to secure the mortgage. While limited and full recourse mortgages do expand the household assets available to absorb equity losses associated with increases in the market value of the mortgage when interest rates decrease, there is no assurance that these additional assets will be interest sensitive, e.g., households may own other real estate assets. Similarly, while some portion of mortgage lender liabilities will be held as assets by households, the aggregate composition of these liabilities is indeterminate.[9] By reducing (increasing) the value of mortgage lender liabilities not held by households, an increase (decrease) in interest rates transfers equity from (into) the greater financial system into (from) the mortgage funding system. In contrast, market value losses (gains) on mortgage lender assets due to interest rate increases (decreases) will be offset by gains (losses) on household liabilities, leaving **aggregate** mortgage funding system equity unchanged from changes in mortgage market value.[10]

The zero duration of systemic surplus condition implicitly assumes both households and mortgage lenders separately seek an interest rate risk immunizing solution. Given this, the following condition applies to mortgage lenders:

Proposition 2: **The zero duration of surplus ($S_L[y]$)condition for lenders**

From the balance sheet for mortgage lenders, $S_L = A_L - L_L$ and the zero surplus condition for lenders is:

$$\frac{1}{S_L}\frac{dS_L}{dy} = 0 = \frac{A_L}{S_L}\, DUR_L{}^A - \frac{L_L}{S_L}\, DUR_L{}^L$$

It follows that:

$$DUR_L{}^A = \frac{L_L}{A_L}\, DUR_L{}^L \quad \rightarrow \quad CON_L{}^A > \frac{L_L}{A_L}\, CON_L{}^L$$

While the classical zero surplus immunization conditions require setting the duration of assets equal to the duration of liabilities, immunization with a positive surplus requires the duration of assets to be equal to the duration of liabilities, multiplied by the loan-to-value ratio for the market values of the mortgage assets and the lender liabilities used to fund the mortgage. This reveals the usefulness of increasing capital requirements on financial institutions to alleviate systemic risk. In conventional presentations of this result, the inability to satisfy this condition in a mortgage funding system based on depository institutions is due to the 'duration gap' confronting depository institutions: the duration of long term fixed rate mortgages is too high relative to the duration of demand and term deposits used to fund the mortgages. Historically, inability of mortgage lender surplus to adequately deal with the duration gap in the face of increasing interest rates was a central issue in the collapse of the S&L industry during the 1980's. If a zero duration of mortgage lender surplus can be achieved, then immunization of interest rate risk for mortgage lenders follows by satisfaction of the convexity condition: $CON_L^{\,A} > (L_L / A_L)\, CON_L^{\,L}$.

The systemic implications of setting term to maturity equal to amortization period for long term mortgage contracts, such as the 30 year fixed rate US mortgage, can be contrasted with a mortgage contract where term to maturity is significantly less than the long term amortization period. In this case, the size of the annual mortgage payment, which is primarily determined by the amortization period of the mortgage, will reset when the unpaid balance on the shorter term to maturity mortgage is refinanced at the end of the term to maturity (and prior to the end of the amortization period). This changes the formulation of the systemic immunization conditions because the value of the mortgage on the household balance sheet is now composed of a funded and an unfunded component. Only the funded component is transferred to the mortgage lender. In turn, changes in the value of the unfunded component due to interest rate changes will systemically act to offset changes in the value of the funded component. For example, while an interest rate decrease will increase the value of the household mortgage liability, households will benefit when the mortgage matures by having a lower payment required to finance the unpaid balance.

To see this, let $L_H = F_H + U_H$ be the funded and unfunded portions of L_H, respectively. Once the amortization period (N) and the interest rate are specified, the value of the annual fixed rate payment (M) on the mortgage can be determined. Given this, U_H is determined as the unpaid balance on the mortgage that needs to be refinanced at time T^*, the maturity date of the short term mortgage:

$$L_H = \sum_{t=1}^{N} \frac{M}{(1 + y)^t} = \sum_{t=1}^{T^*} \frac{M}{(1 + y)^t} + \sum_{t=T^*+1}^{N} \frac{M}{(1 + y)^t} \quad \rightarrow \quad U_H = L_H - \sum_{t=1}^{T^*} \frac{M}{(1 + y)^t}$$

Because U_H represents the unpaid balance due at time T^*, changes in interest rates will alter the intrinsic value of this future liability for households. However, because the unpaid balance will be financed at future market interest rates, from the perspective of mortgage lenders U_H is a floating rate contract where $DUR_H{}^U = 0$. The imposition of a prepayment penalty that is at least equal to 'loss of interest' prevents $T^* < N$ households from capturing any significant gain prior to the maturity of the mortgage.

The implications of this substantive difference in mortgages where term to maturity (T) is equal to the long term amortization period (N) versus mortgage contracts where term to maturity (T^*) is less than the long amortization period can be seen from the change required to the associated zero duration immunization condition for households with short term to maturity mortgages:

$$\frac{A_H}{S}\, DUR_H{}^A - \left(\frac{F_H}{S}\, DUR_H{}^F - \frac{U_H}{S}\, DUR_H{}^U \right) = 0$$

Taking $(A_H^* / S)\, DUR_H{}^{A*} \equiv (A_H / S)\, DUR_H{}^A + (U_H / S)\, DUR_H{}^U$ produces the following:

Proposition 3. **The zero duration of system surplus ($S[y]$)condition when households use mortgages with term to maturity less than amortization period is**:

$$\left(\frac{A_H{}^*}{S}\, DUR_H{}^{A*} - \frac{F_H}{S}\, DUR_H{}^F \right) + \left(\frac{A_L}{S}\, DUR_L{}^A - \frac{L_L}{S}\, DUR_L{}^L \right) = 0$$

Given that $F_H = A_L$ in the $T^* < N$ case, it follows that zero duration of system surplus requires:

$$DUR_H^{A^*} = \frac{L_L}{A_H^*} DUR_L^L .$$

However, in this case the values on the left side of this equilibrium condition will be substantively different than in the $T = N$ case. In particular, the unfunded portion of the household mortgage liability will now be part of $DUR_H^{A^*}$ introducing an element that facilitates achievement of the zero duration of systemic surplus condition.

To illustrate these simplified systemic immunization conditions, assuming annual mortgage payments and no contract contingencies the Macaulay duration (D^* ; $D^* = (1 + y)\, DUR$) of $A_L = L_H$ for a $T = N$ mortgage can be calculated as:

$$\frac{dA_L}{d(1 + y)} = \frac{d}{dy}\left[\frac{1}{y} - \frac{1}{y(1 + y)^T} \right]$$

$$\rightarrow \quad D^* = -\frac{1 + y}{A_L}\frac{dA_L}{d(1 + y)} = \frac{1 + y}{y} - \frac{T}{(1 + y)^T - 1}$$

where T is the term to maturity of a mortgage contract which equals the long amortization period. For example, with no options on the mortgage asset, at a 5% interest rate, $D^* = 11.97$ for A_L with a 30 year term to maturity ($D^* = 2.90$ for the $T^* < N$ benchmark term to maturity of 5 years). This can be contrasted with a duration of zero for the no option floating rate mortgage implicitly associated with U_H for mortgage lenders. Assuming a leverage ratio of $(L_L / A_L) = 0.9$, the zero duration of surplus condition for mortgage lenders produces an immunizing interest elasticity for liabilities of 13.3 for the thirty year $T = N$ mortgage (3.22% for the 5 year T^* benchmark). The historical collapse of the S&L mortgage funding system in the US demonstrated that, when depository institutions are the primary source of mortgage funding, a duration gap problem in the mortgage funding system can have severe implications for the whole financial system.

To see the practical implications of the systemic risk immunization conditions for $T = N$ and $T^* < N$ mortgage contracts, consider the zero duration of surplus implications of a 100 basis point increase (decrease) in mortgage interest rates. Let the value of the residential property being funded with a 30 year term to maturity and amortization period mortgage be A_H = \$125,000. Given (L_H/A_H) = 0.80, this implies a mortgage value of L_H = \$100,000 = A_L and, with a leverage ratio of 0.9, the lender would fund the mortgage with L_L = \$90,000. The 100 bp interest rate increase (decrease) would change the market value of the \$6505.12 annual mortgage payment to L_H = \$89,541.7, ΔL_H = -\$10,458.3 (\$112,487, ΔL_H = \$12,487). These values can be contrasted with a $T^* < N$ mortgage having a 5 year term to maturity and 30 year amortization period.[11] In this case, the same 30 year amortization period and 5% interest rate again produces a \$6505.12 annual mortgage payment. However, because the funded portion of the \$100,000 mortgage is only fixed for 5 years, the 100 basis point increase (decrease) would decrease (increase) the mortgage value by ΔL_H = -\$3200.14 (ΔL_H = \$3214.79). Because the mortgage value is a liability for households, a decrease in mortgage value is a loss for lenders and a gain for households. This leaves the implicit value change for the unfunded liability to be determined.

To calculate the implicit value change for the unfunded liability associated with the $T^* < N$ mortgage, observe that the balance due on the maturity of the mortgage, \$91,683, is determined at origination. Assuming no changes to the mortgage principle pay down, e.g., increasing the unpaid balance or rolling back the amortization period to 30 years, the unpaid balance will then be refunded at maturity using a 5 year mortgage with a 25 year amortization. If interest rates on the maturity date have increased (decreased) to 6% (4%) then a fixed payment of \$7172.06 (\$5868.81) will be needed to refund the mortgage balance due. Evaluating the 5 year annuity of the difference in payments, the associated loss (gain) has a value at T^* of -\$2809.29 (\$2832.74) which becomes -\$2099.34 (\$2116.79) when discounted back to the origination date (t=0) at the prevailing interest rate. Whereas an interest rate increase will benefit households by decreasing the market value of the current mortgage, this gain will be offset by loss associated with the higher payments when the $T^* < N$ mortgage is refinanced at maturity.

Combined with Proposition 3, these simplified comparisons illustrate a number of systemic advantages of the $T^* < N$ mortgage contract term to maturity feature leading to:

Corollary 1: **The systemic risk of long amortization period mortgages is substantively reduced by shortening mortgage term to maturity.**

More precisely, having mortgage term to maturity significantly less than the amortization period enables the mortgage lenders and households comprising the mortgage funding system to more readily approach the zero duration of system surplus required for systemic interest rate immunization of long amortization period mortgages. The longer term to maturity mortgages associated with the simplified (no contingency) mortgage contract have larger mortgage market value changes when interest rates change and a potentially significant duration gap problem. As a consequence, a mortgage funding system using simplified $T = N$ contracts poses greater systemic risk of drawing significant equity from the greater financial system. More importantly in practice, while $T = N$ household gains and lender duration gap losses from interest rate increases could be approximately offset over time by the losses and gains from interest rate decreases leaving system equity approximately unchanged, this possibility is not attainable in practice because of the prepayment option and default option contingencies that are excluded from the simplified mortgage contract used in this section but are integral to the actual contracts used in funding US mortgages.

5. House Prices, Prepayment and Mortgage Default

In the US, an important historical rationale for retaining long term to maturity fixed rate mortgages can be traced to the Great Depression when house price volatility caused severe problems for households with $T^* < N$ mortgages seeking to refinance properties when house prices had fallen dramatically. However, the recent dramatic house price drop associated with the sub-prime financial crisis indicates that funding mortgages through capital markets is also not immune from the devastating impacts of significant house price declines. Extending the basic systemic immunization conditions to include house price changes requires more detailed consideration of the household balance sheet. The introduction of house price growth rates (h) requires the determination of the 'partial duration' ($PDUR$) and 'partial convexity' ($PCON$) of residential house prices for the mortgages used to finance house purchases. In the simplified systemic interest rate immunization solution, the interest elasticity properties of DUR_y and CON_y for the mortgage assets and the liabilities being immunized can be calculated directly from the lender balance sheet. However, in practice, the mortgage pricing function is complicated by contingent claims, the prepayment and default options, that obscure precise calculations for these elasticities. In addition, there is limited knowledge about the cross elasticities for residential house prices and interest rates.[12]

Extending the system surplus function to depend both on mortgage yields and house price growth rates, expansion in a bivariate Taylor series gives:

$$S[y,h] = S[y_0,h_0] + \frac{\partial S}{\partial y}(y - y_0) + \frac{\partial S}{\partial h}(h - h_0)$$

$$+ \frac{1}{2}\left[\frac{\partial^2 S}{\partial y^2}(y - y_0)^2 + \frac{\partial^2 S}{\partial h^2}(h - h_0)^2\right] + \frac{\partial^2 S}{\partial y\,\partial h}(y - y_0)(h - h_0) + \ldots\ H.O.T.$$

$$\frac{S[y,h] - S[y_0,h_0]}{S_0} \cong \left[-PDUR_{Sy}(y - y_0) + PDUR_{Sh}(h - h_0)\right]$$

$$+ \frac{1}{2}\left[PCON_{Sy}(y - y_0)^2 + PCON_{Sh}(h - h_0)^2\right] + \frac{1}{S_0}\frac{\partial^2 S}{\partial y\,\partial h}(y - y_0)(h - h_0)$$

where the subscripts Sy and Sh indicate the partial derivatives of system surplus for h and y, respectively; and, *PDUR* and *PCON* are the associated partial durations and convexities.[13] Observing there are now two random variables in the surplus value function:

Definition: Duration of System Surplus **($S[y,h]$)**

The duration of system surplus with respect to house price change and interest rates is:

$$\frac{S_H}{S}\left(-PDUR_{Hy}(y-y_0) + PDUR_{Hh}(h-h_0)\right) + \frac{S_L}{S}\left(-PDUR_{Ly}(y-y_0) + PDUR_{Lh}(h-h_0)\right)$$

where: $PDUR_{i,\,j} = \frac{1}{S_i}\frac{\partial\ S_i}{\partial\ j}$. Given this, the zero change of surplus conditions for households and mortgage lenders are required to assess the impact of variations in $T = N$ versus $T^* < N$ mortgage contract terms.

In the following, the fund surplus value function for households, $S_H[h,y]$, involves two stylized formulations of the household equity function associated with the $T^* < N$ and $T = N$ mortgage funding systems.

Definition: **The stylized T* < N household surplus (equity) function** is for a full recourse mortgage with a yield maintenance prepayment penalty. This produces the $T^* < N$ household surplus function:

$$S[h_0,y_0] = A_{H,0} - L_{H,0} = A_{H,0} - \left(\sum_{t=1}^{T^*} \frac{(M_0)}{(1+y_0)^t} + \sum_{t=T^*+1}^{N} \frac{(M_{T^*})}{(1+y)^t} \right)$$

$$\rightarrow \quad S[h,y] = A_{H,0}(1+h) - \sum_{t=1}^{T^*} \frac{(M_0)}{(1+y)^t} - U_H = A_{H,0}(1+h) - F_H - U_H$$

where: $A_{H,0}$ is the market value of the residential house asset (other assets not included in this case) and $L_{H,0}$ the market value of the mortgage payment cash flows, both at time $t = 0$; and, M_i is the fixed rate mortgage payment determined at $t=i$. Assuming $PDUR_{Hh}{}^{L} = 0$, the first order term in the Taylor series provides:

Proposition 4: **The zero change in household surplus ($S_H[y,h]$) condition**:

$$\left[-PDUR_{Hy}\ (y-y_0) + PDUR_{Hh}{}^{A}\ (h-h_0) \right] = 0$$

$$\rightarrow \quad \frac{L_H}{S_H}\ PDUR_{Hy}{}^{L}\ (y-y_0) = \frac{A_{H,0}}{S_H}\ (h-h_0)$$

where $L_H = F_H + U_H$. This follows because:

$$\frac{\partial S_H}{\partial h} = A_{H,0}\ (1+h) = A_{H,0} \quad \rightarrow \quad \frac{1}{S_H}\frac{\partial S_H}{\partial h} = PDUR_{Hh}{}^{A} = \frac{A_{H,0}}{S_H}$$

Absent an identifiable functional relationship between house price changes and interest rate changes, the zero change of household surplus condition still cannot be obtained on the household balance sheet. Assuming $PDUR_{L\,h}{}^{L} = 0$ produces:

Proposition 5. **The zero change of systemic surplus ($S[y,h]$) condition for the stylized $T^* < N$ household mortgage is**:

$$\frac{S_H}{S}\left(\frac{A_H}{S_H}\ (h - h_0) - \frac{L_H}{S_H}\ PDUR_{Hy}{}^{L} \right) + \frac{S_L}{S}\left(\frac{A_L}{S_L}\ PDUR_{Ly}{}^{A} - \frac{L_L}{S_L}\ PDUR_{Ly}{}^{L}\ (y - y_0) \right) = 0$$

This can be solved as:

$$\left(\frac{A_H}{S}\ (h - h_0) \right) = \left(\frac{L_L}{S}\ PDUR_{Ly}{}^{L} - \frac{U_H}{S}\ PDUR_{Hy}{}^{U} \right)\ (y - y_0)$$

Because the surplus transfer between households and lenders from interest rate changes is small for the $T* < N$ mortgage, changes in system surplus are largely driven by house price growth rates. Assuming $PDUR_{L\,h}{}^{L} = 0$, these surplus changes, which can be negative or positive, will only be located on household balance sheets. This leads to:

Corollary2: **Systemic risk with $T* < N$ mortgage contracts depends fundamentally on the size of the household surplus to absorb house price changes.**

In contrast to mortgage lenders, households face real restrictions accessing the greater financial system for funds to cover equity losses. A full recourse provision in the $T* < N$ mortgage expands the assets available to protect household surplus against adverse changes in house prices. Having a sufficient household surplus is essential from a $T* < N$ mortgage funding system perspective as $S = S_H + S_L$ with the mortgage lender surplus change being relatively small due to the smaller exposure arising from the use of shorter term to maturity mortgages.

In contrast to the $T* < N$ case, the $T = N$ mortgage contract seeks to insulate the household balance sheet and control the negative impact of interest rate and house price decreases by including contingencies protecting the market value of the $T = N$ household surplus. These options are: for the mortgage liability (L_{H}), the borrower prepayment option (*PPO*); and, for the residential house asset, the borrower mortgage default option (*DO*). The S_{H} from the household balance sheet can now be decomposed as:

$$S_{\mathrm{H}} = (A_H{}^* + DO) - (L_{\mathrm{H}}{}^* - PPO) = A_{\mathrm{H}} - L_{\mathrm{H}}$$

where $A_{\mathrm{H}}{}^*$ and $L_{\mathrm{H}}{}^*$ are the market value of the residential house asset and the fixed rate annuity portion of the household mortgage liability, respectively. Even though *DO* is embedded in the mortgage contract ($L_H = L_H^* - PPO - DO$), the no recourse provision makes the value of this option dependent on house price changes.[14] Calculating the partial durations and convexities of these option payoffs is decidedly less tractable, though there are some helpful studies that address related issues, e.g., Peristiani (2003); Sharp et al. (2008); Downing et al. (2007).

Definition: **The stylized T = N household equity function** for a no recourse mortgage with prepayment and default options is:

$$S_H[h,y] = \left(A_{H,0}^*(1 + h) + DO[h,T,\sigma_h \mid S_0] \right) - \left[\sum_{t=1}^{T} \frac{(M_0)}{(1 + y)^t} - PPO[y,T,\sigma_y \mid y_0] \right]$$

This formulation is stylized because the functional specification of the *PPO* and *DO* options does not capture all relevant variables impacting valuation and exercise.[15]

The stylized $T = N$ household equity function illustrates the intuition of the US mortgage funding system. The contingencies have: complicated payouts; and, difficult to determine exercise decisions. For example, while $\partial DO / \partial A_H^* \rightarrow 0$ (-1) as $h \rightarrow +\infty$ ($-\infty$) with $0 < DO < L^*$, it is not possible to say that *DO* will be exercised when $S_H < 0$. No recourse default is triggered when mortgage payments cease prior to *T* and the residential property is surrendered for the unpaid balance on the mortgage, the payout from lenders to households at that date being: $L_H^* - A_H^*$. This value is undetermined at origination as the default decision date is determined strategically, depending on a number of factors, not limited to having $S_H < 0$. Similarly, $\partial PPO / \partial L_H^* \rightarrow 1$ (0) as $y \rightarrow 0$ ($+\infty$) with $0 < PPO < L_H^*$. However, transactions costs associated with *PPO* exercise could delay exercise if there is an expectation that rates will continue to fall. Upon exercise, the payout from lenders to households is equal to 'loss of interest' calculated as the annuity value of the difference in mortgage payments over the remaining amortization period. Because $\partial DO / \partial T > 0$ and $\partial PPO / \partial T > 0$, it follows:

Corollary 3 **Reducing mortgage term to maturity will reduce the value of the prepayment and default options at origination**.

While mathematically intuitive, straight forward application of Taylor series to solve for the systemic risk immunizing solution is complicated by the contingencies in the $T = N$ household equity function. Unknown *ex ante* default and prepayment probabilities are necessary to the pricing of DO and PPO. Given this, solving for the relevant partial derivatives gives:

Proposition 6: **The partial durations and zero change of household surplus for the stylized $T = N$ household surplus (equity) function:**

$$\frac{\partial S_H}{\partial h} = A_{H,0}^{*} + \frac{\partial DO}{\partial h} \quad \rightarrow \quad \frac{1}{S_H}\frac{\partial S_H}{\partial h} = \frac{A_{H,0}^{*}}{A_{H,0} - L_{H,0}}\left[1 + \frac{\partial DO}{\partial A_H^{*}}\right]$$

$$\frac{\partial S_H}{\partial y} = -L_{H,0}^{*}\left[\frac{1}{L^{*}}\frac{\partial L^{*}}{\partial y} - \frac{1}{L^{*}}\frac{\partial PPO}{\partial y}\right] \quad \rightarrow \quad \frac{1}{S_H}\frac{\partial S_H}{\partial y} = \frac{L_{H,0}^{*}}{A_{H,0} - L_{H,0}}\left[PDUR_{Hy}^{*}\left(1 - \frac{\partial PPO}{\partial L^{*}}\right)\right]$$

where: $A_{H,0}$* is the market value of the residential house asset (with default option value not included) and $L_{H,0}$ *the market value of the mortgage payment cash flows (with prepayment option not included), both at time $t = 0$. These partial derivatives can now be used to determine **the zero change of household surplus for the stylized $T = N$ household equity function** as:

$$\frac{A_{H,0}^{*}}{A_{H,0} - L_{H,0}}\left[1 + \frac{\partial DO}{\partial A_H^{*}}\right](h - h_0) = \frac{L_{H,0}^{*}}{A_{H,0} - L_{H,0}}\left[PDUR_{Hy}^{*}\left(1 - \frac{\partial PPO}{\partial L_H^{*}}\right)\right](y - y_0)$$

$$\rightarrow \quad \left[1 + \frac{\partial DO}{\partial A_H^{*}}\right](h - h_0) = \frac{L_{H,0}^{*}}{A_{H,0}^{*}}\left[PDUR_{Hy}^{*}\left(1 - \frac{\partial PPO}{\partial L_H^{*}}\right)\right](y - y_0)$$

In the absence of the *DO* and *PPO* options, household equity increases (decreases) when both h and y increase (decrease) with the surplus changes being transferred between mortgage borrowers and lenders. With contingencies, whether h increases or decreases, the possible household equity loss associated with a y decrease is protected by exercise of the prepayment option which transfers this loss to lenders. In turn, households capture the market value gain from y increases at the expense of lenders. However, when h decreases and y decreases, equity value loss from the home price decline is mitigated by gains from exercise of the prepayment option. Similarly, when the h decrease is large enough that $(1 + h)\ A_{H,0} < L_H$, then the default option exercise will prevent household surplus from going negative, again at the expense of mortgage lenders.

Casual inspection of the zero change of system surplus condition in Proposition 6 reveals:

Corollary 4: **Mortgage contract contingencies do not substantively impact systemic risk if correctly priced at origination.**

In this case, a higher price will be paid for $L_H = A_L$ to reflect the appropriate option premia. This will increase S_L sufficiently to offset the loss when *PPO* or *DO* is exercised. Unfortunately, obtaining an accurate contingency price at $T = N$ mortgage contract origination is decidedly difficult. In practice, market incentives associated with an 'originate to distribute' method of mortgage issuance has the potential for *PPO* and *DO* being unpriced or, at best, grossly underpriced. In contrast, instead of attempting to embed difficult to price option premia into the mortgage price at origination, yield maintenance penalties involve payment of the option premium upon exercise, with the premium equal to loss of interest on the remaining term to maturity. This leads to the following:

Corollary 5: **Including a yield maintenance prepayment penalty in the mortgage contract is a systemic risk reducing method of pricing the value of the prepayment contingency.**

In effect, systemic risk is reduced because the premium paid for the prepayment contingency is more accurately priced at exercise than at origination.

By insulating the household balance sheet, an important consequence of the *PPO* and *DO* contingencies is that the mortgage funding system surplus available to absorb the impact of changes in interest rates and house prices has been significantly reduced. Adjustment pressure falls on the mortgage lender surplus. This problem is compounded when the prepayment and default options embedded in the $T = N$ mortgage contract are not priced. Assuming $y_0 = E[y \mid \Omega_0]$, in the absence of *PPO* and *DO* the loss (gain) for mortgage lender assets from an increase (decrease) in interest rates would be approximately offset by the gain (loss) when interest rates decreased (increased). Changes in household surplus combine to offset changes in mortgage lender surplus providing for $S = S_H + S_L$. Changes in mortgage lender surplus would be associated with duration gap exposure. In contrast, when the *PPO* and *DO* contingencies are included to insulate the household surplus, the burden of surplus changes associated with systemic immunization falls on $S = S_L$. In this case, the zero change of mortgage lender surplus condition becomes the zero change of system surplus for the stylized $T = N$ mortgage contract.

The consequences of the inclusion of *PPO* and *DO* for systemic immunization can be assessed by evaluating the zero change of mortgage lender surplus condition:

$$\left(-PDUR_{Ly}\ (y - y_0)\ +\ PDUR_{Lh}\ (h - h_0) \right) = 0$$

This solution will depend on whether the contingencies transferred from households to mortgage lenders are priced or unpriced. This produces:

Proposition 7 If the contingencies are unpriced when the mortgage liability is transferred to the mortgage lender balance sheet, the zero change of mortgage lender surplus condition produces:

$$\left[PDUR_L^{A^*} \left(1 - \frac{\partial PPO}{\partial A_L^*} \right) \right] (y - y_0) - \frac{A_H^*}{A_L^*} \frac{\partial DO}{\partial A_H^*} (h - h_0) = \frac{L_L}{A_L^*} \left[PDUR_L^{\,L} \right] (y - y_0)$$

where $S_L = (A_L^* - DO - PPO) - L_L$.

While the market value of mortgage lender liabilities increases when interest rates decrease, the associated gain in lender assets is strangled by exercise of the prepayment option and the gain is captured on household balance sheets. Conversely, when interest rates increase, the loss on lender assets will not be fully balanced by the reduction of lender liabilities if there is a duration gap. In addition, mortgage lenders are now exposed to downward movement in house prices associated with exercise of the default option.

In general, the following holds:

Corollary 8: **When mortgage contracts include an inaccurately priced prepayment option, it is not possible for the mortgage funding system to achieve systemic risk immunization.**

The corollary is only 'in general' because there are a number of qualifications to this result. In particular, when mortgage interest rates are historically low, then the probability of prepayment exercise is small and the actuarially fair *PPO* premium will be small. However, in such a situation there is a higher probability of mortgage lender asset losses from increasing interest rates. Alternatively, if the duration of mortgage lender assets and liabilities is near zero, i.e., floating rate assets and liabilities, systemic immunization can be trivially achieved for mortgage contracts with prepayment options. However, this raises the possibility of a 'reverse' duration gap where the duration of lender assets is ***less*** than lender liabilities. Similarly, a reduction in mortgage term to maturity will also mitigate the value loss associated with prepayment option exercise. For an interest rate decrease from 6% to 5%, the loss of interest from a 30 year term to maturity mortgage will be about 3.5 times the loss from a 5 year term to maturity.

6. US versus Canadian Mortgage Contract Design

Prior to the recent market failure, criticism of the US mortgage system was muted and claims for innovation and superiority over mortgage financing methods used in other countries were common. In contrast, the Canadian mortgage system was criticized for "lack of access to mortgages with fixed rates, penalty-free prepayment and high loan-to-value ratios" (Green and Wachter 2005, p.102). Yield maintenance penalties and relatively low levels of mortgage securitization were also singled out for criticism. Such 'horse race' comparisons deny the complex and unique evolution of the mortgage contract in each country. Despite considerable economic integration of the Canadian and US economies and financial markets, the Canadian mortgage funding system has been comparatively unscathed by the crises that have emerged in the US. The widespread recommendations for reform of the US mortgage market that involve improved valuation methodologies, significant enhancement of regulations (GAO 2009), and strengthening of government oversight generally ignore the implications of mortgage contract design. However, two systemic mortgage funding collapses in a generation, e.g., Benston (1986), Jaffee (1989), Benston and Kaufman (1997), have also led to recent suggestions for reforms in mortgage contract design aimed at achieving reduction of systemic risk.

More precisely, Shiller et al. (2013) and Ambrose and Buttimer (2012) both propose contract designs aimed to manage the systemic default risks of long amortization mortgages. The basic objective is to capture gains associated with reduced cost of default for lenders by allowing adjustments in mortgage payments for borrowers in difficulty. Shiller et al. (2012, p.269) claim the "fragility of financial intermediaries stems from the rigidity of the traditional mortgage contracts, such as the Fixed Rate Mortgages, Adjustable Rate Mortgages and their hybrids". To deal with this fragility, Shiller et al. (2012, p.270) propose a continuous workout mortgage (CRM) that: "share[s] the price risk of a home with the lender and thus provide automatic adjustments for changes in home prices or incomes. Mortgage balances are thus adjusted and monthly are varied automatically with changing home prices". This design is similar to the adjustable balance mortgage (ABM) proposed by Ambrose and Buttimer (2012, p.540) where: "At fixed, preset intervals, the lender and the borrower determine the value of the house. If the house value is lower than the originally scheduled balance for that date, the loan balance is set equal to the house value and the monthly payment is recalculated based on this new value." Ignoring the two-sided character is risk, Ambrose and Buttimer further propose: "If the house retains its initial value or increase in value, then the loan balance and payments remains unchanged, just as in a standard FRM".

Such proposals raise numerous questions. In particular, Corollary 8 indicates that inaccurate pricing undermines the achievement of systemic risk immunization. Both the CWM and the ABM require the assessment of the future path of a (somehow defined) index of house prices and, for the CRM, household income. Why would mortgage lenders be interested in a contract design that is more difficult to price than a conventional mortgage? The ABM proposes that, if house prices move adversely, then the loss is divided between mortgagor and mortgagee when a conventional mortgage would have the mortgagor bear the full loss until the point of default is reached. Ambrose and Buttimer (2012, p.536) claim the ABM will "minimize default risk resulting from changes in the underlying asset value while still retaining contract rates near the cost of a standard fixed-rate mortgage". Given that mortgagor lenders do not participate in house price increases, such an outcome seems fanciful. Why would mortgage lenders not decrease the required loan to value ratio at origination? Being based on an index for both home prices and incomes, the CWM would be even harder to price than the ABM.

In order to determine analytical solutions, this paper has used $T = N$ and $T^* < N$ to motivate stylized mortgage contract design definitions. While the practical realization of each mortgage contract type may differ in individual cases, e.g., US mortgages can have a 15 year term to maturity at primary issue, the objective is to recognize essential characteristics differentiating US from Canadian mortgage contracts. Given this, the three key contract design features of ***the conventional U.S. mortgage contract*** that have persisted through the history of various US mortgage funding system changes are: low or no prepayment penalties; limited or no recourse in the event of default combined with inaccurate pricing by mortgage lenders of the implicit mortgage default premium; and, contracts with a long amortization period that is equal to the term to maturity of the mortgage. In contrast, the three key features of ***the conventional Canadian mortgage contract*** are: term to maturity of the mortgage that is significantly less than the amortization period; full recourse in the event of default; and, 'yield maintenance' prepayment penalties.

In the absence of overwhelming arguments in favour of the initiative, a recommendation to introduce full recourse mortgages with yield maintenance prepayment penalties, restrictions on maximum mortgage term to maturity and accurate pricing of mortgage insurance would not only be politically unpopular, a perceived restriction on the 'freedom to choose', but also illegal under various state laws. US residential mortgage borrowers have demonstrated overwhelming preference for implicitly subsidized, long amortization and term to maturity, usually 30 year, fixed rate mortgages. For the first decade of this century, between 70-90% of mortgages financed by the GSE's – Ginnie Mae, Freddie Mac and Fannie Mae – were long term fixed rate mortgages, e.g., FNMA (2009). Attempts to alter the conventional mortgage contract date back to the FHLBB's attempt to alleviate the duration gap problems of S&L's during the 1980's by encouraging issuance of adjustable rate mortgages (ARMs). Such attempts have not been popular with either mortgage borrowers or lenders, though for different reasons (Bentson 1986, ch. IV). Despite a variety of risk based mortgage contract designs that appeared in the expansion of the sub-prime mortgage market, e.g., Chomsisendgphet and Pennington-Cross (2006); Piskorski and Tchistyi (2010), the systemic risks associated with mortgage contract design have to be addressed. While the residential mortgage financing landscape has changed dramatically since long term, fixed rate mortgages were introduced with government backing in the 1930's, periodic disruptions and collapses in the mortgage market have been addressed by changing financing conduits rather than altering the conventional mortgage contract.

In contrast to the experience in the US, the largely depository institution funded Canadian mortgage system has avoided the history of crisis that has plagued the US system. Mortgage origination and funding is primarily through opaque multi-platform depository institutions, especially the five largest chartered banks (see Appendix for more details). Due to a distinctly different constitutional framework in the US where states have considerably more jurisdiction over mortgage markets than Canadian provinces, such a market structure would be unattainable in the US. The small number of major players permits the Canadian mortgage funding system to be more self-regulatory, with less layering of financial regulation and regulators in comparison with that in the US.[16] As the full recourse and yield maintenance provisions demonstrate, mortgage lenders in Canada have decidedly more market power than lenders in the US. Instead of contract contingencies that shift losses from households to mortgage lenders, the Canadian mortgage contract isolates the impact of interest rate and house price changes onto the household balance sheet. As Corollary 2 demonstrates, this places considerable burden on the surplus (equity) position of households.

7. Conclusion

Failure to originate of residential mortgages at prices consistent with the immunization of systemic risk facilitates the process of contagion where negative shocks to household balance sheets from extreme decreases in house prices or interest rates are transferred to mortgage lenders with insufficient equity to handle such shocks. Much public debate on the management of systemic risk from mortgage funding continues to focus on the difficult to attain goal of improved pricing through better estimates of parameters needed in pricing functions, such as default probabilities and the like. With some exceptions, e.g., Shiller et al. (2013), Ambrose and Buttimer (2012), detailed discussion of mortgage contract design change has been muted. The conventional US 30 year, fixed rate mortgage contract is the product of a funding system dominated by GSE's. From collapse of Regulation Q to the recent bankruptcy of the GSE's and assorted mortgage lenders, government policy has been unable to contain the systemic risk associated with the conventional 30 year fixed rate ($T = N$) mortgage contract.

This paper demonstrates the systemic risk management benefits of a funding system based on $T^* < N$ mortgage contracts where the maximum mortgage term to maturity is substantively less than the amortization period. Proposition 3 and Corollary 1 demonstrate this. In the absence of prepayment and default options, an increase in interest rates will reduce the market value of the mortgage but will increase the value of the mortgage to be obtained at the end of the term to maturity. This acts to contain the impact of interest rate changes to the household balance sheet. As demonstrated in Corollary 5, Proposition 7 and Corollary 8, the systemic risk management benefits of yield maintenance prepayment penalties arise from the more accurate pricing of this contingency at exercise than at origination. Finally, the full recourse provision substantially reduces systemic default risk by making default more costly for mortgagors.

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APPENDIX: A Primer on the Canadian Mortgage Market

The Canadian mortgage system is based on full recourse mortgage contracts with term to maturity at primary issue significantly less than amortization period. Standard contracts have a 'loss of interest' (yield maintenance) provision to dampen the impact of interest sensitive prepayment option exercise and a 'three times monthly payment' penalty impacting other factors influencing the exercise decision.[16] In the case of a house sale prior to the end of term, mortgages are assumable with no prepayment penalty if the house purchaser elects and qualifies to assume the mortgage or the mortgage holder qualifies to transfer the mortgage to their next residence. At the end of mortgage term to maturity, the unpaid balance on the mortgage is due. It is at this point that the mortgage borrower may reset the terms of the mortgage if desired, this can include changing both the amortization period and unpaid balance. It is also possible at this time to transfer the mortgage without penalty to another lender, though this does involve some legal costs that may be waived by the next lender or added to the unpaid balance. Within this framework, rates vary by term.

Ultimately, borrowers would prefer a system that provides the lowest all-in borrowing costs over the amortization period of the mortgage. Recognizing differences in capital markets, currency and the like, one measure of pricing efficiency is the spread of the mortgage rate over the comparable US Treasury or Government of Canada 30 year bond yield. For example, on June 4, 2009 the 30 year Government of Canada bond rose +.06 to 4.02 while the 30 year U.S. Treasury rose +.15 to 4.60. Recognizing that Royal Bank and TD are the largest mortgage lenders, the spread between the Royal variable rate mortgage and 30 year Canada bond was -.77 bp, while Bloomberg quotes for this date have the US 5/1 year ARM and 30 year Treasury spread at 0 bp. The degree of competition in the Canadian market is evidenced by the 50 bp spread between the largest lenders and next largest mortgage lenders, the CIBC and BMo. As Canadian variable rate mortgages and a variety of other depository institution interest rates float off the individual bank prime rate, the difference on this date is a consequence of steeper prime rate reductions due to inter-bank competition for consumer lending business.

Closer inspection of these Canadian and US mortgage rates reveals a number of differences and similarities in contract design and pricing. The most remarkable difference is the spread between the variable rate and 25 year term to maturity quote for the Royal (8.05 - 3.25 = 480 bp) -- one of the few chartered banks to offer such a long term to maturity -- and the 5/1 year ARM and the conventional 30 year fixed (5.35 - 4.55= 80 bp). This 400 bp difference in spreads does not adjust for the 'yield maintenance' and other prepayment penalties on the Canadian mortgage which would make the 'true' spread difference significantly wider.[17] Both US and Canadian mortgage markets give explicit recognition to underwriting concerns related to credit scores and loan to value ratios. The US 'jumbo' and 'prime' terminology roughly corresponds to the Canadian 'posted' and'better-than-posted' or 'special offer' rates. In the US, the 'jumbo' difference of 117 bp for a 15 year term to maturity is roughly comparable to the125 bp difference at the ten year term to maturity for Canada. Explicit pricing of embedded prepayment options in the Canadian mortgage occurs with the 'open' and 'closed' features. The effective difference between a 'variable flex' and 'variable rate open' mortgage is the option to prepay a floating rate mortgage without penalty which is priced at 50 basis points for the five year term. Because interest sensitive prepayment is not priced due to the variable interest rate provision, this rate difference captures the value of the 5 year prepayment option from other factors.

Further examination of the spreads between variable rate and longer term to maturity fixed mortgages rates confirms the bias in the US mortgage funding system to encouraging long term to maturity, fixed rate mortgage borrowing. In the US, the 1 year ARM-15 year term spread is 37 basis points while in Canada the comparable spread for the not-open, variable rate (reset with bank prime rate changes) against a fixed rate, 5 year term to maturity is 113 basis points. Casual inspection of Canadian rates reveals more accurate pricing of the lender's balance sheet risk associated with the longer term to maturity mortgages. Households can lower borrowing costs by originating shorter term to maturity and variable rate contracts that, in turn, locate the risk of surplus changes associated with interest movements on the household balance sheet. In exchange for assuming the associated risks of borrowing with a long amortization period, shorter term to maturity mortgage liability, households are typically rewarded with a substantial discount on the mortgage rate charged. Households unwilling or unable to assume variable interest risk do have access to longer term to maturity financing at mortgage rates that capture the additional risks to the lender surplus.

NOTES

1. By allowing periodic review of borrower creditworthiness, term to maturity restriction also enhances achievement of adequate underwriting standards. The approach of restricting mortgage contract term to maturity to manage systemic risk of residential mortgage funding is not new. Included in the long history of studies up to the S&L crisis advocating some variation of this approach are Guthmann (1938), Muth (1962), Clickner (1967), Findlay and Capozza (1977) and Eskridge (1984).

2. This follows from the distribution free property of options that the price of an option with a longer term to maturity cannot be less than an option with the same contract features but with a shorter term to maturity. This is not a statement about the total value of these options over the full amortization period. More precisely, for a mortgage with a term to maturity that is less than the amortization period, the contract options can be divided into those that are priced over the initial term to maturity and those that are priced over the term remaining between the initial maturity date and the end of the amortization period. It is possible that the value of options priced at originati

when term to maturity and amortization period are equal may be less than the sum of option values over the amortization period for the shorter term to maturity mortgage.

3. Older sources on the history of mortgage contracts includes Anonymous (1856), Frederisksen (1894), Sakolski (1932), Fahey (1934), Rabinovitz (1945) and Skilton (1946).

4. A definitive historical study on the US mortgage market on the eve of the Great Depression is unavailable. The description given by Jaffee and Quigley (2008) differs from Guthmann (1938) where a fully amortizing 10-12 year mortgage term to maturity is identified for the building and loan societies. Guthmann (1938, p.31) refers to "life insurance companies, mutual savings banks, and building and loan associations" as the "backbone of the urban mortgage market". Evidence provided in Jaffee and Quigley and similar sources identify the mortgage contracts used by commercial banks.

5. Though the commercial banking sector was also an important source of mortgage financing during the period where depository institutions directly funded mortgages, the problems with the S&L's were substantively greater. While over 25% of all S&L's failed between 1983 and 1990, only 8% of commercial and savings banks failed (Bentson and Kaufman 1997). Commercial banks also had considerable latitude in asset selection that was not available to S&L's which were largely confined to mortgage lending until various states and the FHLBB loosened these restrictions in the early to mid-1980's.

6. For example, Green and Wachter (2005, p.112) observe: "the US mortgage system – with the implicit government guarantee for Fannie Mae and Freddie Mac – has solved the problem of how to persuade low-risk borrowers to join with higher-risk borrowers in broad mortgage pools, which provide the basis for mortgage-backed securities which can then be sliced up in financial markets."

7. Shin (2009, p.309) observes: "Securitization by itself may not enhance financial stability if the imperative to expand assets drives down lending standards." In turn, such driving down of lending standards would be more difficult if mortgage insurance was accurately priced. In some cases, the systemic risk generated by the mortgage mis-pricing was legislatively mandated, e.g., GAO (2006, 2007); Jaffee and Quigley (2008).

8. From that beginning, a number of improvements to Redington's classical immunization rules have been proposed, aimed at relaxing limitations in this classical formulation (Poitras 2007, 2013). While some limited progress has been made toward incorporating default risk into the immunization problem, e.g., Fooladi et al. (1997); Chance (1990), particular attention has been given to generalizing the classical model to allow for non-parallel shifts in the yield curve. Most available studies aim to identify rules for specifying optimal portfolios that are immunized against instantaneous non-parallel shifts. In contrast, Reitano (1992, 1996) and Poitras (2007) explore the properties of the immunization bounds for fund surplus applicable to exogenously specified non-parallel shifts. In particular, partial durations and convexities are exploited to identify bounds on portfolio gains and losses for an instantaneous unit shift in the term structure..

9. The Mortgage Market Statistical Annual is an essential source for information on mortgage lenders in the US. This source reveals that the largest holder of mortgage related securities by investor type is the GSE portfolios, with about 25-30%, followed by depository institutions with 20-25%. The largest non-depository institutional investors are life insurance companies with about 10% followed by public and private pension funds with a slightly smaller percentage and mutual funds with 5-7%. Information about the aggregate balance sheet composition of households and mortgage lenders is available in a number of sources, e.g., Goodman and Ho (2004); Perli and Sack (2003).

10. In order for aggregate mortgage funding system equity to be unchanged as households and mortgage lenders transfer changes in mortgage market value arising from interest rate changes, it is assumed that the unaggregated balance sheet surplus for households and mortgage lenders is sufficient to absorb such changes. For example, if interest rate increases are 'too large' for mortgage lender equity to absorb, then there will have to be a transfer of equity from the greater financial system before these losses are offset by the mortgage lender gains when interest rates fall, even though there is substantial aggregate equity located on household balance sheets. To deal with this theoretically would require introducing capital constraints and reaction functions that would make the problem too complicated for present purposes.

11. The use of a 30 year amortization period is for ease of exposition. The current maximum allowable amortization period in Canada is 25 years for insured mortgages though some uninsured mortgages are issued with 30 year amortization periods.

12. The study of residential mortgage default has a long history including von Furstenberg (1969), Vandell (1978) and Kau et al. (1994). Most recent studies deal with estimation of default probabilities, e.g., Dalglish (2009) finds for sub-prime mortgages that "default probabilities are highly sensitive to changes in interest rates and house prices" with "large numbers of defaults, when interest rates rose and house prices declined". Studies dealing with interest rate sensitivity of house prices typically also consider other variables such as household income. McQuinn and O'Reilly (2008) observe "empirical models of house prices struggle to achieve credible results concerning the impact of interest rates with coefficients that are frequently insignificant or of the wrong sign". Himmelberg et al. (2005) also find asymmetry where: "the sensitivity of house prices to changes in fundamentals is higher at times when real, long-term interest rates are already low and in cities where expected price growth is high". Peek and Wilcox (1991) examine 40 years of house price data to find: "Real house prices are estimated to decline with increases in real after-tax interest rates, and rise with both cyclical and permanent income increases and increases in the relative cost of materials. Demographic factors, such as the size and age distribution of the population, are also significant determinants of house prices."

13. The standard convention is to express the 'duration' of variables that vary inversely with a minus sign, so that a given variable change will have the correct sign for the dependent variable change. For example, if the $PDUR_{Sy}$ is 5 then an 1% change in yields results in a -5% change in surplus ($-PDUR_{Sy}$ (.01) = -.05). Because the impact of house price changes on surplus is positive, $PDUR_{Sh}$ does not require a minus sign.

14. The value of the prepayment option may also depend on house price changes. The use of prepayment option exercise to liquidate household equity gains has been recognized for many years, e.g., Giliberto and Thibodeau (1989). However, there is still considerable confusion about the embedded prepayment option. For example, studies of the optimal refinancing decision, e.g., Fortin, et al. (2007); Kalotay et al. (2008), ignore the equity liquidation option and focus exclusively on the after-tax gain to option exercise associated with interest rate decreases. Hurst and Stafford (2004) and Wetmore and Ndu (2006) are recent empirical studies demonstrating the significance of equity liquidation motivated refinancing. For a 1991-94 sample, Hurst and Stafford report the troubling result: "households experiencing an unemployment shock and having limited initial liquid assets to draw upon are shown to have been 25% more likely to refinance ... On average, such liquidity-constrained households converted over 2/3 of every dollar of equity they removed into current consumption as mortgage rates plummeted ... producing an estimated expenditure stimulus of *at least* $28 billion dollars." Canner et al. (2002, p.479) estimate that $132 billion in home equity was liquified in the 2001 to early 2002 refinancings.

15. In particular, there is strong empirical evidence that house price increases also increase the exercise probability of the prepayment option. In addition, second order and implicit function complications have been ignored. For example, interest rate increases negatively impact house price changes over time by reducing the amount of mortgage debt that can be assumed with a given household cash flow available to fund mortgage debt. As a consequence, h is an implicit function of interest rates, which impacts both the market value of house prices and the default option. However, this effect is spread over a time line that is long enough to involve changes in the aggregate supply of household residences and other factors, requiring the introduction of additional variables.

16. The primary regulator in Canada is the Office of the Superintendent of Financial Institutions (OSFI) that is an independent agency of the Government of Canada, created in 1987, and reporting to the Minister of Finance. OSFI is responsible for supervising and regulating banks, insurers and some federally registered pension plans. OSFI is also the home for the Office of the Chief Actuary which is responsible for oversight of the Canada Pension Plan, Old Age security and the Student Loans program. In addition, the Canada Mortgage and Housing Corporation also plays a regulatory role as the provider of mortgage insurance and, since 2001, has been a conduit for Canada Mortgage Bonds issued through the CMHC-run entity the Canada Housing Trust. In contrast to the US MBS market, the benchmark MBS in Canada has a 5 year term and there are limits on the aggregate issuance amounts. In addition to such mortgage backed security issuance, selected chartered banks also offer a limited amount of covered bonds. Kiff (2009) provides a further comparison of US and Canadian mortgage markets. Through the CMHC and OSFI, the federal government has authority over the maximum amortization period of mortgages qualifying for CMHC insurance. For example, in a move to control house price increases in major urban centers, in June 2012 the federal government reduced the maximum amortization period to 25 years from 30 years. This is a significant reduction from the maximum 40 year amortization period briefly reached in 2006. The allowable loan to value ratio permitted to avoid mortgage insurance when a mortgage is refinanced was also reduced to 80% from 85%.

16. Typical contract language for the prepayment penalty is reflected in the standard Canadian mortgage contract for the Royal Bank. The contract permits a once a year payment, without penalty, of 10% of the initial principal amount. Payments in excess of 10% incur a penalty on the entire amount of the prepayment. The 10% penalty free prepayment amount cannot be carried forward to the next year. It is also possible to 'double-up' on payments subject to some restrictions. The prepayment penalty is specified as: "The Prepayment charge will be the greater of: i) three month's interest on the amount you want to prepay; and, ii) interest for the remainder of the term on the amount prepaid calculated using the 'interest rate differential'". The 'loss of interest' calculation is based on the rate for a similar mortgage with term to maturity equal to the remaining term to maturity of the mortgage being prepaid.

17. Canadian variable rate mortgages typically have a conversion-to-fixed rate option that allows the variable rate borrower to convert to a fixed rate without penalty during the 5 year term to maturity of the variable rate mortgage. Any of the fixed rate mortgages is available for conversion. For example, after two years of a five year variable rate mortgage a borrower is worried about a significant upward move in rates, there is a one time option to convert to a fixed rate mortgage for any term to maturity mortgage – say 7 years – offered by the lender. In other words, variable rate borrowers are able without penalty to avoid the risk of an expected upward movement in interest rates by locking into a fixed rate mortgage.