Currency Swaps, Fully Hedged Borrowing and Long Term Covered Interest Arbitrage

Currency swaps and fully hedged borrowings offer alternative contracting methods for raising funds directly in a target currency. In plain vanilla form, both contracting methods involve raising an initial principal value in a source currency and exchanging for a principal value and associated sequence of future cash flows denominated in the target currency. Significantly, for the same initial principal value, the pattern of future cash flows for a currency swap and a fully hedged borrowing will differ. The primary objective of this paper is to demonstrate how these differences can be exploited to derive covered interest arbitrage conditions that are applicable to both currency swap rates and long term forward exchange (LTFX) rates. Perfect market conditions are developed connecting spot interest rates derived from the domestic and foreign debt markets with the implied zero coupon interest rates embedded in LTFX rates. Among other uses, the conditions provided can be used to determine when currency swap rates are consistent with interest rates observed in the foreign and domestic debt markets. It is demonstrated that when yield curves have significant shape, information from LTFX rates has value added in determining currency swap rates that are consistent with absence of arbitrage.

I. Introduction and Background

Covered interest arbitrage for money market securities – short-term covered interest arbitrage – has been studied extensively, e.g., Clinton (1988), Taylor (1989), Thatcher and Blenman (2001), Peel and Taylor (2002). It is widely recognized that, in markets not subject to substantial "turbulence", covered interest parity (CIP) holds to within transactions costs for actively traded Eurodeposits. Deviations from CIP have been reported for other types of money market securities such as treasury bills, consistent with restrictions on the ability to execute the underlying covered interest arbitrage trades, e.g., Poitras (1988). Extending CIP to securities with longer term maturities is complicated by a number of factors, such as the market preference for trading coupon instead of zero-coupon bonds. In addition, there are decided complications introduced by the simultaneous presence of LTFX contracts and currency swaps, that provide two different methods for acquiring cash flows denominated in one currency in exchange for cash flows denominated in another currency.[1] The resulting long term covered interest arbitrage conditions are decidedly more difficult, both to derive and to interpret. Previous attempts to specify covered interest parity conditions for currency swaps, e.g., Popper (1993), Fletcher and Taylor (1994, 1996), have stated conditions which are not derived using absence of arbitrage methods. This paper provides appropriate absence-of-arbitrage conditions connecting currency swaps and fully hedged borrowings.

The simplest form of arbitrage pricing relationship for LTFX involves extending the conventional covered interest parity formula for pricing short term forward exchange contracts. In the case of long term, T year forward contracts, covered interest arbitrage gives the following perfect markets result:

Proposition I: **Zero Coupon Long-term Covered Interest Parity (LTCIP)**

Assuming perfect capital markets with riskless zero coupon lending and borrowing available for all maturities greater than one year, then the following absence-of-arbitrage condition holds:

$$F(0,T) = \frac{(1 + zz_T)^T}{(1 + zz_T^*)^T} S_0 \equiv \theta_T S_0 \qquad (1)$$

where: $F(0,T)$ is the long term forward exchange rate observed at time t=0 for delivery at $t=T$ quoted in domestic direct terms, S_0 is the spot exchange rate observed at t=0 quoted in domestic direct terms, zz_T is the annualized T year domestic implied zero coupon interest rate associated with $F(0,T)$, zz^*_T is the T year foreign implied zero coupon interest rate associated with $F(0,T)$.[2]

The short arbitrage support for this condition is derived by observing that the cost of borrowing $\$Q$ for T years using a domestic zero coupon borrowing, at rate zz_T, must be not less than the return on a $\$Q/S_0$ investment for T years in a foreign zero coupon bond where the maturity value of $(\$Q/S_0)(1 + zz_T^*)^T$ is sold forward at $F(0,T)$. Similarly, the long arbitrage condition requires that the return on $\$Q$ invested for T years in a domestic zero coupon bond, at rate zz_T, must not be greater than the fully covered cost of a T year foreign borrowing for the $\$Q$. With perfect markets, combining the short and long arbitrage conditions gives (1).

It is possible for empirical analysis to proceed by substituting observed interest rates into (1) and determining whether the calculated deviations lie within boundaries determined by transactions costs and, possibly, other factors, e.g., Poitras (1988, 1992). In the case of long term covered interest arbitrage, this process is complicated by the market preference for using coupon bonds to raise long term funds. Hence, while (1) is applicable to evaluating deviations for short term money market instruments, such as Eurodeposits, BA's, treasury bills and commercial paper, which feature zero coupons, it is not immediately applicable to markets where coupon bearing securities are actively traded. Even though there are zero coupon instruments which are traded in the long term market such as US Treasury strips (Gregory and Livingston 1992) and long term zero coupon Eurodeposits, the bulk of market liquidity is focussed on trading coupon bearing securities.[3] Compared to the matching of single future cash flows – the return of principal at maturity – used to derive Proposition 1, coupon bonds are decidedly more difficult to use in specifying absence of arbitrage conditions due to the need to reconcile a sequence of future cash flows. In addition, currency swaps and LTFX contracts provide distinct methods for hedging the relevant cash flows.

While a number of studies have recognized a connection between the LTFX and currency swap markets, e.g., Iben (1992), Mordue (1992), Usmen (1994), Mason et al. (1995), these studies did not exploit the potential equivalence between fully hedged borrowings and currency swaps to develop covered interest parity conditions for these transactions. Instead, these and other studies have been concerned with pricing LTFX, e.g., Poitras (1992), Iben (1992), Das (1994), or pricing currency swaps, e.g., Melnik and Plaut (1992), Popper (1993), Fletcher and

Taylor (1994, 1996), Takezawa (1995), Duffie and Huang (1996), Hubner (2001) or with practical problems of identifying the most cost-effective method of financing, e.g., Mason et al. (1995). Despite this attention, the absence of arbitrage requirements connecting fully hedged borrowings and currency swaps have been depicted using long term "swap-covered interest parity", e.g., Popper (1993), Fletcher and Taylor (1994, 1996), Takezawa (1995) which compares the nominal yield on a fixed coupon domestic liability (asset) with the nominal yield on a portfolio combining a fixed coupon foreign liability (asset) with a fixed-to-fixed currency swap. Heuristically, combining the foreign liability with a currency swap transforms the uncovered currency exposure on the foreign liability into a covered domestic currency liability. It is claimed in a number of studies that absence of arbitrage – swap-covered interest parity – requires the foreign yield covered with a currency swap to be equal to the domestic yield.

More precisely, let r_T be the fixed coupon yield on the domestic liability maturing at T, r_T* be the fixed coupon yield on the foreign liability maturing at T, rs_T be the yield for the domestic component of the fixed-to-fixed currency swap maturing at T and rs_T* be the yield for the foreign component of the fixed-to-fixed currency swap maturing at T.[4] On the decision date, domestic debt can be issued at r_T. An alternative method is to issue foreign debt at r_T* and to enter a fixed-to-fixed currency swap which involves making payments based on rs_T and receiving payments based on rs_T*. By incorrectly assuming that the individual cash flows being compared are equal in domestic currency value when evaluated at the initiation date spot FX rate, an 'arbitrage-free' equilibrium condition for the periodic cash flows is specified as: $r_T = r_T^* - rs_T^* + rs_T$. However, as demonstrated in Section V, this condition ignores the mismatch between the principal values of the borrowings when the foreign coupon cash flows are fully covered and, as a result, does not adequately specify an appropriate absence of arbitrage condition. In addition, because this approach involves comparing yields to maturity instead of spot interest rates, the approach may also be subject to the conventional criticisms of traditional yield spread analysis, e.g., Fabozzi (2000).

II. Currency Swap Arbitrage[5]

The various absence of arbitrage and equilibrium conditions to be derived in Sections II-IV depend on equating sets of cash flows which are denominated in the same currency. For simplicity of exposition, plain vanilla fixed-to-fixed currency swaps (Abken 1991) will be assumed with the ***domestic fixed rate borrower*** being Canadian, seeking to acquire a ***fixed rate*** US dollar (*US$*) liability cash flow. While calculation of arbitrage conditions will usually proceed under the assumption of ***perfect markets***, in some cases a difference between borrowing and investing rates will be permitted.[6] For both the currency swap and the fully hedged borrowing par bonds will be used to raise the borrowing amount.[7] In the case of a currency swap, the fundamental condition for initiating the swap is that the Canadian dollar (*C$*) amount of the borrowing being exchanged in the swap, PVC_0, be equal to the domestic currency value of the fixed rate *US$* liability being received in the swap, PVU_0, at time zero, when the swap is initiated. In other words: $PVC_0 = (PVU_0) \cdot S_0$. This condition requires the further exchange of a sequence of future fixed coupon payments and exchange of principals at maturity. A key feature of the currency swap is that the *C$/US$* spot FX rate, S_0, governs the valuation of ***all*** the cash flows in the swap.

A currency swap is an exchange of borrowings denominated in different currencies. Because the exchange of future cash flows embeds a sequence of forward foreign exchange transactions which are valued using the current spot exchange rate, the interest rates quoted on the borrowings in a currency swap will not typically equal the interest rates for borrowing done directly in the domestic and foreign capital markets. Recognizing this, it is also possible to fund the PVC_0 by making a fixed coupon borrowing of PC_0 directly in the domestic capital market, exchanging at the spot exchange rate to get PU_0 and fully hedging each of the fixed *C$* coupon cash flows using a sequence of LTFX contracts. This ***fully hedged borrowing*** will produce a sequence of *US$* coupon payments which will differ in size, both from adjacent fully hedged coupon payments and from the coupon payments in the fixed-to-fixed currency swap. Recognizing that there will be a mismatch of the cash flows in the currency swap and the fully hedged borrowing makes it analytically difficult to derive equilibrium and absence of arbitrage relationships between currency swap rates, domestic interest rates and the implied zero coupon rates embedded in LTFX. In order to reduce complexity, it is expedient to consider an absence of arbitrage solution that does not involve LTFX transactions.

The use of four different funding values – PC_0, the value raised by borrowing (investing) in the domestic debt market; PU_0, the value raised by borrowing (investing) directly in the foreign debt market; PVC_0, the domestic currency value raised (paid) by the currency swap; and, PVU_0, the foreign currency value raised (paid) by the currency swap – is to facilitate the construction of arbitrage relationships involving currency swaps. If the conditions $PC_0 = PVC_0$ and $PU_0 = PVU_0$ could be used, this would satisfy the $t=0$ self-financing requirement for arbitrage. However, differences in currency swap rates and direct borrowing rates undermine this solution by creating a coupon and principal mismatch. For example, a dealer could borrow PVC_0 doing a par bond currency swap to acquire PVU_0. This amount can be invested in a par bond borrowing, with principal value PU_0 with each of the cash flows being sold forward in a fully hedged investment to acquire terminal principal value of $PC_0 = F(0,T)\ PU_0$. Even ignoring the terminal principal value mismatch, insofar as domestic borrowing (investing) rates differ from currency swap rates there will be a coupon mismatch. This coupon mismatch can be eliminated by appropriate adjustment of the principal value underlying the currency swap.

To add some numbers to the description of the cash flows, consider the cash flows associated with PU_0 = *US$*100,000 ***borrowed*** at r_T^* = 5% that is exchanged at S_0 = 1.5 to get PC_0 =

*F$*150,000 which is ***invested*** at i_T = 9%. Now, consider the cash flows from this position combined with a currency swap. Take the prevailing fixed-to-fixed swap rates to be pay rs_T = 9.5% and receive is_T* = 4.5%.[8] If the par bond swap is done with principal $PVC_0 = M = PC_0$ and $PVU_0 = M^* = PU_0$, then $(rs_T)(PVC_0)$ = *F$*14,250 and $(is_T^*)(PVU_0)$ = *US$*4500 and at $t = T$ the maturing investment will return principal of PVC_0 which is exchanged for PVU_0 to settle the swap. The PVU_0 is then used to liquidate the principal on the initial *US$* borrowing of PU_0. However, if M and M^* are the principal values for the swap, then there will be mismatching of both the US and non-US coupon cash flows because rs_T does not equal i_T and r_T^* does not equal is_T^*. The non-US investment will be earning $(i_T)(PC_0)$ = *F$*13,500 while the swap will be paying *F$*14,250 and receiving *US$*4500. To fully cover the non-US coupon cash flows, the swap principal must be adjusted to $M = (i_T / rs_T)(PC_0)$ that will produce a coupon cash flow of $i_T\ PC_0$ from the swap. This adjustment creates a principal mismatch between the swap and the borrowing, again undermining the arbitrage requirements.

To derive an absence of arbitrage condition, it is sufficient to recognize that it is possible to adjust the principal value in the swap to offset either the domestic or the foreign coupon payments, with the other coupon payment streams being mismatched. For example, adjusting the principal to equate the domestic coupons involves reducing the principal on the swap to $PVC_0 = PC_0\ (i_T/rs_T)$ which would be \$142,105.25 = \$150,000 (9/9.5) in the example. While this would provide a match of the \$13,500 coupon on the investment with the coupons received from the swap, the US \$100,000 borrowing would require a \$5000 coupon payment with the swap receiving \$4263.16 = $is_T^*\ PVU_0 = is_T^*\ (i_T / rs_T)\ (PC_0 / S_0)$ = (.045)\$94,736.84. At maturity, the *C$* investment would pay C\$7,894.75 = $(1 - (i_T/rs_T))\ PC_0$ more than was required to settle the *C$* payment on the swap and the *US$* swap payment would be (*US$*5,263.16) = $((i_T / rs_T) - 1)\ (PC_0 / S_0)$ less than was required to settle the *US$* borrowing. Because these end-of-period values are known at maturity, the *C$*7894.75 can be sold forward at *F(0,T)*. Absence of arbitrage requires that ***when appropriately discounted***, the value of the coupon stream of *US$*500 and principal mismatch of *US$*5,263,16 not be less than the *C$*7894.75/*F(0,T)* payment.

Assuming that rates for direct lending and borrowing are equal ($i=r,\ is^*=rs^*$), the general cash flow pattern of the trade with the adjusted domestic principal value can be captured as:[9]

	Foreign Debt	Swap Receipt	Swap Payment	Domestic I.
t=1	$-r_T^*\ PU_0$	$rs_T^*(r_T/rs_T)PU_0$	$-rs_T\ (r_T/rs_T)PC_0$	$r_T\ PC_0$
t=2	$-r_T^*\ PU_0$	$rs_T^*(r_T/rs_T)PU_0$	$-rs_T\ (r_T/rs_T)PC_0$	$r_T\ PC_0$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
t=T	$-(1+r_T^*)\ PU_0$	$(1+rs_T^*(r_T/rs_T)PU_0$	$-(1+rs_T)(r_T/rs_T)PC_0$	$(1+r_T)\ PC_0$

As constructed, the swap coupon payments and the coupons on the *C$* investment cancel. This leaves the residual principal value paid at maturity to be $(1 - (r_T/rs_T))PC_0$. This amount can be given a certain value by selling (buying for $r > rs$) the *C$* amount forward at *F(0,T)*. This certain *t=T US$* value can be equated with the future value of the stream of net coupon payments on the swap receipt/foreign debt position $(-r_T^* + rs_T^*(r_T/rs_T))PU_0$ plus the net return of principal $((r_T/rs_T) - 1)PU_0$. Using this information, a CIP condition for currency swaps now can be derived by manipulation of these known residual cash flows.

More generally, the present value of coupon payments for the case with matched *C$* coupons and the *US$* coupon mismatch would be:

$$\sum_{t=1}^{T} \frac{\left\{ \frac{r_T}{rs_T}\, rs_T^* \;-\; r_T^* \right\}}{(1\;+\;z_t^*)^t}\ \frac{PC_0}{S_0}$$

where z_t^* (z_t) is the spot interest rate associated with cash flows at time t in the foreign (domestic) debt market. Substituting for the value of the LTFX rate from (1), the ***present value*** of the principal mismatches at maturity (which are to be added to the present value of the coupon mismatch) gives:

$$\frac{(1 - \frac{r_T}{rs_T})}{(1 + z_T^*)^T} \left\{ \frac{PC_0}{F(0,T)} - PU_0 \right\} = (1 - \frac{r_T}{rs_T}) \left\{ \frac{(1+zz_T^*)^T}{(1+z_T^*)^T} \frac{1}{(1+zz_T)^T} - \frac{1}{(1+z_T^*)^T} \right\} PU_0$$

From the assumption of perfect markets, combining the long and short absence of arbitrage conditions requires that the sum of these two expressions must be zero which leads to the condition:

$$\sum_{t=1}^{T} \frac{\frac{r_T}{rs_T} rs_T^* - r_T^*}{(1 + z_t^*)^t} = (1 - \frac{r_T}{rs_T}) \left\{ \frac{1}{(1 + z_T^*)^T} - \frac{(1 + zz_T^*)^T}{(1 + z_T^*)^T} \frac{1}{(1 + zz_T)^T} \right\}$$

A similar absence of arbitrage condition would apply when the *US$* coupon payments are equated and the *C$* coupons are mismatched:

$$\sum_{t=1}^{T} \frac{\frac{r_T^*}{rs_T^*} rs_T - r_T}{(1 + z_t)^t} = (1 - \frac{r_T^*}{rs_T^*}) \left\{ \frac{1}{(1 + z_T)^T} - \frac{(1 + zz_T)^T}{(1 + z_T)^T} \frac{1}{(1 + zz_T^*)^T} \right\} \qquad (7)$$

As previously, the equality is required because it is possible to take either short or long positions in the relevant cash flows.

Recalling from (1) that $F(0,T) = \theta_T S_0 \equiv \{(1 + z_t) / (1 + z_t^*)\}^T S_0$, the absence of arbitrage conditions for both the mismatched *C$* coupon cash flows and the mismatched *US$* coupon cash flows can be manipulated to get two CIP agios associated with mismatching of either the domestic or foreign coupon stream. This leads to the following:

Proposition II: Currency Swap Covered Interest Parity[10]

Assuming perfect capital markets, absence of arbitrage in *T* period currency swap rates produces the following interest agios, $\theta_{T,I}$ for domestic coupon matching and $\theta_{T,II}$ for foreign coupon matching:

$$\theta_{T,I} = \left\{ 1 - \frac{r_T rs_T^{\,*} - rs_T r_T^{\,*}}{(rs_T - r_T) r_T^{\,*}} \left[(1 + z_T^{\,*})^T - \frac{(1 + z_T^{\,*})^T}{(1 + r_T^{\,*})^T} \right] \right\}^{-1} \qquad (I)$$

$$\theta_{T,II} = 1 - \frac{r_T^{\,*} rs_T - rs_T^{\,*} r_T}{(rs_T^{\,*} - r_T^{\,*}) r_T} \left\{ (1 + z_T)^T - \frac{(1 + z_T)^T}{(1 + r_T)^T} \right\} \qquad (II)$$

Absence of arbitrage is obtained when $\theta_{T,I} = \theta_{T,II}$.

While it is tempting to simplify Proposition II to a single equation by equating (I) and (II) and solving, this only provides a more complicated relationship between the various r, rs, r^*, rs^*, z and z*, i.e., it is not possible to derive a significant general simplification. Given this, it is necessary to recognize the solutions given in Proposition II have singularity points. In (I), the singularity point occurs when $r_T = rs_T$ and in (II) there is a singularity point where $r_T^* = rs_T^*$. In other words, when the domestic interest rate equals the domestic swap rate and the foreign interest rate equals the foreign swap rate, it is not possible to use Proposition II to solve for an absence of arbitrage equilibrium for θ. At these values, absence of arbitrage is automatically satisfied. Another singularity point occurs when $zz^*_T = zz_T$, which implies $\theta_T = 1$. This restriction requires that $(r_T^* \, rs_T) = (rs_T^* \, r_T)$ in (I) and, equivalently, $(r_T \, rs_T^*) = (rs_T \, r_T^*)$ in (II). These equality conditions can be satisfied at the singularity point.

To illustrate the absence of arbitrage conditions for $\theta_{T,I}$ and $\theta_{T,II}$ in Proposition II, consider two sets of example interest rates, selected to satisfy the swap-covered interest parity conditions: r_T^* = 5%, rs_T = 10%, rs_T^* = 6%, r_T = 9%; and, r_T^* = 5%, rs_T = 6.5%, rs_T^* = 5.5% and r_T = 7%, which have less difference in foreign and domestic rates than the first set of rates. For simplicity, further assume that the foreign and domestic yield curves are flat, permitting $z_T = r_T$ and $z_T^* = r_T^*$. For these values:

r_T*=5%, rs_T=10%, rs_T*=6% r_T=9% r_T*=5%, rs_T=6.5%, rs_T*=5.5%, r_T=7%

T	$\theta_{T,I}$	$\theta_{T,II}$	T	$\theta_{T,I}$	$\theta_{T,II}$

2	1.08932	1.0836	2	1.03173	1.03097
3	1.1443	1.13112	3	1.04963	1.04799
4	1.20832	1.18293	4	1.06912	1.06611
5	1.28374	1.23939	5	1.09038	1.0854
10	2.01254	1.60772	10	1.23254	1.20242

The divergence in the θ_T calculated from (I) and (II) indicates that the set of rates used, $\{r_T*, rs_T, rs_T*, r_T\}$, that were selected to satisfy the "swap-covered interest arbitrage" condition, are actually not consistent with absence of arbitrage. Because both (I) and (II) must be satisfied in order to solve for θ_T, it is necessary to adjust at least two rates. Because r and $r*$ will be determined by factors outside the currency swap market, this implies that rs and $rs*$ have to be determined consistent with maintaining absence of arbitrage in the LTFX market.

Given r_T and r_T* together with the assumption that the foreign and domestic yield curves are flat, Table 1 provides relevant solutions for rs_T, rs_T* and θ_T over a range of interest rate scenarios such that $\theta_{T,I} = \theta_{T,II}$. Significantly, it is found that, for a specific (r_T, r_T*), θ_T ***does not vary as arbitrage-free currency swap rates vary***. The implication is that if absence of arbitrage is satisfied in currency swap rates then LTFX can be determined independent of (r_T - rs_T) and (r_T* - rs_T*). Examination of ΔSCIP, the deviation of the calculated by substituting the arbitrage free currency swap rates into the swap-covered interest parity condition, is also revealing. Deviations can be measured in basis points with the absolute value of deviations increasing with maturity. This implies that, while it is possible to identify swap rates which satisfy the swap-covered interest parity condition and which also admit arbitrage opportunities, swap rates which are consistent with absence of arbitrage do approximately satisfy the swap-covered interest parity condition when the swap rate is close to the domestic borrowing rate and the difference between foreign and domestic interest rate levels is small, conditions which are often satisfied in practice. This explains the empirical results reported by Fletcher and Taylor (1994, 1996). However, when the difference between foreign and domestic interest rate levels is large, then deviations from swap covered interest parity can be large, especially for long maturity swaps, e.g., 45.78 basis points for r_T = 15% and r_T* = 5%.

As noted, one significant feature of θ_T derived from Proposition II is that the LTFX rate is virtually independent of the difference between arbitrage-free currency swap rates and domestic interest rates. More precisely, tiven S_0 , r_T and r_T*, the theoretical LTFX rate calculated using $F(0,T) = \theta_T\ S_0$ does not change as arbitrage-free currency swap rates change. Because Proposition II provides absence of arbitrage restrictions on the currency swap rates, it follows that (I) and (II) in Proposition II also implicitly specify restrictions on the relationship between (zz_T, zz_T*) and (r_T , r_T*). This is reflected in the change in θ_T when (r_T, r_T*) change. As expected, as the difference between (r_T , r_T*) narrows, θ_T gets closer to one. Yet, the theoretical absence of arbitrage conditions in Proposition II are not fully sufficient because the connection between fully hedged borrowings and currency swaps is ignored. The Proposition only provides absence of arbitrage restrictions associated with using a currency swap to hedge a foreign investment (borrowing) financed with a domestic borrowing (investment). In Section 4, it is demonstrated that extending the transactions to allow individual coupon and principal cash flows to be hedged using LTFX contracts permits the term structure of LTFX to be considered and, as a consequence, provides a connection between LTFX rates and the shape of the domestic and foreign yield curves.

While Proposition II does not directly incorporate the term structure of LTFX, by allowing for differences between r_T and z_T and r_T* and z_T* it does allow for cases where the domestic and foreign yield curves are not flat. Recognizing that the examples from Table 1 suppressed consideration of yield curve shape by assuming (r=z) and (r*=z*), Table 2 considers the implications of introducing yield curve shape into the calculations. The values used for z and z* were selected to 'stress test' Proposition II, rather than to reflect specific empirical situations.

This table demonstrates that allowing for yield curve shape does have a significant impact on θ_T. For example, when the domestic yield curve is upward sloping and the foreign yield curve is inverted (upward sloping), then θ_T will be lower (higher) for $r^* < r$. There are also small differences in the calculated *rs*. Though ΔSCIP deviations are generally slightly larger when a significant amount of yield curve shape is introduced, there are a few cases where the differences are smaller.

III. Fully Hedged Borrowing and LTFX Equilibrium

To illustrate equilibrium in the LTFX market taken in isolation from the currency swap market, consider the following sequence of *C$/US$* FX rates quoted by the Royal Bank of Canada, for Aug. 8, 1994:

$$S_0 = 1.3778 \qquad F(0,1) = 1.3960$$
$$F(0,2) = 1.4198 \qquad F(0,3) = 1.4428$$
$$F(0,4) = 1.4633 \qquad F(0,5) = 1.4833$$

where *F(0,T)* is the forward exchange observed at t=0 for delivery at $t=T$ where t is measured in years. Assume, for example, that the Canadian borrower raised *C$*137,780 using a 5 year fixed coupon borrowing. When translated at the spot FX rate, the principal value of the borrowing would provide *US$*100,000. If the fixed *C$* borrowing had a coupon of, say, 10% paid annually then the resulting fully hedged *US$* cash flows would be:

t=1: *C$*13,778/1.3960 = *US$*9869.63
t=2: *C$*13,778/1.4198 = *US$*9704.18
t=3: *C$*13,778/1.4428 = *US$*9549.49
t=4: *C$*13,778/1.4633 = *US$*9415.70
t=5: (*C$*13,778 + *C$*137,780)/1.4833 = *US$*102,176

This sequence of uneven *US$* cash flows can be compared with the *US$* cash flows from the fixed-to-fixed currency swap which would be: five ***equal*** annual coupon payments of (*US$*100,000)*($rs_T$) each period plus the return of principal *US$*100,000 at maturity.

Compared to a fully hedged *C$* borrowing, the currency swap would require lower *US$* cash flows at the beginning, with the difference progressively narrowing to the point where the currency swap cash flows are higher than the fully hedged borrowing with the highest differential occurring at maturity. The opposite situation would apply for the fully hedged *US$* borrower. Comparing a fully hedged borrowing in *US$* to acquire *C$* with a fixed-to-fixed currency swap out of *US$* and into *C$*, the fully hedged borrower would issue fixed coupon *US$* debt, exchange at the spot exchange rate to acquire *C$*, and fully hedge each of the *C$* cash flows required to make payments on the *US$* issue. If the *US$*100,000 issue were made at, say, 9.25%, then using the 8/8/94 LTFX rates the cash flows would be:[11]

t=1: (*US$*9250)(1.3960) = *C$*12,913
t=2: (*US$*9250)(1.4198) = *C$*13,133.20
t=3: (*US$*9250)(1.4428) = *C$*13,345.90
t=4: (*US$*9250)(1.4633) = *C$*13,535.50
t=5: *US$*(100,000 + 9250)(1.4833) = *C$*162,051

Compared to a fully hedged borrowing, the associated (rs_T *C$* fixed)-to-($rs_T$* *US$* fixed) currency swap would have higher *C$* cash flows at the beginning, narrowing to the point where the currency swap cash flows become lower at maturity.

As before, the perfect markets assumption is retained.[12] It is further assumed that there is no funding advantage for either the foreign or domestic fully hedged borrowing arising from implicit differences between quoted interest rates for direct borrowings (investing) in the cash market or the implied zero coupon rates associated with LTFX. This ***assumption***, that there is ***no funding advantage*** permits differences in interest rates to be analyzed as deviations from the equilibrium conditions. For the fully hedged borrowing to acquire *US$*, the cash flows are denominated in *US$* even though the actual debt payments are made in *C$*. Hence, the fully hedged borrower is providing a sequence of *US$* cash flows in exchange for PUX_0. All cash flows are in *US$* and can be discounted using US spot interest rates, appropriately adjusted for credit risk. Because the perfect markets assumption involves ***no default risk in LTFX*** and, as before, the relevant default risk associated with the inherent risk of the borrower generating sufficient *US$* cash flows (that, due to institutional rigidities, may be different than that of the borrower's ability to generate *C$* cash flows) is ignored.

Initially consider a *T* period par bond borrowing done directly in the Canadian debt market at r_T:

$$PC_0 = \sum_{t=1}^{T} \frac{r_t \, PC_0}{(1 + z_t)^t} + \frac{PC_0}{(1 + z_T)^T} \qquad (8)$$

Now consider a fully hedged borrowing, where the par bond is issued directly in the Canadian debt market and the cash flows are fully hedged into *US$*. For a par bond, the associated fully hedged *US$* cash flows used to pay the *C$* borrowing are discounted using the *US$* spot interest rates:

$$\frac{PC_0}{S_0} = PU_0 = \sum_{t=1}^{T} \frac{[r_t \, PC_0 \; / \; F(0,t)]}{(1 + z_t*)^t} + \frac{[PC_0 \; / \; F(0,T)]}{(1 + z_T*)^T} \qquad (9)$$

The discounting is done with z_t* because a sequence of *US$* cash flows is generating a *US$* denominated borrowing unconstrained by the requirement of a specific yield to maturity. It is now possible to derive an ***equilibrium*** condition for LTFX.

This discussion leads to the following:

Proposition III: **No funding advantage equilibrium condition for LTFX**

Assuming there perfect markets and no funding advantage for either the domestic or foreign fully hedged borrowing, then for each t=1 to T it follows:[13]

$$\frac{r_t \, PC_0 \; / \; S_0}{(1 + z_t)^t} = \frac{r_t \, PC_0 \; / \; F(0,t)}{(1 + z_t*)^t} \quad \Rightarrow \quad F(0,t) = \frac{(1 + z_t)^t}{(1 + z_t*)^t} S_0$$

From Proposition 1, the equilibrium condition for LTFX derived from the fully hedged borrowing becomes, for each $t \in [1,T]$:

$$\frac{1 + z_t}{1 + z_t*} = \frac{1 + zz_t}{1 + zz_t*} \qquad (10)$$

In words, for each t, ***the spot interest rate agio derived from the foreign and domestic debt markets must equal the implied zero coupon interest agio from the LTFX market***.

In the short-term market, the corresponding version of (10) – short term CIP – is automatically satisfied because the traded securities have zero-coupons. Various studies have identified empirically that short term CIP is most applicable to Eurodeposits. Extending the analysis to long-term securities introduces complications associated with coupon-bearing securities. This raises at least two ***empirical*** questions. First, are implied zero coupon interest rates (spot interest rates) calculated from the coupon yield curve the appropriate rates to use in evaluating long-term covered interest parity? Second, if spot interest rates are the appropriate rates to use, which particular security yield curve is most applicable? Considerable confusion was created in early studies of short-term covered interest parity surrounding the interpretation of observed significant deviations from short term CIP when treasury bill rates were used, e.g., Frenkel and Levich (1975). In the long-term case, observed deviations using spot interest rates are somewhat more difficult because (10) is only an equilibrium condition and not an absence of arbitrage condition. Poitras (1992) presents empirical evidence for some significant deviation from (10) using spot interest rates calculated from the government securities market.

IV. Currency Swaps and Fully Hedged Borrowing

The more complicated absence of arbitrage connection between currency swaps and fully hedged borrowings can now be identified by comparing the cash flows associated with entering a currency swap raising PVU_0 by exchanging for PVC_0 through a currency swap at rs_T. Using par bonds, this swap will generate coupon payments of $rs_T* \ PVU_0$ and coupon receipts of $rs_T \ PVC_0$ together with the exchange of principal at initiation and maturity. The *C$* coupon cash flows from the swap can be fully matched by borrowing PC_0 in the Canadian market at r_T where $PVC_0 = (r_T / rs_T)PC_0$. Simultaneous full hedging for each of the *US$* cash flows required to settle the *C$* borrowing generates a sequence of *US$* cash flows which are mismatched with the *US$* cash flows from the currency swap. In addition, because PC_0 does not equal PVC_0, there is a mismatch in the principal values at maturity which has to be taken into account. However, adjusting for the principal mismatch by adding a LTFX transaction for $(1 - (r_T / rs_T))\, PC_0$ at maturity, both the fully hedged borrowing and the currency swap can be constructed to generate the same sequence of *C$* cash flows. It follows that the present values of the *US$* cash flows must be equal in order to satisfy absence of arbitrage.

Much as in the case of (I) and (II) from Proposition II, for currency swaps there will also be two conditions for fully hedged borrowing (see Proposition IV below). In this case, the two conditions will be for matched *C$*- mismatched *US$* and matched *US$*- mismatched *C$* borrowings, respectively. In either case, it is not possible to construct an uncomplicated absence of arbitrage condition due to the mismatching of the relevant cash flows on the two borrowings,. Given this, for the matched *C$* case, consider a par bond borrowing done in the domestic market with coupon payments of $(r_T)(PC_0)$ that is equated with the cash flows from a currency swap that raises PVC_0. The currency swap cash flows are divided by S_0, where $M = PVC_0 = (r_T / rs_T)PC_0$, adjusted by an appropriate LTFX transaction to equate the market values of the borrowings. This gives:

$$\sum_{t=1}^{T} \frac{(r_T\, PC_0)\ /\ F(0,t)}{(1 + z_t*)^t} + \frac{PC_0\ /\ F(0,T)}{(1 + z_T*)^T} =$$

$$\sum_{t=1}^{T} \frac{(rs_T^*\ PVC_0)\ /\ S_0}{(1 + z_t*)^t} + \frac{PVC_0\ /\ S_0}{(1 + z_T*)^T} + \frac{(1 - \frac{r_T}{rs_T})}{(1 + z_T^*)^T}\, \frac{PC_0}{F(0,T)}$$

Manipulation provides:

$$\sum_{t=1}^{T} \frac{rs_T\,(\theta_t^{-1} - [rs_T^*/rs_T])}{(1 + z_t*)^t} = \frac{(1 - \theta_T^{-1})}{(1 + z_T*)^T}$$

where θ_t is the implied zero coupon interest agio for *F(0,T)* and primes have been dropped for ease of notation. A similar condition can be derived for the matched *US$* case:

$$\sum_{t=1}^{T} \frac{rs_T^*(\theta_t - [rs_T/rs_T^*])}{(1 + z_t)^t} = \frac{1 - \theta_T}{(1 + z_T)^T}$$

As with currency swaps, these two conditions can be solved for θ_T to provide restrictions on the relevant variables.

Solving the absence of arbitrage conditions gives:

Proposition IV: Absence of Arbitrage between Currency Swap Rates and LTFX

Assuming perfect capital markets, absence of arbitrage between currency swap rates and LTFX produces the following interest agios, $\theta_{T,a}$ for the domestic market and $\theta_{T,b}$ for the foreign market:

$$\theta_{T,A} = 1 - (1 + z_T)^T \sum_{t=1}^{T} \frac{rs_T^* \left\{ \theta_t - \dfrac{rs_T}{rs_T^*} \right\}}{(1 + z_t)^t} \qquad (A)$$

$$\theta_{T,B} = \left[1 - (1 + z_T^*)^T \sum_{t=1}^{T} \frac{rs_T \left\{ \theta_t^{-1} - \dfrac{rs_T^*}{rs_T} \right\}}{(1 + z_t^*)^t} \right]^{-1} \qquad (B)$$

Absence of arbitrage is obtained when $\theta_{T,a} = \theta_{T,b}$.

Compared to Proposition II, the interest agios in Proposition IV require more information to evaluate due to the presence of the θ_t on the rhs of the equations. It is possible to simplify Proposition IV by exploiting the ***equilibrium*** condition (10) for LTFX, but this involves substituting an equilibrium condition into an absence of arbitrage condition. This changes the substantive interpretation of the interest agio expressions in Proposition IV. Substituting (10) for

$t < T$ into (A) and (B) in Proposition IV and manipulating gives:

Proposition V: Equilibrium between Currency Swaps and LTFX

Assuming (10) is a valid equilibrium condition for LTFX, then (A) and (B) from Proposition 3 can be determined as:

$$\theta_{T,a} = 1 - (1 + z_T)^T \left\{ \left[\frac{rs_T^*}{r_T^*}(1 - \frac{1}{(1 + r_T^*)^T}) \right] - \left[\frac{rs_T}{r_T}(1 - \frac{1}{(1 + r_T)^T}) \right] \right\}$$

$$\theta_{T,b} = \left\{ 1 - (1 + z_T^*)^T \left\{ \left[\frac{rs_T}{r_T}(1 - \frac{1}{(1 + r_T)^T}) \right] - \left[\frac{rs_T^*}{r_T^*}(1 - \frac{1}{(1 + r_T^*)^T}) \right] \right\} \right\}^{-1}$$

where the interest agios have been derived using the perfect markets assumption.

Assuming $z_T = r_T$ and $z_T^* = r_T^*$, evaluating $\theta_{T,a}$ and $\theta_{T,b}$ using the inputs from Table 1 reveals identical values for θ_T and rs_T. It follows that (10) is a valid result when foreign and domestic yield curves are flat, a corollary which can be investigated empirically. As illustrated in Table 3, the validity of $\theta_{T,a}$ and $\theta_{T,b}$ does not carry over to cases where the yield curve has significant shape, supporting the related results provided in Table 2.

Table 3 compares two types of results for some selected cases where $z_T \neq r_T$ and $z_T^* \neq r_T^*$. Though the relevant rates selected are may appear to be empirically extreme, the rates were chosen to illustrate differences in the two sets of conditions. Recalling that the results given in Table 1 were derived under the assumption that the foreign and domestic yield curves were flat, the results given in Table 3 verify that giving significant shape to the yield curves does not have substantive impact on currency swap rates calculated from (I) and (II), though LTFX does change when there is significant differences in the slopes of the foreign and domestic yield curves. Changes in rs_T, rs_T^* and θ_T can be measured in basis points. However, this is not the case for solutions obtained from $\theta_{T,a}$ and $\theta_{T,b}$. For the most extreme slopes considered, deviations in swap rates calculated using (A) and (B) differ by over 80 basis points from rates calculated using (I) and (II). Sizable differences in θ_T are also observed. These differences occur with various combinations of inverted and upward sloping yield curves. Interpretation of these differences is aided by observing that for (A) and (B) $\theta_{T,a} = \{(1 + z_T)/(1 + z_T^*)\}^T$. For a given spread between yield and spot interest rate, the swap rate differences tend to be larger for shorter maturities. This indicates that, while (10) may be an appropriate condition when yield curves are relatively flat, there may be difficulties when yield curves have significant shape. These difficulties will appear as substantial deviations when spot interest rates are used to calculate (2).[14]

V. Swap-Covered Interest Parity[15]

Popper (1993, p.441) describes the trading mechanics underlying the conventional approach as:

> An investor may either invest in a domestic currency asset and earn the per-period return of (r_T) or invest abroad and cover for foreign exchange risk with a currency swap. The swap-covered foreign return is the sum of the uncovered foreign-currency return and the net currency swap payments. Denote the fixed dollar rate exchanged in the currency swap as (rs_T) and denote the fixed non-dollar rate exchanged in the currency swap as (rs_T^*). In the completed foreign transaction, the investor earns the per-period uncovered foreign-currency return r_T^*, while paying the foreign-currency swap rate, rs_T^* and receiving the domestic currency swap rate, rs_T. Thus, the net foreign covered return is: ($r_T^* + rs_T - rs_T^*$). Arbitrage equates the two returns and gives a swap-covered parity condition: $r_T = (r_T^* + rs_T - rs_T^*)$

A virtually identical motivation appears in Fletcher and Taylor (1994, p.461) and Fletcher and Taylor (1996, p.530). The discussion in Sections II-IV above demonstrates that, while the underlying transactions are inconsistent with an absence of arbitrage argument, the swap-covered interest parity condition is reasonably accurate in most practical situations.[16]

Is is possible to develop an arbitrage foundation for the swap-covered interest parity condition? An arbitrage can be heuristically defined as: a riskless trading strategy involving no ***net*** investment of funds that generates only zero or positive profit in all feasible future states of the world. Based on the discussion presented in Popper (1993) and Fletcher and Taylor (1994, 1996), it is difficult to identify the underlying transactions needed to satisfy the requirements for an arbitrage trading strategy. To attempt to make sense of the Popper (1993) approach to swap-covered interest parity, consider the value of a fixed coupon par bond being issued directly in the domestic (Canadian) market to raise PC_0:

$$PC_0 = \sum_{t=1}^{T} \frac{CR}{(1 + r_T)^t} + \frac{PC_0}{(1 + r_T)^T} = \sum_{t=1}^{T} \frac{r_T\, PC_0}{(1 + r_T)^t} + \frac{PC_0}{(1 + r_T)^T} \qquad (11)$$

This borrowing is apparently being directly compared to the ***domestic currency cash flow*** from a portfolio which combines a foreign (US) par bond investment of PU_0, financed with PC_0 exchanged at S_0, which is fully covered with a currency swap. ***Assuming incorrectly*** that the principal values of the foreign investment and foreign component of the currency swap cancel at maturity, the discussion in Popper (1993) seems to imply:

$$PC_0 = S_0\, PU_0 = \sum_{t=1}^{T} \frac{(S_0\, CR^*) - (S_0\, SR^*) + SR}{(1 + r_T)^t} + \frac{PC_0}{(1 + r_T)^T}$$

$$= \sum_{t=1}^{T} \frac{(S_0\, PU_0\, r_T^*) - (S_0\, PU_0\, rs_T^*) + (rs_T\, PC_0)}{(1 + r_T)^t} + \frac{PC_0}{(1 + r_T)^T}$$

$$= \sum_{t=1}^{T} \frac{(r_T^* - rs_T^* + rs_T)\, PC_0}{(1 + r_T)^t} + \frac{PC_0}{(1 + r_T)^T} \qquad (12)$$

where CR^*, SR^* and SR are the appropriate coupons on the foreign investment and the foreign and domestic components of the currency swap, respectively. Because covered interest arbitrage requires that the cash flows being exchanged are of equal value at the time the swap is initiated, comparison of (11) with (12) provides the swap-covered interest parity conclusion that $r_T = (r_T^* + rs_T - rs_T^*)$ is an appropriate absence of arbitrage restriction to impose on currency swap pricing. Because the denominator in the present value calculation cancels out in the comparison of the cash flows, this condition does ***not*** depend on the relationship between z_t^* and z_t.

Unfortunately, ***the above argument is incomplete*** as stated because the appropriate method of fully covering all the foreign cash flows is not adequately addressed. Uncovered future foreign cash flows involve the need to use uncertain future exchange rates to convert the cash flows to domestic currency. This makes the return on the trading strategy uncertain, eliminating the use of arbitrage arguments to derive the condition. Without a more accurate statement of the cash flows involved, it is difficult to make sense of the conventional "arbitrage" condition. To be an arbitrage, the foreign currency denominated cash flows, both coupons and principal, must be fully covered. Because $r_T^* \neq rs_T^*$, in general, this will require the underlying principals for the foreign investment and the currency swap to be different in order to fully cover the coupon cash flows. However, if this is done, then the principal values exchanged at maturity will not be fully covered. While it would be possible to cover the residual principal amounts with a LTFX position, this is not incorporated into the conventional analysis and would require a restatement of the $r_T = (r_T^* + rs_T - rs_T^*)$ condition.

The situation becomes more complicated if dealer intermediation in currency swap quotes is taken into account. Consider a theoretical cash flow diagram for the various transactions involved in the intermediation process:

Example of Cash Flow Patterns
for Dealer Currency Swap Intermediation

US\$ Transactions	**Swap Transactions**		**C\$ Transactions**
	Receive	**Pay**	
Borrow at r^* = 5%	is^* = 4.5%	rs = 9.5%	Invest at i = 9%
	Pay	**Receive**	
Invest at i^* = 4.5%	rs^* = 5%	is = 9%	Borrow at r = 9.5%

As constructed, the *C\$* borrower would be indifferent between borrowing directly in the Canadian debt market at r = 9.5% or paying rs = 9.5% on the swap (receiving 4.5% *US$*). The *US$* borrower would be indifferent between borrowing at r^* = 5% directly or paying rs^* = 5% on the swap. A motivation for doing the swap would be the inability of foreign borrower seeking *C$* to access the Canadian debt market at the competitive rate of 9.5% due to differential credit assessment imposed on foreign borrowers. On the receive side of the swap, there may be funding advantages for banking intermediaries created in a specific currency due to excess demand from interest rate insensitive retail investors seeking to make a currency play.

In practice, the swap intermediary quoting a fixed-to-fixed swap rate of receiving 4,5% *US$* and paying 9.5% *C$* would try to match this trade with another fixed-to-fixed swap trade that is paying 5% *US$* and receiving 9% *C$*. This would provide a 50bp *US$* + 50bp *C$* for both sides of the trade. If the swap dealer was unable to match with another trade, the fixed-to-fixed 4.5% *US$* for 9.5% *C$* trade would be covered by borrowing *C$* at 9.5%, exchanged through the swap to get the *US$* principal and investing at 4.5% *US$*. Similarly, the fixed-to-fixed 5% *US$* for 9% *C$* trade would be matched by borrowing *US$* at 5%, exchanging through the swap to get *C$* principal that is invested at 9% *C$*. It follows that "swap-covered interest parity" needs to be evaluated using the domestic and foreign borrowing rates, r and r^*, and the two paying rates, rs and rs^*.

VI. Conclusions

The limited number of previous studies on long term covered interest parity, both for swaps and LTFX contracts, have a number of shortcomings. In particular, the pricing condition typically associated with currency swaps, "swap-covered interest parity", is not formally consistent with absence of arbitrage. Under perfect market assumptions, this paper derives a number of absence of arbitrage and equilibrium conditions relevant to currency swaps and LTFX. The main absence of arbitrage proposition provides restrictions on LTFX imposed by currency swap rates. Another proposition extends the covered interest arbitrage condition for money market securities by identifying a connection between spot interest rates derived from the domestic and foreign debt markets with the implied zero coupon interest rates embedded in LTFX rates. The cash and carry arbitrage condition that correspond to CIP for money market securities is found to be more difficult to identify when applied to the coupon bonds associated with long term borrowings. Two propositions are given where interest agios consistent with absence of arbitrage are derived, one for the domestic and ne for the foreign debt market. Absence of arbitrage is determined by equating the two agios. Using simulation analysis it was found that if currency swap rates satisfy absence of arbitrage, as determined by equality of the domestic and foreign interest agios, then LTFX rates can be determined independently of the difference between currency swap rates and domestic and foreign borrowing rates.

Table 1*

Currency Swap Rates and Interest Agios
Consistent with Absence-of-Arbitrage
Assuming Flat Yield Curves

Inputs				Results		
T	r_T*	r_T	rs_T*	rs_T	θ_T	ΔSCIP
10	5%	15%	5.85%	16.3078%	2.48363	.4578
5	5%	15%	5.85%	16.0978%	1.57595	.2478
3	5%	15%	5.85%	16.0138%	1.31379	.1638
2	5%	15%	5.85%	15.9722%	1.19955	.1222
10	5%	15%	5.25%	15.3846%	2.4836	.1346
5	5%	15%	5.25%	15.3229%	1.57595	.0729
3	5%	15%	5.25%	15.2982%	1.31379	.0482
2	5%	15%	5.25%	15.2859%	1.19955	.0359
10	5%	9%	5.85%	10.0227%	1.4534	.1727
5	5%	9%	5.85%	9.9461%	1.20556	.0961
3	5%	9%	5.85%	9.91446%	1.11869	.06446
2	5%	9%	5.85%	9.89846%	1.07764	.04846
10	5%	9%	5.25%	9.3008%	1.45336	.05008
5	5%	9%	5.25%	9.2783%	1.20551	.02783
3	5%	9%	5.25%	9.26896%	1.11869	.01896
2	5%	9%	5.25%	9.26425%	1.07764	.01425
10	5%	9%	4.75%	8.6992%	1.45336	-.05008
5	5%	9%	4.75%	8.72173%	1.20555	-.02817
3	5%	9%	4.75%	8.73104%	1.11869	-.01896
2	5%	9%	4.75%	8.73575%	1.07764	-.01425
10	5%	9%	4.15%	7.97728%	1.45336	-.17272
5	5%	9%	4.15%	8.05389%	1.20556	-.09611
3	5%	9%	4.15%	8.08554%	1.11869	-.06446
2	5%	9%	4.15%	8.10154%	1.07764	-.04846
10	5%	6.5%	5.85%	7.41301%	1.1524	.06301
5	5%	6.5%	5.85%	7.38555%	1.0735	.03555
3	5%	6.5%	5.85%	7.374%	1.04347	.024
2	5%	6.5%	5.85%	7.36811%	1.02878	.01811
10	5%	6.5%	5.25%	6.76853%	1.15241	.01853
5	5%	6.5%	5.25%	6.76046%	1.07349	.01046
3	5%	6.5%	5.25%	6.75706%	1.04347	.00706
2	5%	6.5%	5.25%	6.75533%	1.02877	.00533
10	5%	6.5%	4.75%	6.23147%	1.15211	-.01853
5	5%	6.5%	4.75%	6.23954%	1.07349	-.01046
3	5%	6.5%	4.75%	6.24294%	1.04347	-.00706
2	5%	6.5%	4.75%	6.24467%	1.02877	-.00533
10	5%	6.5%	4.15%	5.58699%	1.1524	-.06301
5	5%	6.5%	4.15%	5.61445%	1.0735	-.03555
3	5%	6.5%	4.15%	5.62600%	1.04347	-.024
2	5%	6.5%	4.15%	5.63189%	1.02878	-.01811

* ΔSCIP = $(r_T^* + rs_T - rs_T^*) - r_T$

Table 2*

Currency Swap Rates and Interest Agios
Consistent with Absence-of-Arbitrage
Assuming Shaped Yield Curves

	Inputs					Results		
T	r_T*	z_T*	r_T	z_T	rs_T*	rs_T	θ_T	ΔSCIP
10	5%	6.5%	15%	12%	5.85%	16.5298%	2.17361	.6798
5	5%	6.5%	15%	12%	5.85%	16.2415%	1.51313	.3915
3	5%	6.5%	15%	12%	5.85%	16.1087%	1.29291	.2587
2	5%	6.5%	15%	12%	5.85%	16.0384%	1.19062	.1884
10	5%	6.5%	15%	12%	5.25%	15.4499%	2.14918	.1999
5	5%	6.5%	15%	12%	5.25%	15.3652%	1.50715	.1152
3	5%	6.5%	15%	12%	5.25%	15.3621%	1.29192	.1121
2	5%	6.5%	15%	12%	5.25%	15.3054%	1.18967	.0554
10	5%	6.5%	15%	18%	5.25%	15.3941%	2.92914	.1451
5	5%	6.5%	15%	18%	5.25%	15.3207%	1.65493	.0707
3	5%	6.5%	15%	18%	5.25%	15.2942%	1.33884	.0422
2	5%	6.5%	15%	18%	5.25%	15.2822%	1.21001	.0322
10	5%	6.5%	15%	18%	5.85%	16.3435%	2.92878	.4935
5	5%	6.5%	15%	18%	5.85%	16.0904%	1.65453	.2404
3	5%	6.5%	15%	18%	5.85%	16.0002%	1.33855	.1520
2	5%	6.5%	15%	18%	5.85%	15.9596%	1.20981	.1096
10	5%	5.0%	9%	12%	4.75%	8.7294%	1.60080	-.0206
5	5%	5.0%	9%	12%	4.75%	8.7447%	1.23701	-.0053
3	5%	5.0%	9%	12%	4.75%	8.7473%	1.12935	-.0027
2	5%	5.0%	9%	12%	4.75%	8.7475%	1.08223	-.0025
10	5%	5.0%	9%	6%	4.15%	7.8382%	1.32697	-.3118
5	5%	5.0%	9%	6%	4.15%	7.9618%	1.17399	-.1882
3	5%	5.0%	9%	6%	4.15%	8.0204%	1.10720	-.1296
2	5%	5.0%	9%	6%	4.15%	8.0581%	1.07257	-.0919
10	5%	5.0%	9%	6%	5.85%	10.1618%	1.35894	.3118
5	5%	5.0%	9%	6%	5.85%	10.0382%	1.18357	.1882
3	5%	5.0%	9%	6%	5.85%	9.9761%	1.11102	.1261
2	5%	5.0%	9%	6%	5.85%	9.9419%	1.07428	.0919
10	5%	6.5%	6.5%	8%	5.50%	7.0439%	1.17634	.0439
5	5%	6.5%	6.5%	8%	5.50%	7.0233%	1.07897	.0233
3	5%	6.5%	6.5%	8%	5.50%	7.0152%	1.04537	.0152
2	5%	6.5%	6.5%	8%	5.50%	7.0112%	1.02960	.0112
10	5%	6.5%	6.5%	8%	4.50%	5.9561%	1.17422	-.0439
5	5%	6.5%	6.5%	8%	4.50%	5.9767%	1.07868	-.0233
3	5%	6.5%	6.5%	8%	4.50%	5.9848%	1.04530	-.0152
2	5%	6.5%	6.5%	8%	4.50%	5.9888%	1.02958	-.0112
10	5%	6.5%	6.5%	5%	5.25%	6.8160%	1.13780	.0660
5	5%	6.5%	6.5%	5%	5.25%	6.7900%	1.07003	.0400
3	5%	6.5%	6.5%	5%	5.25%	6.7763%	1.04225	.0263
2	5%	6.5%	6.5%	5%	5.25%	6.7686%	1.02824	.0186

* ΔSCIP = $(r_T^* + rs_T - rs_T^*) - r_T$

Table 3

Comparison of Interest Agios and Currency Swap Rates for Equilibrium versus Absence-of-Arbitrage Conditions*

Input: r_T*=5% z_T*=6.5% r_T=9% z_T = 10% rs_T*=5.85%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	$\Delta SCIP$
10	1.50493		10.0721	-.9433	.2211
10		1.38175	9.33213	0	-.5179
5	1.2167		9.9708	-.7608	.1207
5		1.17548	9.31276	0	-.5372
3	1.12247		9.9288	-.6815	.0789
3		1.10187	9.31714	0	-.5329
2	1.07927		9.90781	-.6407	.0578
2		1.06681	9.32224	0	-.5278

Input: r_T*=5% z_T*=5% r_T=9% z_T = 8% rs_T*=5.25%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	$\Delta SCIP$
10	1.41514		9.31307	-.7099	.0631
10		1.32539	8.66532	0	-.5847
5	1.19678		9.2868	-.8409	.0368
5		1.15126	8.49023	0	-.7598
3	1.11564		9.27478	-.9009	.0248
3		1.08819	8.41371	0	-.8363
2	1.07631		9.26839	-.9326	.0184
2		1.05796	8.3741	0	-.8759

Input: r_T*=5% z_T*=5% r_T=9% z_T = 8% rs_T*=4.75%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	$\Delta SCIP$
10	1.41174		8.68693	-.6838	-.0631
10		1.32539	8.06372	0	-.6863
5	1.19581		8.7132	-.8232	-.0368
5		1.15126	7.9337	0	-.8163
3	1.11527		8.72522	-.8887	-.0248
3		1.08819	7.8758	0	-.8742
2	1.07614		8.73161	-.9124	-.0184
2		1.05796	7.84559	0	-.9044

Input: r_T*=5% z_T*= 5% r_T= 9% z_T = 10% rs_T*= 4.75%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	$\Delta SCIP$
10	1.49855		8.71029	.6657	-.0397
10		1.59233	9.27367	0	.5237
5	1.21566		8.7298	.8178	-.0202
5		1.26188	9.46756	0	.7176
3	1.12218		8.73665	.8869	-.0134
3		1.14977	9.55547	0	.8055
2	1.07916		8.73977	.9233	-.0102
2		1.09751	9.60173	0	.8517

Input: r_T*= 5% z_T*= 6% r_T= 9% z_T = 10% rs_T*= 4.75%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	$\Delta SCIP$
10	1.49572		8.69326	-.3547	-.0567
10		1.44833	8.40858	0	-.3414

5	1.21503		8.71965	-.2105	-.0304
5		1.20347	8.53514	0	-.2149
3	1.12196		8.73012	-.1451	-.0199
3		1.11753	8.59871	0	-.1513
2	1.07906		8.73526	-.1106	-.0147
2		1.0769	8.63346	0	-.1165

Input: r_T*= 5% z_T*= 5% r_T= 6.5% z_T = 7.5% rs_T*= 4.75%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	ΔSCIP
10	1.16922		6.24423	.8456	-.0058
10		1.26529	6.89266	0	.6427
5	1.07753		6.24822	.9201	-.0018
5		1.12485	7.04139	0	.7914
3	1.0449		6.24883	.9514	-.0012
3		1.07314	7.10716	0	.8572
2	1.02941		6.24884	.9674	-.0012
2		1.04819	7.14144	0	.8914

Input: r_T*= 5% z_T*= 6% r_T= 6.5% z_T = 7.5% rs_T*= 4.75%

Results:

T	$\theta_{T,I}$	$\theta_{T,a}$	rs_T	Dev_T	ΔSCIP
10	1.16700		6.22923	-.1497	-.0208
10		1.15087	6.12037	0	-.1296
5	1.07697		6.23875	-.08368	-.0113
5		1.07279	6.16866	0	-.0813
3	1.0447		6.24259	-.05632	-.0074
3		1.04306	6.9273	0	-.0573
2	1.02931		6.24449	-.04245	-.0055
2		1.0285	6.20588	0	-.0441

* $Dev_T = z_T - z_T^* - (((\theta_{T,I})^{1/T} - 1)(1 + z_T^*))$

$\Delta SCIP = (r_T^* + rs_T - rs_T^*) - r_T$

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Currency Swaps, Fully Hedged Borrowing and Long Term Covered Interest Arbitrage

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ABSTRACT

This paper provides equilibrium and absence of arbitrage conditions related to currency swaps and fully hedged borrowings. The main absence of arbitrage condition identifies the relationship between currency swap rates and long term forward exchange rates. The main equilibrium condition provides the restriction that, at each relevant maturity date, the interest rate agio in the long term forward exchange market will equal the spot interest rate agio calculated from the foreign and domestic debt markets.

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NOTES

1. Long term in this case refers to maturities greater than one year. This convention is consistent with market practice of making primary issues of securities with maturities of one year and less in zero coupon form. Primary issues with maturities greater than one year typically, but not always, are made in coupon-bearing form.

2. Domestic direct terms is defined as units of domestic currency to units of foreign currency. As discussed in Section II, the rates zz and zz^* are not necessarily equal to the spot interest rates from the foreign and domestic bond markets.

3. In addition, due to pricing anomalies, there are difficulties associated with using the observed zero coupon yields quoted for US Treasury strips, Daves and Ehrhardt (1993). Even if the problems of correctly evaluating the Treasury strip yield quote could be solved, the rates are still not applicable to the underlying arbitrage transactions, because only the government can borrow in the strip market.

4. In practice, fixed-to-fixed currency swap rates have to be calculated to account for the market convention of quoting prices using fixed-to-floating currency swaps. This implies that to get the fixed-to-fixed rates, an floating-to-fixed interest rate swap must be incorporated into the pricing, e.g., Fletcher and Taylor (1994).

5. The connection between currency swaps and fully hedged borrowings was recognized in the trade literature shortly after an active market in long term forward exchange rates emerged in the 1980's. Mason et al. (1995) provides a useful illustration of the approach used by practitioners in comparing fully hedged borrowings and currency swaps. Examples from the trade usually serve to recognize transactions costs, which invariable favour the use of currency swaps over fully hedged borrowings.

6. In the present context, the assumption of perfect markets includes: 1) no transactions costs, such as bid/offer spreads, commissions or 'shoe leather'; 2) equal lending and borrowing rates in a given currency; 3) instantaneous execution; 4) no taxes; and, 5) no default risk in the LTFX and debt securities. Unlike the perfect markets encountered in other studies, it is ***not*** always assumed that the domestic and foreign term structures are flat.

7. Par bonds provide the analytical simplification that the coupon can be reexpressed as the yield to maturity times the par value. Adjustment to include discount or premium bonds is tedious and does not add substantively to the analysis.

8. These particular swap rates do not satisfy the stated requirement for swap-covered interest parity, i.e., it is not true that 5% = 9% + (4.5% - 9.5%). However, as discussed in Section V, this is an implication of dealer swap trading.

9. Observe that the $t=0$ transactions involve an exchange of principal values in the currency swap that will cancel.

10. The derivation of this Proposition requires the substitution of $(1/r^*) - (1/[r^* (1 + r^*)^T])$ for the sum of discount spot interest rates. Making this substitution at the appropriate point, the

derivation of the Proposition follows from (7) without considerable algebraic complexity.

11. The rate of 9.25% is not necessarily consistent with absence-of-arbitrage for the 10% *C$* offering used in the previous example. Given the sequence of forward exchange rates for different maturities, the precise fixed coupon *US$* interest rate which is consistent with absence-of-arbitrage will depend on the sequence of spot interest rates in the US and Canadian debt markets so, at this point, the arbitrary rate of 9.25% can be chosen without significant loss of intuition.

12. The perfect markets assumption does suppress some important issues involved in comparing currency swaps and fully hedged borrowings. For example, assuming that ***transactions costs are zero*** favours the fully hedged borrowing. This because the currency swap involves only one transaction while the fully hedged borrowing involves a sequence of forward exchange transactions together with the initial and terminal debt market and foreign exchange transactions. This is an extension of a similar result observed for short-term CIP, e.g., Clinton (1988).

13. This follows from dividing (8) by S_0 and equating with (9). This condition must hold in equilibrium because the cash flows in (8) and (9) have been constructed to be equal in *C$* terms at each point in time.

14. The derivation of $\theta_{T,a}$ and $\theta_{T,b}$ only substituted (9) for $t < T$. It is also possible to eliminate θ completely by substituting for the $t = T$ case as well. For completeness, this further simplification provides the following result for the mismatched *US$* case:

$$rs_T \sum_{t=1}^{T} \left\{ \frac{1}{(1 + z_t)^t} - \frac{rs_T^*/rs_T}{(1 + z_t*)^t} \right\} = \frac{1}{(1 + z_T*)^T} - \frac{1}{(1 + z_T)^T}$$

Combining this result for the mismatched *C$* case gives:

$$\frac{(1 + r_T)^T}{(1 + r_T^*)^T} = \frac{rs_T \; r_T^*}{rs_T^* \; r_T}$$

The errors produced by the equation are considerably larger than when the t=T simplification is not used.

15. In addition to the arbitrage between a fixed-to-fixed currency swap and a fully hedged borrowing, there are also arbitrages connecting a fixed-to-floating currency swap and a floating-to-floating currency swap with a sequence of foreign exchange swaps featuring different maturity dates and par values, e.g., Iben (1992). In this terminology, a foreign exchange swap is a zero coupon currency swap, which combines a spot foreign exchange transaction with an offsetting forward exchange transaction. These arbitrages will not be examined here.

16. In order to handle the apparent problem associated with having an uncovered return in an arbitrage transaction, Popper (1993, p.447) claims in a note that: "The amount of notional swap principal may be chosen to achieve a fully hedged position". Even if this implies that only cross-currency annuity swaps are being used to fully cover the foreign cash flows, e.g., Mordue (1992), this does not eliminate the difficulties with the arbitrage argument.