Physica A 507 (2018) 89–98

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The pre-history of econophysics and the history of economics: Boltzmann versus the marginalists

Geoffrey Poitras [1]

Simon Fraser University, Vancouver B.C., Canada V5A lS6

HIGHLIGHTS

- A comparative intellectual history of econophysics and economic science is provided to demonstrate why and how econophysics is distinct from economics.
- The history and role of the ergodicity hypothesis of Ludwig Boltzmann is considered.
- The use of phenomenological methods in econophysics is detailed.
- The role of ergodicity in empirical estimates of models in economics is identified.

ARTICLE INFO

Article history:
Received 7 January 2018
Received in revised form 17 March 2018
Available online 21 May 2018

Keywords:
Ergodicity
Ludwig Boltzmann
Econophysics
Neoclassical economics
Marginal revolution
Phenomenology

ABSTRACT

This paper contrasts developments in the pre-history of econophysics with the history of economics. The influence of classical physics on contributions of 19th century marginalists is identified and connections to the subsequent development of neoclassical economics discussed. The pre-history of econophysics is traced to a seminal contribution in the history of statistical mechanics: the classical ergodicity hypothesis introduced by L. Boltzmann. The subsequent role of the ergodicity hypothesis in empirical testing of the deterministic theories of neoclassical economics is identified. The stochastic models used in modern economics are compared with the more stochastically complex models of statistical mechanics used in econophysics. The influence of phenomenology in econophysics is identified and discussed.

> "Rational mechanics gives us a first approximation to theory of the equilibrium and of the movements of bodies. In the same way the theories of Jevons, Walras, Marshall, Fisher, and others present us with a first approximation to the theory of economic phenomena."
>
> Vilfredo Pareto [1]

1. Introduction

At least since Mirowski [2], it has been widely recognized that important theoretical elements of neoclassical economics were adapted from mathematical concepts developed in 19th century classical physics. Much effort by historians of economic

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E-mail address: poitras@sfu.ca.

[1] The author is Professor of Finance at Simon Fraser University. In addition to helpful comments from the reviewers and numerous discussions with John Heaney over the years, participants in the History of Econophysics sessions at the annual meeting of the History of Economics Society in 2012 and a joint session of the American Finance Association and the History of Economics Society at the Allied Social Science Association annual meeting in 2018 have also contributed to the paper.

https://doi.org/10.1016/j.physa.2018.05.058
0378-4371/© 2018 Published by Elsevier B.V.

thought and others has been dedicated to exploring the validity and implications of "Mirowski's thesis" [3–7], especially the connection between the deterministic 'rational mechanics' approach of classical physics and the subsequent development of neoclassical economic theory [8–10]. This well traveled literature connecting classical physics with neoclassical economics is seemingly incongruent with emergence of the distinct subject of econophysics during the last decade of the twentieth century. Appearing primarily in physics journals, econophysics is now a "recognized field" [11] within physics that involves the application of empirical and theoretical methods from statistical mechanics to the analysis of economic phenomena, e.g., [12–18]. This paper uses comparative intellectual history to illustrate how econophysics differs from mainstream economics.

Contributions to econophysics range from empirical studies on scaling and power law distribution properties of financial data to theoretical studies of multi-agent order flow models, e.g., [19,20]. The common theme is application of methods from statistical mechanics to economic phenomena. Significantly, there are contributions by economists stretching back to the work of Pareto on the scaling law for wealth that overlap with this theme. There are also some contributions in economics journals that do not differ substantively from contributions to econophysics appearing in physics journals. Is it possible that 'econophysics' differs from 'economics' not on specific content or methodology but, rather, on the discipline of the scholarly journals publishing the research? Contrasting the history of economics with the pre-history of econophysics, this paper demonstrates how and why econophysics is methodologically and philosophically distinct from mainstream economics. The introduction into economics by Francis Ysidro Edgeworth (1845–1926) and other marginalists of mathematical methods adapted from classical physics is connected to the contemporaneous introduction of ergodicity to statistical mechanics by Ludwig Boltzmann (1844–1906). Tracing the subsequent historical evolution of stochastic concepts in economics reveals a specific form of the ergodicity hypothesis being adopted in econometric modeling while statistical mechanics was in the process of developing a richer variety of stochastic models that are now being phenomenologically employed in econophysics.

As Mirowski [21–24] is at pains to emphasize, physics has evolved considerably from the deterministic classical approach which inspired neoclassical economics. In discussing historical developments in physics, Mirowski, Sornette [17] and others often jump from the determinism of rational mechanics to the stochastic behavior of Ising models and quantum mechanics to recent developments in chaos theory, overlooking the relevance of initial steps toward stochastic modeling of physical phenomena by Ludwig Boltzmann, James Maxwell (1831–1879) and Josiah Willard Gibbs (1839–1903).[2] Introduction of the 'ergodicity hypothesis' by Boltzmann around the time that *Mathematical Psychics* by Edgeworth appeared in 1881 provides an important connection between the history of economics and the pre-history of econophysics. The subsequent emergence and development of stochastic concepts in econometrics adopted a form of ergodicity to empirically test the deterministic models of neoclassical economic theory. In this process, ergodicity permitted statistical techniques designed for repeatable experiments of the natural sciences to be extended to the non-experimental data of economic phenomena.

Early contributors to the econometrics project featured training in physics or mathematical statistics, subjects where ergodic concepts are employed. Consequently, stochastic generalization of the deterministic and static equilibrium models of neoclassical economic theory adopted 'time reversible' ergodic probability models, such as the likelihood functions associated with (possibly transformed) stationary Gaussian error distributions. By this time, stochastic representations in statistical mechanics had evolved considerably from the early ergodic models of Boltzmann to include Ising models of interacting elements and quantum mechanics. The stochastic models of statistical mechanics now available to econophysics include a wide variety of non-linear, irreversible and chaotic stochastic models aimed at capturing key empirical characteristics of different physical and economic phenomena. In addition to scaling and power laws, these models include truncated Levy processes as well as the fractals and chaos theory popularized by Mandlebrot. Application of these models often employ phenomenological methods that vary substantively from the methods associated with the stochastic representations of modern economic theory that are based on constrained optimization solutions for consumers, producers and investors derived from axiomatic choice theory.

Comparing the history of economics with the pre-history of econophysics reveals substantive differences in methodology that restrict straightforward adoption in mainstream economics of models used econophysics. As Roehner [11] observes: "From a methodological perspective, the strong emphasis econophysics has put on the search for regularities is probably one of its most important innovations. In contrast, for most economists a quantitative regularity is considered of no interest unless it can be interpreted in terms of agents' motivation and behavior and has a clearly defined theoretical status". The process of adapting stochastic models from statistical mechanics to econophysics involves letting the data determine the appropriate stochastic model, e.g., cross correlation analysis, before proceeding to theoretical explanation, if any. In contrast, in mainstream economics stochastic representation of economic phenomena involves initial development of a theoretical model based on axiomatic propositions about rational economic behavior, before proceeding to determine the statistical 'fit' of the data to the theoretical model.

Schinckus [27] explores the positivist philosophical foundation of econophysics identifying the fundamental role of empirical observation: "The empiricist dimension is …the first positivist feature of econophysics". For McCauley [28] and others, this concern with empiricism involves the use of phenomenological methods to identify macro-level statistical regularities that are characterized by scaling and power laws, such as those identified by Mandelbrot [29] and Mandelbrot

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[2] Cercignani [25] discusses the connection between Boltzmann and the energetists of that time. Volovich [26] details the differences between classical and quantum mechanics.

and Hudson [30] for financial data. In the equilibrium models of economics, the ergodic hypothesis is an essential methodological link to confront the 'non-repeatable' experiments that characterize most observed economic and financial data. The quandary posed by having only a single observed *ex post* time path to estimate the distributional parameters for the ensemble of *ex ante* time paths is mirrored in the ergodicity hypothesis initially arising from the kinetic theory of gases. In contrast to the natural sciences, in general, and physics, in particular, the human sciences provide no assurance that *ex post* statistical regularity translates into *ex ante* forecasting accuracy. Adopting 'time reversible' ergodic processes in econometrics was seen as the method for overcoming the difficulties of non-experimental data leading to the subsequent evolution from the deterministic models of neoclassical economic theory to the stochastic models of modern economics.

2. Classical physics, marginalists and neoclassical economics

Presenting the marginalist founders as a cohesive group that led to the subsequent emergence of neoclassical economics disguises fundamental issues dividing the distinct marginalist approaches of Jevons, Walras, Menger, Edgeworth and Marshall, e.g. [31]. In the history of economic thought, such divisions have created difficulties in the search for a cohesive 'marginal revolution', e.g., [32–35]. These difficulties are compounded if the search for a cohesive and consistent connection with 19th century classical physics is introduced. Despite considerable variation in usage of 'marginalist' and 'neoclassical', it is still possible to identify certain relevant elements connecting seemingly disparate approaches. De Vroey [35] captures one key element: "Equilibrium is at the core of both Marshall's and Walras' reasoning. Both of them adopted the stationary equilibrium conception in which equilibrium is defined as a state of rest". One of the leading figures in neoclassical economics, Paul Samuelson [36], extended this approach to include both static and dynamic theories. While static neoclassical analysis "described equilibrium as resulting from economic agents solving maximization problems taking prices as parameters …Samuelson's analysis of dynamics rested on the concept of stationary equilibrium, which holds that equilibrium constitutes a state of rest" [37]. Lucas [38] provides further elaboration of this point:

> The underlying idea seems to be taken from physics, as referring to a system 'at rest'. In economics, I suppose such a static equilibrium corresponds to a prediction as to how an economy would behave should external shocks remain fixed over a long period, so that households and firms would adjust to facing the same set of prices over and over again and attune their behavior accordingly.

Lucas attacked "the idea that an economic system is in any sense 'at rest' [as] simply an anachronism", advancing the 'modern' system of economic theory that relies on "the equilibrium discipline, rational expectations and Walrasian microfoundations" [37].

Missing from much of the discussion surrounding the emergence of neoclassical economics from the marginalist contributions is a temporal perspective. Recognizing that the first marginalist steps involved the hedonic calculus of utility, the precursor of axiomatic choice theory, Frank Knight [39] captures the tenor of the times:

> The utility theory should be seen as the culmination, historically and logically, of the rationalistic and individualistic intellectual movement of which the competitive economic system itself is one aspect and modern science and technology are others. To its admirers it comes near to being the fulfillment of the eighteenth-century craving for a principle which would do for human conduct and society what Newtonian mechanics had done for the solar system.

The desire to see 'Darwinian' continuity in the development of economic thought represented in the marginalist contributions of Marshall masks an essential feature of the "new theories of economics" that separated classical political economists from a mathematical group of marginalists. Pareto [1] provides an excellent illustration of the perspective from this subset:

> Rational mechanics gives us a first approximation to theory of the equilibrium and of the movements of bodies. In the same way the theories of Jevons, Walras, Marshall, Irving Fisher, and others present us with a first approximation to the full theory of economic phenomena. It must be clearly understood that it is only an approximation; it is similar to that just made in the case of the heavy body supposed to fall in a vacuum. Pure economics has no better way of expressing the concrete economic phenomenon than rational mechanics has for representing the concrete mechanical one. It is at this point that there is a place for mathematics. The problem of pure economics bears a striking likeness to that of rational mechanics. Now, in point of empirical fact, men have as yet not succeeded in treating the latter problem without the aid of mathematics. It therefore appears quite legitimate to appeal also to mathematics for assistance in the solution of the economic problem.

In recognizing Marshall, Pareto is apparently not averse to seeing marginalist contributions as an extension of classical political economy. It is the use of mathematics, and the associated connection to the developed mathematical theories of classical physics, that is the defining characteristic of the 'new theories of economics' introduced by those the marginalists that adopted the "mathematical" approach of the "new theories of economics", e.g., [40].

Because the overwhelming theme in discussions of the early marginalists is the unifying notion of utility, the differing emphasis on mathematics and connection to classical physics can be overlooked, e.g., [31]. It is not surprising that some of the early marginalists sought to emulate the profound advances that mathematics provided in classical physics. For example, Jevons [41] observes: "Just as the gravitating force of a material body depends not alone upon the mass of that body, but upon the masses and relative positions and distances of the surrounding material bodies, so utility is an attraction between a wanting being and what is wanted''. However, Jevons is primarily concerned with establishing the principles of equilibrium and utility, undermining the dominance of the classical political economy of Smith, Ricardo and Mill. Jevons is not concerned much with developing connections to classical physics. It is with Edgeworth [42] where the most emphatic marginalist connecting classical physics with the application of mathematics to economics appears:

> An analogy is suggested between the Principles of Greatest Happiness, Utilitarian or Egoistic, which constitute the first principles of Ethics and Economics, and those Principles of Maximum Energy which are among the highest generalizations of Physics, and in virtue of which mathematical reasoning is applicable to physical phenomena quite as complex as human life.

Significantly, as reflected in Irving Fisher [43], if the use of mathematics and the symbiotic connection with classical physics is emphasized, then the perception of a 'marginalist revolution' that begins with Jevons, Walras and Menger loses cohesiveness.

3. Ludwig Boltzmann and the origins of ergodic theory

The kinetic gas model of Boltzmann is concerned with formulating dynamic properties of the stationary Maxwell distribution—the velocity distribution of gas molecules in thermal equilibrium. Starting in 1871, Boltzmann made the essential step of invoking the ergodicity hypothesis to determine the evolution equation for the distribution function, [44–46]. The Maxwell distribution provides the probability for the relative number of molecules with velocities in a certain range. Using a mechanical model that involved molecular collision, in 1867 Maxwell [47] was able to demonstrate that, in thermal equilibrium, this distribution of molecular velocities was a 'stationary' distribution that would not change shape due to ongoing molecular collision. Boltzmann aimed to determine whether the Maxwell distribution would emerge in the limit whatever the initial state of the gas. In order to study the dynamics of the equilibrium distribution over time, Boltzmann introduced the probability distribution of the relative time a gas molecule has a velocity in a certain range while still retaining the notion of probability for velocities of a relative number of gas molecules. Under the ergodicity hypothesis, the average behavior of the macroscopic gas system, which can be empirically measured over time using observable variables, can be interchanged with the average value calculated from the ensemble of unobservable and highly complex microscopic molecular motions at a given point in time. In the words of Wiener [48]: "Both in the older Maxwell theory and in the later theory of Gibbs, it is necessary to make some sort of logical transition between the average behavior of all dynamical systems of a given family or ensemble, and the historical average of a single system''.

With hindsight, it is unfortunate that the seminal contributions of Ludwig Boltzmann to statistical mechanics were, initially, fiercely resisted by a physics establishment wedded to the results of classical physics. Though much of the resistance at the time was associated with a reluctance to accept notions of atoms and molecules, it is the concept of ergodicity that sits at the intersection between the history of economics and the pre-history of econophysics. Initially developed in the context of gas molecules colliding in a glass cylinder, Boltzmann hypothesized that the process of mechanical collision would lead to a state of maximum microscopic disorder that corresponds to maximum entropy and, as a consequence, macroscopic uniformity. To derive this result, Boltzmann introduced the ergodicity hypothesis to model the non-equilibrium velocity distributions for groups of molecules moving at the same speed and in the same direction. Put differently, in a world of mechanically colliding particles, Boltzmann derived the result that molecules moving "at the same speed and in the same direction'' is "the most improbable case conceivable ... an infinitely improbable configuration of energy'' [49]. The ergodicity hypothesis permits the average properties of the empirically measurable macroscopic system, such as temperature, pressure, volume and viscosity, to be theoretically determined from the infinite disorder of the microscopic system dynamics.

At a number of different levels, the results produced by Boltzmann were, and still are, profound. In relation to the differing methodological approaches of modern economics and econophysics identified by Roehner [11] and others, the classical ergodicity hypothesis initially introduced by Boltzmann directs attention to the stochastic properties of the macroscopic system. For econophysicists this implies that, even if the theoretical microeconomic foundations of a macrosystem cannot initially be determined due to the complex interaction of relevant economic agents, there is still inherent value in determining the empirical properties of the 'global' macrosystem, such as the stochastic dynamics, correlation effects, self-organization, self-similarity or scaling properties, e.g., [19]. However, at least since the critique by Koopmans [50] of statistical business cycle models estimated by institutional economists [51], such empirical phenomenological estimations undertaken without prior theoretical *explanans* is often derided by economists as naive, 'measurement without theory' [52]. As a consequence, ergodicity in economics plays a different methodological and philosophical role than in econophysics.

The insights of Boltzmann capture fundamental philosophical differences between statistical mechanics – the foundation of econophysics – and the classical physics embedded in neoclassical economics. Developing contributions of Jevons, Walras, Edgeworth and other mathematical marginalists, neoclassical economic theory inherited essential features of mid-19th century physics: deterministic rational mechanics; conservation of energy; and the non-atomistic continuum view of matter that inspired the energetics movement later in the 19th century. Jevons [53] reflects the entrenched Laplacian determinism of these marginalists:

> We may safely accept as a satisfactory scientific hypothesis the doctrine so grandly put forth by Laplace, who asserted that a perfect knowledge of the universe, as it existed at any given moment, would give a perfect knowledge of what was to happen thenceforth and for ever after. Scientific inference is impossible, unless we may regard the present as the outcome of what is past, and the cause of what is to come. To the view of perfect intelligence nothing is uncertain.

The transition from rational to statistical mechanics inspired by Boltzmann transformed theoretical physics from the microscopic mechanistic models of Rudolf Clausius (1822–1888) and James Maxwell to the macroscopic probabilistic theories of Josiah Willard Gibbs and Albert Einstein (1879–1955) [54]. With Boltzmann, the philosophical view that 'for perfect intelligence nothing is uncertain' is replaced by the view that 'perfect intelligence has created order from infinite uncertainty'.

The ergodicity hypothesis has evolved considerably from Boltzmann and the kinetic theory of gases relevant to the early pre-history of econophysics, e.g., [55–57]. The *Encyclopedia of Mathematics* [58] now defines ergodic theory as the "metric theory of dynamical systems. The branch of the theory of dynamical systems that studies systems with an invariant measure and related problems", e.g., [59]. This modern definition implicitly identifies the birth of ergodic theory with proofs of the mean ergodic theorem by von Neumann [60] in 1932 and the pointwise ergodic theorem by Birkhoff [61] in 1931. These early proofs have had significant and varied impact in a wide range of subjects, including the emergence of econometrics to test the theories of neoclassical economics. Building on a seminal contribution to probability theory by Kolmogorov [62], in the years immediately following it was recognized that the pointwise ergodic theorem generalizes the strong law of large numbers, an essential building block of many theoretical results in econometrics. Similarly, the equality of ensemble and time averages – the essence of the mean ergodic theorem – is necessary to the concept of a stationary stochastic process, another fundamental concept in econometrics.

The time period from the mathematical contributions by Edgeworth to the early empirical tests in econometrics coincided with significant developments in the pre-history of econophysics. One advance involved the formulation of quantum mechanics by the likes of Max Born, Wolfgang Pauli, Werner Heisenberg and Erwin Schrödinger, with many path-breaking results appearing in a brief window from 1924–1928. In addition, Ernst Ising produced the initial one-dimensional solution for the 'Ising model' of ferromagnetism in 1925, a model that was ultimately solved in the important two-dimensional phase transition case by Lars Onsager in 1944. These developments have had numerous extensions into econophysics, e.g., [63,64], but not into economics. In addition, the pre-history of econophysics also witnessed contributions dealing with inherent difficulties in the classical statistical mechanics of Boltzmann, such as the introduction of the Kac ring to resolve the relationship between reversible and irreversible processes, [56], and development of statistical mechanics without ergodic theory, [65], that are awaiting complementary efforts in econophysics.

4. Evolution of ergodicity in economic science

The fundamental transformation in physics from rational to statistical mechanics inspired by Boltzmann and Gibbs had little impact on the evolution of economics until the appearance of contributions to discrete time empirical testing of neoclassical demand theory by Tinbergen, Wold and others during the interwar period. The slow progress of ergodic notions initially into econometrics and, somewhat later, into economic theory has not overwhelmed the philosophical and methodological hold of classical physics inherited from the marginalists. In particular, the methodology of rational mechanics associated with Laplacian determinism is well suited to the axiomatic formalization of microeconomic theory provided initially in the von Neumann and Morgenstern expected utility approach to modeling 'uncertainty' and the subsequent fascinations with game theory, rational expectations equilibrium and the Bourbaki inspired Arrow–Debreu general equilibrium theory, e.g., [66].

Using contributions of the marginalists as an initial historical reference point, the progression of ergodic notions into economics has been uneven and opaque. Ergodicity is an implicit assumption in the general application to the non-experimental data of economic phenomena of empirical methods developed for the experimental data of the natural sciences. Early applications appear initially in econometrics. Important contributors to early empirical estimation of economic theories, especially neoclassical demand theory, were strongly influenced by methods used in the natural sciences, especially physics, and mathematical statistics. These contributors include the first two winners of the Nobel prize in Economics – Ragnar Frisch (1895–1973) and Jan Tinbergen (1903–1994) – as well as the statistician *cum* econometrician Herman Wold (1908–1992). The etymology for "econometrics" begins with Frisch [67], which appeared the same year Frisch obtained a Ph.D. in mathematical statistics. Together with Irving Fisher, Frisch was responsible for founding the Econometric Society in 1930. In the history of neoclassical economic theory, Frisch [68] provided an important extension of work on commodity supply and demand curves initiated by Henry Schultz [69,70]. In addition to work on the measurement of utility, the relevance of Frisch to the evolution of neoclassical theory was described by R.G.D. Allen [71]: "The future progress of economic science depends largely upon the work of investigators, like Professor Frisch, who realise the importance of subjecting the concepts and conclusions of abstract economic theory to the test of statistical determination and verification".

Concern with the implications of estimating empirical relationships for non-experimental data is a common theme in early contributions to econometrics. This concern culminated with Wold [72] where it is recognized that "experimental versus nonexperimental model building is a dualism that goes to the core of the scientific method". Wold [72] summarizes received opinion at the time: "Recognition is due – and perhaps overdue – for nonexperimental analysis as an active partner to the method of controlled experiments, and for the integration of both experimental and non-experimental approaches in the scientific method". Building on Wold [73] which presented the Wold decomposition theorem for (covariance) stationary time series, Wold and Jureen [74] explicitly develops the ergodic foundations of demand theory, establishing the connection between ergodicity and stationarity for empirically testing neoclassical economic theories. As such, this extends Wold [75] where the theoretical connection is made only by referencing theorems in Wold [73]. As a consequence, a relatively restricted form of ergodicity initially entered econometrics as a stationarity assumption imposed on the error term added to a deterministic model, permitting testing for the consistency of regression parameter estimates. To further regression hypothesis testing, it was natural to further assume the stationary error term had a Gaussian distribution.

Acceptance of ergodic notions to facilitate empirical testing in economics was not universal. Regression techniques translated readily to another consuming issue of the time: the estimation of macroeconomic models initiated by Tinbergen [76] in 1939. Unlike the estimation of commodity demand and supply functions central to neoclassical economic theory, these macroeconomic models were akin to 'complex systems' arising in statistical mechanics. As Wold [75] observes:

> a serious obstacle in econometric analysis lies in the complicated structure of economic life, in the fact that if we wish to study the interdependence between two economic factors, we should pay regard to a great many other factors which are more or less correlated with the phenomena under analysis. To demonstrate the complications which may arise it is sufficient to recall J. Tinbergen's studies of business cycles; for the United States, his analysis takes into consideration 70 different economic factors and about 50 relations between them.

The analogy to complex systems theory references an important theme in econophysics [11]. This empirically driven phenomenological approach to macroeconomics is decidedly different than the Laplacian determinism of neoclassical economics. Using *ad hoc* economic theory to identify relevant endogenous and exogenous variables, the approach initiated by Tinbergen also differed methodologically from the measurement driven approach to business cycles of the institutional economists, such as Arthur Burns and W.C. Mitchell [51]. Contrasting the empirically driven ideal of Tinbergen and the empirical skepticism of J.M. Keynes illustrates the wide philosophical and methodological diversity of different schools within economics.

As Boumans [9] details, Tinbergen studied mathematics and physics at the University of Leiden and produced a Ph.D. thesis in 1929 "*Minimumproblemen in de natuurkunde en de economie*" (Minimization problems in Physics and Economics) that was strongly influenced by the physicist Paul Ehrenfest, a doctoral student of Boltzmann. Yet, despite impressive training in methods of the natural sciences by key figures in the early history of econometrics, the lack of mathematical sophistication by the bulk of economists at the time contributed to unevenness in the acceptance, use and development of ergodic notions. Reluctance to accept the phenomenological use of regression analysis for establishing the validity of macroeconomic theory inspired fierce competition for intellectual attention space in economics captured in Keynes critical review of Tinbergen [76]. The epistemology of the anti-phenomenology camp is set out by Keynes [77]:

> Prof. Tinbergen is obviously anxious not to claim too much. If only he is allowed to carry on, he is quite ready and happy at the end of it to go a long way towards admitting, with an engaging modesty, that the results probably have no value. The worst of him is that he is much more interested in getting on with the job than in spending time in deciding whether the job is worth getting on with. He so clearly prefers the mazes of arithmetic to the mazes of logic, that I must ask him to forgive the criticisms of one whose tastes in statistical theory have been, beginning many years ago, the other way round.

Striking at fundamental assumptions underpinning the use of time reversible (stationary) ergodic processes, Keynes questioned the validity of "passing from statistical description to inductive generalisation". This schism between alternative epistemologies persists to the present in the writings of Post Keynesian and other heterodox economists arguing against the use of 'reversible' stationary error terms to model the 'true uncertainty' of future events. Despite paying limited attention to econophysics, this camp is almost certainly opposed to any empirical results in economics derived using methods of the natural sciences.

Gradual acceptance of stochastic models in mainstream economics was heavily influenced by the empirical methods employed to test the theories of neoclassical economics that involved adding an ergodic, stationary, usually Gaussian, error term to a deterministic model and estimating a general linear model (GLM). Following seminal contributions by Tinbergen, Wold and Koopmans, there were iterations and extensions of the GLM to deal with complications arising in empirical estimates that dominated subsequent work in econometrics, e.g., [78] and [79]. This led to application of generalized least squares estimation techniques that encompassed autocorrelated and heteroskedastic error terms. Employing L_2 vector space methods with combinations of stationary, i.e., independently, identically distributed (iid), error term distributions ensured these early stochastic models implicitly assumed a restricted form of ergodicity. Generalization of this discrete time estimation approach to the class of ARCH and GARCH error term models by Engle and Granger, e.g., [80], was of such significance that a Nobel memorial prize in economics was awarded for this contribution in 2003. By modeling the evolution of stationary error term volatility, this approach permitted a limited degree of non-linearity to be modeled providing a typically better *ex ante* fit to observed economic time series. Only recently has the ergodicity of the GARCH model and related methods been considered in detail, e.g., [81].

Where similar methodologies are employed, it is not surprising that econophysics overlaps with modern economics. Connection of ARCH models with econophysics is explicitly examined in [15]. Emergence of ARCH, GARCH and related models was part of a general trend toward employing linear time series models of transformed economic variables, e.g., [82]. In particular, at least since Dickey and Fuller [83], it has been found that estimates of univariate time series models for many economic times series reveals evidence of 'non-stationarity', seemingly undermining applicability of the ergodicity hypothesis. A number of approaches have emerged to deal with this apparent empirical quandary. In particular, transformation techniques for bivariate and multi-variate time series models have received considerable attention. Extension of Box–Jenkins methods led to the concept of economic time series being I(0) – stationary in the level – and I(1)—non-stationary in the level but stationary after first differencing. Two I(1) economic variables could be cointegrated if differencing the two series produced an I(0) process, e.g., [82], permitting testing for 'causality'—a research area overlapping with studies in econophysics. Extending early work on distributed lags, long memory processes have also been employed where time series are subject to fractional differencing, another topic appearing in econophysics. Significantly, recent contributions on Markov switching processes and exponential smooth transition autoregressive processes in econometrics have demonstrated the "possibility that nonlinear ergodic processes can be misinterpreted as unit root nonstationary processes" [84].

5. Phenomenology, econophysics and economics

How and why is econophysics methodologically and philosophically distinct from mainstream economics? Given the discussion to this point, a definite answer to this question is illusive. Both economics and physics are large tents with many distinct components. A shared concern with the study of economic phenomena dictates that some overlap between econophysics and parts of economic science is expected and unavoidable. However, being intimately connected to the Laplacian determinism of neoclassical economic theory, associated contributions to mainstream economics are either purely logical, derived from first principles of rational behavior, or are concerned with subjecting a logical theory to subsequent empirical testing. In contrast, being influenced by the methods of experimental physics, empirical studies in econophysics employ a phenomenological approach to the study of economic phenomena that involves searching for stochastic models that fit the available data. Subsequent development of a logical theory to explain the empirical results may or may not be part of the project.

The distinct interpretations of the term 'phenomenology' in philosophy and the natural sciences, such as physics, is confusing. Philosophical interpretation of phenomenology in the social sciences makes a distinction between the natural sciences, where knowledge is linear and cumulative, and the human sciences, where "the real problem that the human sciences present to thought is that one has not properly grasped the nature of the human sciences if one measures them by the yardstick of an increasing knowledge of regularity. The experience of the socio-historical world cannot be raised to a science by inductive procedure of the natural sciences" [85]. From this perspective, phenomenology argues against 'thinking' that the study of human interaction in markets can only be interpreted 'scientifically' and maintains that the 'phenomena' of reality – objects and events – are perceived or understood in the 'first person' of the human consciousness, and are not independent of that consciousness. Anathema to econophysics, this perspective has some adherents in heterodox economics.

At least since criticism by Edmund Husserl (1859–1938) of the 'descriptivism' advanced by Ernst Mach (1838–1916), there has been inherent conflict between the conception of phenomenology in philosophy and physics [86]. The loose connection of the philosophical perspective with phenomenology in physics is that understanding is primarily a process of discovery by looking and perceiving before making assumptions and logical deductions. Phenomenological methods in physics involve computing specific predictions of a stochastic model for comparison with previously accumulated experimental measurements, recognizing that there are constraints on available modes of measurement. In other words, the choice of stochastic model is motivated by measurable and measured quantities. An excellent example of phenomenology in physics is provided by the close collaboration between the theoretician Max Born and the experimentalist James Franck at Göttingen, starting in 1921, that led to important breakthroughs in quantum mechanics. As Im [87] observes: "Born's close collaboration with Franck was well suited to his research style: a formal and mathematical description of nature based upon plentiful observational data". Theories that are concerned with quantities that are only logical and not measurable – such as the utility function in neoclassical economics – are not phenomenological. By allowing Boltzmann to connect the stochastic properties of unobservable molecular collisions with available experimental measurements of temperature, volume and pressure, the ergodicity hypothesis is representative of phenomenology in statistical mechanics.

Given the general recognition of Boltzmann as a founder of statistical mechanics, and the acknowledgment that econophysics is "a pure subject in statistical mechanics", tracing differences in specific applications of the ergodicity hypothesis has relevance to identifying the fuzzy boundary between econophysics and modern economic science. Casual inspection of scholarly outlets in econophysics reveals a wide variety of topics, including but not limited to: non-linear stochastic dynamics, with models involving chaos, Hurst exponents and stochastic volatility; scaling and application of fractal and multi-fractal methods; and, complex systems, random dynamic systems and network models. Recalling the position of Roehner [11] that: "for most economists a quantitative regularity is considered of no interest unless it can be interpreted in terms of agents' motivation and behavior and has a clearly defined theoretical status", the phenomenological approach to modeling the *ex ante* empirical properties of macroscopic variables of statistical mechanics and econophysics is concerned with generating statistical information from experimental or non-experimental observations on measurable variables of interest and employing a range of stochastic models to fit the empirical evidence.

The capital asset pricing model (CAPM) for stock returns provides a useful illustration of the differing methodological approaches of empirical econophysics and modern economics, e.g., [88]. Commencing with theoretical derivation from 'microscopic' expected utility maximizing behavior of economic agents, the logical truth of the CAPM is then confronted with *ex post* empirical evidence for observed stock returns and interest rates. As it turns out, the CAPM in various guises has not generated sufficiently credible *ex ante* performance due to instability in regression parameter estimates. By contrast, econophysicists have engaged in numerous attempts to devise stochastic models designed to fit the sample path properties of stock returns or stock prices, often without making substantive reference to the underlying microscopic motivations of economic agents. Instead of iterating the structure of the microscopic maximizing model to attempt a better *ex post* fit to the empirical evidence as in numerous contributions to the CAPM of financial economics, in econophysics weak *ex ante* performance of a stochastic model due, say, to random *ex ante* instability in the sample paths begs phenomenological questions such as: are there alternative stochastic models – ergodic or otherwise – that are a better fit to the *ex ante* properties of observed economic data? For example, is the random instability in the observed *ex ante* sample paths identified in stock return time series consistent with, say, stochastic models that admit bifurcation of an ergodic process, e.g., [89]; [90]?

In contrast to the absence of CAPM studies in econophysics, there are numerous contributions to option pricing theory in both econophysics and financial economics that are more-or-less similar. At first glance, this appears to be an area where the phenomenological approach of empirical econophysics is abandoned and replaced with theoretical modeling that is largely divorced from empirical data. Despite appearing to connect with empirical applications by providing 'new', more precise models for estimating prices or solving for the price of a novel type of option, the bulk of contributions on option pricing in econophysics (and financial economics) involve theoretical extensions that are seemingly not grounded in empirical data or applications, e.g., [91,92]. This highlights the relevance of an overlooked dichotomy in econophysics: the division of labor between empirical econophysics and theoretical econophysics. Option pricing models fall largely within the realm of theoretical econophysics. As such, claims by Roehner [11] and others about the empirical orientation of econophysics are incomplete. As in the larger domain of physics, studies in theoretical econophysics are not necessarily concerned with providing logical explanations for an accumulation of results obtained from phenomenological fitting of stochastic models to (non-experimental) empirical data.

Despite the theoretical orientation of option pricing models in econophysics, ergodicity again plays a central, albeit different, role. Ergodicity is embedded in the use of continuous time diffusion processes—a familiar theoretical construct in statistical mechanics since the early contributions of Norbert Wiener (1894–1964). The log normal diffusion, i.e., geometric Brownian motion, for the price state variable was used in early contributions to continuous time finance starting in the 1960's that culminated in heat equation solutions for the European option pricing models of Black and Scholes [93] and Merton [94]. This was a major advance in the domain of economic science. However, despite the appearance of providing a solution for the price of an option contract, because the option price is an observed variable in the option market a Black–Scholes model provides a solution for the only unknown variable in the equation, the volatility. As volatility is an unobserved variable, using more complicated stochastic processes, e.g., diffusions with stochastic volatility, only provides a different theoretical mapping from the option price to some unobserved variable. Similarly, different specifications for an option contract, e.g., barrier or Asian options, only change the details of the theoretical mapping; the option price is still being mapped to some unobservable.

The distinction between theoretical and empirical econophysics speaks to the difference between methodology and methods. Instead of, say, fitting a power law distribution or determining the characteristic exponent of a stable distribution for empirical data on a economic variable such as stock returns, providing the theoretical solution for an option pricing model is similar, say, to providing a solution for the stable distribution with an inverse Gaussian subordinator [95]. Phenomenological methodology requires methods for implementation. In the same fashion that the method of estimating the Hurst exponent for a pairs trade [96] represents an observed variable in terms of a statistical parameter, option pricing models provide a method of mapping the observed option price to another form such as the implied volatility. Use of absence of arbitrage to derive an option pricing model is a significant difference from a statistical estimator. However, the result is not the same as deriving a theoretical model based on rational choice behavior that is then subject to empirical testing. As with a statistical estimator, ergodicity embedded in the option pricing method is not introduced after a theoretical model has been derived.

6. Conclusion

Confronted with non-experimental data for economic variables, such as stock prices, wage rates, commodity prices, interest rates and the like, the dominant methodology of modern economics initially develops theoretical models based on maximizing behavior and then observes the statistical fit between the exogenous and endogenous model variables. While comparison of 'stylized facts' with predictions of theories initially deduced directly from the 'first principles' of constrained maximizing behavior for individuals and firms is recommended practice in modern economics, such theories often have poor *ex ante* empirical performance resulting in an on-going horse race of specification searches. At least since Tinbergen [76], when confronted with the problem of modeling complex 'macroscopic' economic variables, such as exchange rates or GNP or unemployment, where it is difficult or not possible to derive an empirically reliable theory from known neoclassical principles about the (microscopic) rational behavior of individuals and firms, a resolution of such problems has encouraged development of an inductive approach to empirical modeling in modern economics that is similar to the phenomenological approach of econophysics.

Being a large tent, it is not surprisingly that modern economics has developed both deductive – logical and theoretical – and inductive – empirical and phenomenological – lines of inquiry. As such, phenomenology is not a distinguishing methodological feature separating econophysics from economics. However, the evolution from the deterministic models of neoclassical economics to stochastic models of modern economics has focused on obtaining empirical estimates by adding a stationary error term to a linear model of equilibrium relationships derived from axiomatic choice theories of rational behavior for individuals, firms and investors. Recognizing that empirical limitations of reversible ergodic processes have been recognized in statistical mechanics for almost a century, the various stochastic models employed in econophysics have potential to capture more complex non-linear dynamics, e.g., multi-fractal models; chaos theory; truncated Levy processes; bifurcating processes.

Emergence of the 'new' subject of econophysics in the mid-1990s begs an obvious question: if physics has exerted considerable and long lasting influence on economics, what is 'new' about econophysics? This paper addresses this question by tracing the pre-history of econophysics back to the origins of statistical mechanics and the seminal contributions of Ludwig Boltzmann. The evolution of an essential element of that pre-history – the ergodicity hypothesis – in the subsequent history of economics is identified and contrasted with the influence that the rational mechanics of classical physics exerted initially on the marginalists and, subsequently, on neoclassical economics and modern economic science. Examination of this history reveals that the enduring philosophical determinism of rational mechanics undermines the use of phenomenological methods in much of modern economics allowing econophysics to provide a distinct approach to modeling and predicting the stochastic behavior of economic phenomena, especially financial prices.

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