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Hedging Canadian Treasury Bill Positions with U.S. Money Market Futures Contracts

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The absence of a viable futures market for Government of Canada Treasury bills (T-bills) poses a legitimate problem for Canadian money market participants seeking to hedge cash positions.[1] While, in some circumstances, hedging objectives can be achieved through the use of when-issued T-bill positions, for many hedging situations a potentially more practical alternative is to make use of instruments traded on U.S. exchanges, specifically, Canadian dollar, U.S. T-bill, and Eurodollar futures contracts. However, use of U.S. money market futures by Canadian hedgers is complicated by basis risk considerations and by the hedge profit function being denominated in two different currencies. The objective in this paper is to compare a number of strategies for hedging cash Canadian Treasury bill positions. The strategies examined exhibit differing position sizes for the U.S. money market futures used in forming the hedge. Theoretically, it is demonstrated that the minimum-variance hedge ratios can be interpreted and estimated as coefficients in an appropriately specified *multivariate* regression. Section I outlines some institutional detail and the fundamentals that drive hedge performance. Section II explains how covered parity boundaries can be used for determining the hedging instrument. Section III examines the empirical performance of the hedges based on a number of hedging instrument and hedge ratio selection criteria. Finally, Section IV contains a summary of the relevant results.

I. Institutional Background and Fundamentals

The ability of Canadian money market participants to hedge cash positions is severely limited by the lack of feasible hedging instruments. The Government of Canada Treasury bill futures contract, introduced on the Toronto Futures Exchange (TFE) in September 1980, has suffered a number of setbacks culminating in the contract ceasing trading in early 1987.[2] Presently, Canadian dollar-denominated instruments available for hedging cash T-bill positions are limited to exchange-traded bond options and OTC-style when-issued markets.[3] Unfortunately, bond options have considerable basis risk when used to hedge T-bill positions and use of when-issued T-bill contracts is restricted because contract maturities are almost always less than one week.[4] Consequently, hedge trades using foreign (particularly U.S.) debt and currency futures contracts are an important potential alternative. However, use of these instruments requires accurate hedge design. In addition, even if the hedge is properly constructed, there is still a residual basis risk problem that is compounded because returns from

[1]While the discussion here is in terms of Canadian Treasury bills, the analysis is extendable to hedging other types of Canadian dollar-denominated money market instruments.
[2]The TFE is a part of the Toronto Stock Exchange. Even though government debt futures no longer trade, the contracts are still listed on the TFE and may resume trading at a later date. While there are no hard and fast answers as to why the TFE contracts ceased trading, the absence of a well-developed base of market "locals" severely hampered the liquidity of the contracts. As a result, inability to unwind trades discouraged many of the institutions and dealers willing to support the contract.
[3]In addition, in the fall of 1986 a Government of Canada Treasury bill option contract began trading on the Montreal Exchange. To date, trade in T-bill options has been characterized by limited liquidity and almost no open interest. As a result, this instrument is also not a feasible hedging vehicle.
[4]On the when-issued market for Government of Canada T-bills, see G. Poitras [forthcoming].

the futures positions are denominated in U.S. dollars. As a result, the hedge is at least partly dependent on exchange rate behavior.

To accurately hedge a cash Canadian T-bill position with U.S. money market futures, hedge design should account for the interest and exchange rate relationships implied by covered interest parity (CIP). For arbitrage involving securities identical in all respects except currency denomination, the annualized CIP relationship is often stated as:

$$\frac{F}{S} = \frac{1 + r}{1 + r*} \tag{1}$$

Extending this result to currency futures, the nearby contract can be taken as a replacement for the spot position:

$$\frac{F(0,T)}{F(0,N)} = \frac{1 + i(0,T-N)}{1 + i*(0,T-N)} \tag{2}$$

where

F = the one-year forward exchange rate in domestic direct terms[5]
S = the spot exchange rate in domestic (U.S.) direct terms
r = the domestic (U.S.) interest rate (on a 365-day basis)
$r*$ = the foreign (Canadian) interest rate (on a 365-day basis)
$F(0,t)$ = the price of a currency futures contract at time 0 for delivery at time t; T is the date for deferred delivery and N is the date for nearby delivery
$i(0,T-N)$ and $i*(0,T-N)$ = the time 0 domestic (U.S.) and foreign (Canadian) interest rates adjusted by $(T-N)/365$ to account for the trading horizon (This notation takes no account of the term structure of rates implied by currency futures.)

Even though for institutional reasons, Equations 1 and 2 may not be exact, these formulations are sufficient for designing the hedge strategies developed here.[6]

The CIP conditions Equations 1 and 2 provide the framework for hedging Canadian T-bill positions with U.S. money market futures. The basics of the hedge strategy follow from the textbook-covered interest arbitrage trade for one-year securities where the covered Canadian rate exceeds the rate on a comparable U.S. security:

[5]*Domestic direct terms* is defined as units of domestic currency to units of foreign currency, e.g., for Canadian investors this would be \$C/\$US (Stigum [1981]). However, the Canadian dollar futures contract is traded in \$US/\$C. Hence, to avoid complications, the U.S. is considered the "domestic" country for present purposes.

[6]Among the reasons why Equations 1 and 2 may not be exact are clearing lags and holidays. Some care should be taken to note that Equation 1 expresses CIP on an *annualized* basis. When evaluating a trade for a shorter period of time, it is necessary to correct the formulas to get the appropriate basis points to be gained from doing the trade as in Equation 2.

U.S. asset	*Exchange Market*	*Canadian asset*
Borrow \$$Q$ for one year at r	Buy \$$Q/S(0)$ Canadian dollars	Invest \$$Q/S(0)$ for one year at $r*$
	Sell [\$$Q/S(0)$]$(1+r*)$ Canadian dollars forward at $F(0,1)$	

Omitting transactions costs, this trade will generate an arbitrage profit by assumption. *Converting the CIP trade to a hedge involves substituting a short U.S. money market futures position for the borrowed U.S. funds and, to reflect the currency transactions, a tailed currency spread that is short the deferred and long the nearby.* Given this, the hedge design problem is concerned with specifying appropriate hedge ratios for the futures positions. In turn, hedge ratio specification depends on the behavior of basis risk.

Basis risk in the hedge arises from two factors: deviations from CIP for the relevant Canadian and U.S. instruments and discrepancies arising from the use of futures positions as replacements for cash transactions. Of these sources of basis risk, the replacement of cash transactions with futures positions is the more straightforward. Futures prices for U.S. money market instruments are closely tied to cash prices through cash-and-carry arbitrages. Almost all systematic deviations of futures from cash prices are due to a combination of fluctuations in carry costs, expectations, term structure effects, distortions arising from delivery, and the time to maturity of the futures contract. These factors do impose some limitations on hedge design. For example, because of the impact that delivery considerations have on futures prices during the delivery month of the contract, the nearby contract selected should not enter the delivery month during the anticipated life of the hedge (unless "rolling the hedge forward" is incorporated as part of the hedging strategy). Ignoring delivery effects, a number of studies have shown that there is reasonably close correspondence between movements in cash and futures prices for U.S. money market futures. Hence, the substitution of futures positions for cash transactions introduces only limited basis risk.

Basis risk arising from CIP deviations is another matter. CIP holds as an equality for instruments that are identical in all respects except for currency of denomination. This is not the case when Canadian T-bills are compared with either Euros or U.S. T-bills. Each of these instruments possesses differential risk characteristics. As demonstrated below, due to a combination of risk and arbitrage considerations, Euro and U.S. T-bill rates can be combined to provide effective inequality bounds on the covered Canadian rate, i.e., the covered Canadian rate is bounded above by the Euro rate and below by the U.S. T-bill rate. This implies that deviations from CIP are also bounded and, hence, variation of the hedge position is constrained. Fortunately, the relationship between Euro and T-bill rates is (locally) quite stable because these two rates are actively arbitraged (Kreicher [1982], Kaen, et al. [1983], Krol [1987]).[7] This provides some scope for designing hedging

[7]Historically, the cash U.S. T-bill/Euro spread has fluctuated between 35 and 200 basis points, with occasional "spikes" outside this range.

strategies that exploit the systematic portion of the basis risk to improve hedge performance. To understand how this is done, it is necessary to examine the specific mechanics of how covered interest arbitrage applies to the relevant instruments under consideration.

In general, for equality to hold in Equations 1 and 2 the arbitrage must be "two-sided." In the Euro/Canadian T-bill case, while the arbitrage trade is apparent when the Canadian T-bill rate exceeds the Euro rate, the arbitrage trade for the Euro rate exceeding the Canadian T-bill rate is not so clear. In this case, when the T-bill rate falls below the covered Euro rate, the implied covered interest arbitrage trade would involve borrowing at the Canadian T-bill rate, converting to U.S. dollars and buying Euro deposits—simultaneously covering forward the funds to be received at the maturity of the Euro deposit. This trade cannot be executed because only the Canadian government has the ability to issue liabilities in the Canadian T-bill market. Hence, the Euro rate only provides an upper boundary on covered Canadian T-bill rates. Based on this analysis, a weak inequality can be substituted in Equation 2. Manipulating and solving gives the following expression (omitting time-dating for convenience):

$$i^e - [i* + \{[F(0,T) - F(0,N)]/F(0,N)\}(1 + i*)] >= 0 \qquad (3)$$

where, for present purposes, $i*$ is the Canadian T-bill rate and i^e is the Euro-U.S. dollar rate. In other words, the covered Canadian T-bill rate is bounded above by the Euro rate. There is not strict equality because arbitrageurs are unable to borrow at the Canadian T-bill rate.[8]

A lower bound on the covered Canadian T-bill rate has, historically, been provided by the U.S. T-bill rate, i.e., when i^u is the U.S. T-bill rate, Equation 3 extends naturally:

$$i^u - [i* + \{[F(0,T) - F(0,N)]/F(0,N)\}(1 + i*)] =< 0 \qquad (4)$$

U.S. T-bills are perceived by the market to be less risky than Canadian T-bills. Hence, substitution between U.S. and Canadian T-bills by asset holders would be based on covered parity plus an adjustment for the differential risk characteristics of the two instruments. For the lower bound, T-bill investors and other important players are required to react to the size of the covered differential with U.S. T-bills primarily by making portfolio adjustments. The adjustment process is conditioned by the level of overnight foreign and Canadian interest rates, tax rates, foreign exchange market conditions, Canadian monetary policy, and so on. The lower bound is not driven by the "pure" arbitrage activity that drives the upper boundary. Given that the Euro rate will almost always lie above the U.S. T-bill rate because of the differing risk characteristics of those two instruments, this implies that the covered Canadian T-bill rate will fluctuate within boundaries provided by the Euro and U.S. T-bill rates.

[8]As illustrated in the Appendix, Equations 3 or 4 can be used to analytically describe the hedge specification.

Specifying dge Positions

The key practical questions in implementing the hedge are which U.S. money market futures to use and how to determine the appropriate hedge ratios for the interest rate futures and currency futures spread positions. Selection of the appropriate interest rate futures contract is primarily an empirical question. A number of different methodologies are available for deriving hedge ratios (e.g., Toevs and Jacob [1986]). Because the objective here is not to provide an exhaustive comparison of the performance of different hedge ratio specification procedures, only two hedging strategies are examined: dollar value and minimum variance. The dollar-value approach computes the hedge ratio directly from the value of the cash market transactions used in the "textbook" covered interest arbitrage trade. This approach produces accurate hedging outcomes when the price behavior of the futures contracts used closely matches that of the cash instruments. The greater the basis risk, the more likely it is that hedges based on the dollar-value approach will be ineffective. The dollar-value approach is used here primarily as a benchmark against which outcomes of the minimum-variance hedge ratios can be compared.

Following Ederington [1979], the accepted method for deriving minimum-variance hedge ratios is to exploit the first-order conditions for hedge variance minimization.[9] For the case at hand, this requires specification of the expected profit and variance of profit for the hedge position. For a hedge done with Euros:

$$E(\pi) = Q\{i^{*\prime}(0) - E[i^{*}(1)'']\} + A\{E[\text{FPS}(1)] - \text{FPS}(0)\} + B\{eu(0) - E[eu(1)]\} \tag{5}$$

$$\text{var}(\pi) = Q^2\sigma_{*}^2 + A^2\sigma_{f}^2 + B^2\sigma_{e}^2 + 2[QA\sigma_{*f} - (QB\sigma_{*e} + AB\sigma_{fe})] \tag{6}$$

where: Q is the value of the cash position, A is the value of the legs of the currency futures spread (FPS), B is the value of the interest rate futures position and σ is either a variance or a covariance defined by the subscripts f for futures spread, e for Euro and $*$ for Canadian T-bill. The $'$ and $''$ superscripts allow for possibly differing times between settlement and maturity for the cash instrument.

Given Equation 6, generalized hedge ratios for both A and B unconstrained can be derived by solving the two first-order conditions (foc) for the minimum-variance problem. As demonstrated in the Appendix, solving the foc produces a generalization of the conventional minimum-variance hedge ratio result; the minimum-variance hedge ratios are regression coefficients in a *multivariate* regression of Canadian T-bill rates on the Euro (or U.S. T-bill) rate and the futures spread.

[9]The hedge ratio calculation and estimation problem is examined in a number of sources, e.g., Toevs and Jacob [1986], Bell and Krasker [1986], Hill and Schneeweis [1981], Gemmill [1985], Cecchetti et al. [1986]. Benninga et al [1984] show that, if futures prices are unbiased, then the minimum-variance hedge ratio is also an optimal hedge ratio for hedgers with quadratic utility functions.

Use of regression to estimate hedge ratios raises a number of technical points. In particular, in the specification of the variables in the regression: should levels or first differences be used? Recent work by Toevs and Jacob [1986] attempts to cast some doubt on the conventional wisdom (e.g., Hill and Schneweiss [1981]) that first-difference regressions are superior to regressions done in levels. However, regressions based on levels typically have undesirable statistical properties (e.g., nonstationarity as reflected in Durbin-Watson values significantly different from two). As a result, the minimum-variance hedge ratios provided here use regressions based on *changes* in rates (or prices) and futures spreads.[10] Because the ultimate objective is to minimize the variation in *prices,* regression results based on rates require some manipulation before being used as hedge ratios. Specifically, the following coefficient transformation is required:

$$-\Delta P = \Delta r/(1 + r(0) + r(1) + r(0)r(1))$$

In practice, rates must be adjusted to be comparable in maturity to the maturity of the currency spread. In addition, the coefficient on the interest rate futures position must be adjusted by the ($US/$C) exchange rate to get dollar equivalency.

On a more technical level, as demonstrated by Cecchetti, et al. [1986], hedge ratio estimation is complicated because both the means and the variance-covariance matrix of the relevant variables are not typically constant through time as required for ordinary least squares (OLS) regression.[11] In addition, because the minimum-variance hedge ratio does not take the mean return on the hedge portfolio into account, regression-based hedge ratios will not necessarily be optimal for all types of hedger-expected utility functions. Theoretically, correct estimation of hedge ratios requires both specification of the hedger's expected utility function and the use of sophisticated estimation procedures.[12] To date, there is little empirical evidence indicating that ARCH techniques substantially improve OLS-based hedge ratio estimates. Cecchetti, et al. [1986] present results for U.S. Treasury bonds that OLS estimates may differ from hedge ratios derived from estimation procedures based on *log utility* expected utility functions. However, while it is possible to incorporate information on the mean returns of the hedge portfolio into the estimation procedure, this does lead to considerably more complicated estimation procedures. Hence, to account for ARCH behavior, tests for the ARCH specification are conducted. In addition, the means of the hedge portfolio are reported in the empirical results of Section III as a measure of the cost of hedging.

[10]Theoretical specification of the parameters is provided in Appendix Equation A.3.
[11]More precisely, the variables exhibit autoregressive condition heteroscedastic (ARCH) behavior. Estimating and testing in ARCH models is currently an active area of research in econometrics (e.g., Engle [1982]). In addition to the ARCH problem, conventional estimates of hedge ratios typically confuse ex ante and ex post distributions.
[12]Wolff [1987] shows that ARCH models are a specific form of random coefficient model.

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In order to compare the hedging strategies, results are presented for monthly and quarterly fixed-maturity anticipatory hedges over distinct trading horizons covering 1983-1986. Specifically, for the monthly case, hedges are put on one month prior to the *start* of the delivery month of the front contract in the currency spread. In the quarterly case, hedges are initiated three months prior to the starting month for front contract delivery. (In all cases, the delivery month for the interest rate future and the front contract in the currency spread are the same.) The Canadian T-bill position being hedged involves an anticipated purchase (or sale) of the topical three-month T-bill at a prespecified future date. The hedging objective is to lock in the current three-month T-bill rate for a purchase to take place either one month or three months in the future. With slight adjustment, the results for this type of hedge readily extend to other types of cash trades, e.g., for hedging a "ride on the yield curve." By focusing on the outcomes of longer-term fixed-maturity hedges, this approach to evaluating hedging effectiveness is more applicable to the types of hedges that would be used in practice than the typical approach (e.g., Hill and Schneeweiss, Toevs and Jacob) which compares the variability of hedged and unhedged positions over a daily or weekly time series.

In addition to the size of the hedging horizon, a number of other adjustments were made to make the hedge results more comparable to actual hedging outcomes. Most importantly, in calculating the hedge returns, it was assumed that fractional position sizes could be established and that the underlying cash position was C$20 million (par value). In practice, rounding has the greatest impact on the value of the interest rate futures position. A C$20 million position was selected because a position approaching this size is needed for the tail on the currency spread to be close to one contract. To see this, recall that the dollar-value approach determines the number of currency spreads directly from the values implied in the corresponding covered interest arbitrage trade. Based on the spread analytics contained in Section A of the Appendix, the resulting tail would be equal to (i(0,3 months) $-$ $i*$(0,3 months)) times the nearby position. If U.S. rates are, say, 8 percent and Canadian rates are 10 percent then i(0,3) would equal .02 and i*(0,3) is .025 indicating that approximately 200 nearby contracts are required to augment the deferred position to 201.

Another practical feature concerns the use of a tail on the minimum-variance hedge. Because of the more favorable margin requirements and transactions costs on spreads versus open positions, hedges without tails are typically less costly than hedges with tails. However, when the hedge is not tailed, then the resulting basis variation must be accounted for by appropriately adjusting the spread and interest rate futures positions. Theoretically, this involves solving the minimum-variance problem where the variance of the hedge is defined

[13]The data used are daily settlement prices for IMM Euro, U.S. T-bill and Canadian dollar futures contracts together with (closing) cash Canadian T-bill, Euro, and U.S. T-bill prices as well as forward and spot exchange rates. The futures prices were obtained from Data Resources, while the cash prices are from the Bank of Canada. Where appropriate, the Euro and U.S. T-bill rates were adjusted to a 365-day basis.

without a currency futures position—as is the case in Equation 6 and Appendix Equation A.3. In order to accurately assess the validity of this procedure, the results for the untailed minimum-variance spread would have to be compared with a tailed minimum-variance spread, where the hedge ratios for the tailed minimum-variance spread have been derived from a regression of Canadian T-bill prices (or rates) on Euro (or U.S. T-bill) prices (or rates), the currency spread, and the exchange rate. However, the in-sample regressions (not reported here) indicated that the coefficient for the exchange rate was insignificant and close to zero for both Euros and U.S. T-bills. On the basis of this information, tailed minimum-variance hedges were not considered.

Two approaches were used to estimate the minimum-variance hedge ratios. The first set of regressions was based on data for Canadian dollar equivalent position sizes for the actual instruments used to construct the hedges. While this approach identifies the hedge ratios that produce the most reduction in the value of the cash position, the estimates suffer from being based on in-sample information. As an adjunct to the in-sample information, regressions were also estimated using *cash market data* for daily, weekly, and monthly frequencies over the 1984-1987 sample period (see Tables 1 and 2). This approach was used both to indirectly assess the validity of hedges based on in-sample information and to see if cash market data can be used as a substitute for the actual futures data. The differing data frequency (i.e., daily, weekly, monthly) was used to assess the validity of estimating hedge ratios with a data frequency different from the hedging horizon. If coefficient estimates are affected by the sampling frequency, then this information substantially enhances the out-of-sample applicability of the results.

Examining these results reveals a number of interesting points. Most significantly, dollar-value and minimum-variance strategies correspond quite closely for the quarterly and monthly horizons, with a small difference between the dollar-value and minimum-variance hedge ratios emerging as the estimation horizon is shortened. As is evident in the cash data and in sample regressions, these hedge ratios do get substantially smaller as the estimation horizon is shortened. Regarding the appropriate interest rate future, using Euros produced much the same fit and coefficient estimates as using U.S. T-bills—a result that should be reflected in hedge performance. The addition of the change in exchange rates did not improve the fit significantly, indicating that tailing the minimum-variance hedge is not likely to noticeably improve hedge performance. As expected, regressions for the daily frequency exhibited ARCH behavior indicating that there is some room for potential improvement in the efficiency of the daily estimates. However, there was little evidence of significant ARCH behavior for the longer frequencies indicating that this source of estimation inefficiency is not of importance to the hedges considered here.[14]

[14]The cash data regressions were also run in terms of levels with no significant change in the coefficient estimates.

Table 1A*. Cash market estimates: Eurodollars

$dTBC = a_0 + a_1\,dEU + a_2\,dFWD$**

Sample: 84/01/01-87/10/31

	a_0	a_1	a_2	R^2	DW	SEE	ARCH***
Monthly NOB = 43	.0015	1.05	.69	0.96	2.15	.038	1.21
	(0.30)	(22.5)	(27.8)				
Weekly NOB = 173	−.0009	0.65	0.56	0.73	2.36	.032	5.27
	(0.39)	(13.5)	(18.7)				
Daily NOB = 878	−.0003	0.31	0.34	0.35	2.09	.017	17.11
	(0.42)	(12.29)	(19.2)				

$dTBC = b_0 + b_1\,dEU + b_2\,dFWD + b_3\,dS$

	b_1	b_2	b_3	R^2	DW	SEE	ARCH***
Monthly NOB = 43	1.05	0.69	−0.07	0.97	2.16	.032	1.26
	(2.05)	(23.7)	(0.17)				
Weekly NOB = 173	0.64	0.56	−0.02	0.73	2.36	0.032	5.29
	(12.3)	(15.3)	(6.06)				
Daily NOB = 878	0.29	0.31	0.58	0.35	2.08	0.018	12.17
	(11.3)	(15.4)	(3.30)				

*t value in parentheses; coefficients have to be exchange rate adjusted to be comparable to coefficients in Table 2.
**The difference between the spot and 90 day forward
***Expressed as an F test

Table 1B*. Cash market estimates: U.S. T-bills

$dTBC = a_0 + a_1\,dTBU + a_2\,dFWD$

Sample: 84/01/01 - 87/10/31

	a_0	a_1	a_2	R^2	DW	SEE	ARCH**
Monthly NOB = 43	−.005	0.97	0.69	0.88	2.14	.060	0.26
	(0.54)	(10.27)	(14.24)				
Weekly NOB = 173	−.0006	0.61	0.51	0.64	2.04	.037	0.05
	(0.23)	(9.51)	(14.8)				
Daily NOB = 878	−.0003	0.28	0.33	0.30	1.94	0.018	12.02
	(0.41)	(9.03)	(18.01)				

$dTBC = b_0 + b_1\,dTBU + a_2\,dFWD + a_3\,dS$

	b_1	b_2	b_3	R^2	DW	SEE	ARCH**
Monthly NOB = 43	0.95	0.64	1.46	0.89	2.20	0.057	0.17
	(10.31)	(12.44)	(2.20)				
Weekly NOB = 173	0.57	0.47	0.67	0.65	7.06	0.037	0.08
	(8.45)	(11.63)	(1.70)				
Daily NOB = 878	0.27	0.29	0.80	0.31	1.95	0.018	7.41
	(8.42)	(14.04)	(4.47)				

*t value in parentheses; coefficients have to be exchange rate adjusted to be comparable to coefficients in Table 2.
**Expressed as an F test

Table 2A*. In-sample estimates: Euros

$dP_{ctb} = a_0 + a_1\,(dP_{eu}/FX) + a_2\,[dF(0,N)/FX] + a_3\,[dF(0,T)/FX]$

Sample: 83/02-87/02

	a_1	a_2	a_3	R^2	DW	SEE
Quarterly NOB = 17	.97	1.29	1.27	0.85	2.47	.0012
	(6.25)	(4.96)	(5.23)			
Monthly NOB = 49	0.71	0.65	0.65	0.89	2.09	.0004
	(6.28)	(5.40)	(4.92)			

*FX is the spot exchange rate, P_{ctb} is price of Canadian T-bill, P_{eu} is price of Euro; t value in brackets. Equation is estimated in first differences with spread positions unconstrained.

Table 2B*. In-sample estimates: U.S. T-bills

$dP_{ctb} = a_0 + a_1\,(dP_{tbu}/FX) + a_2\,[dF(0,N)/FX] + a_3\,[dF(0,T)/FX]$

Sample: 83/02-87/02

	a_1	a_2	a_3	R^2	DW	SEE
Quarterly NOB = 17	1.07	1.21	1.21	0.82	2.22	.0013
	(6.01)	(4.64)	(4.95)			
Monthly NOB = 49	0.73	0.54	0.56	0.78	2.15	0.0006
	(3.71)	(2.96)	(3.35)			

*FX is the spot exchange rate; t value in brackets. Equation is estimated in first differences with spread positions unconstrained.

With appropriate adjustments, these regression estimates were used to calculate the minimum-variance hedge ratios that appear in Tables 3 and 4. Results for all hedges are given in terms of the percentage reduction in the (price) variance of the cash position. Percentage reduction in variance is arrived at by the following process: (1) calculate the change in the *price* of a three-month Canadian T-bill between the start date and the end date for the hedge; (2) calculate the change in the value of the hedged position using the appropriate hedge ratios, where the hedged position includes both futures and cash values; (3) *assuming that the expected change in the value of the cash position is zero,* calculate the variance of the relevant position by squaring the values, summing and dividing by the number of observations; and (4) calculate the reduction in variance by taking a ratio of the variance of the hedged position to the unhedged position and convert to a percentage. In turn, Table 5 provides the mean return on the hedged and unhedged positions arrived at by summing the value change and dividing by the number of observations. No account is taken of either transactions or variation margin costs.

The results in Tables 3-5 reveal a number of points of interest to hedgers. Specifically, dollar-value hedges underperformed relative to minimum-variance hedges for the monthly and quarterly horizons. For both horizons, the minimum-variance hedges were able to reduce the variance of the cash position substantially with a slight preference for the use of Euros over U.S.

Table 3. Percentage reduction in the variability at the cash position—Eurodollar futures

Type of Hedge	Monthly	Quarterly
Dollar-Value Untailed	72%	82%
Dollar-Value Tailed	71%	82%
Minimum-Variance In-Sample Estimates	88%	85%
Minimum-Variance Cash-Market Estimates	73%	

Table 4. Percentage reduction in the variability at the cash position—U.S. T-bill futures

Type of Hedge	Monthly	Quarterly
Dollar-Value Untailed	64%	82%
Dollar-Value Tailed	64%	83%
Minimum-Variance In-Sample Estimates	79%	85%
Minimum-Variance Cash-Market Estimates	76%	

Table 5*. Mean returns from hedging strategies

	Monthly	Quarterly
Unhedged Cash Position	−$4,516	$11,808
Euros		
Dollar-Value Untailed	−$1,264	−$6,276
Dollar-Value Tailed	−$1,052	−$6,036
Minimum-Variance (In-Sample)	−$2,580	−$4,840
U.S. T-bills		
Dollar-Value Tailed	−$ 964	−$4,096
Dollar-Value Untailed	−$ 752	−$3,856
Minimum-Variance (In-Sample)	−$1,540	−$4,268

*Assumes C $20 million par value cash position.

T-bill futures. For the case of Euros, variance reduction for the in-sample estimates was well over 80 percent. While minimum-variance hedge ratios based on cash market data were not as effective as in-sample estimates, their performance was still acceptable. Not surprisingly, the means returns from hedging were generally of opposite sign to the change in the value of the cash position with mean returns for hedges based on U.S. T-bill futures distinctly lower in absolute value terms than hedges based on Euros. Finally, these results implicitly assume that hedges are unmanaged. If the hedge is put on selectively, i.e., only in anticipation of adverse price movements, then the hedge performance results would be different. An examination of the raw data on the hedge returns reveals that the minimum-variance hedges were particularly successful at reducing position volatility in the face of large moves in the market.

IV. Conclusions

The absence of a viable futures market for Government of Canada T-bills poses a significant hedging problem for Canadian money market participants. In this paper, it was demonstrated that, if hedges are appropriately specified, U.S. money market futures contracts can be used to effectively hedge over 80 percent of the anticipated variability of the cash position for both monthly and quarterly hedging horizons. Regarding hedge ratio specification, while the dollar-value approach was effective for both hedging horizons, the dollar-value hedges were generally less variance-reducing than the minimum-variance hedges. In general, for accurate hedge design, the minimum-variance approach is preferred.

Appendix

A. Analytics of the Spread

To evaluate the returns from a particular hedge strategy, further analysis of Equations 3 and 4 is required. For ease of exposition, assume that the relevant variables satisfy the conventional calculus regularity conditions. (In practice, this assumption requires that the hedge positions be continuously adjusted.) Specifically, consider the total derivative of Equations 3 and 4 assuming equality:

$$di - \{di* + (1 + i*)\, d[F(0,T) - F(0,N)]/F(0,N) + [F(0,T) - F(0,N)]/F(0,N)\}\, d(1 + i*) = 0 \quad \text{(A.1)}$$

Observing that the last term can be safely ignored for present purposes, expanding and manipulating gives an approximation to the condition for a desired hedge:

$$di = \left\{ di* + (1 + i*)\left[\frac{d(\text{FPS})}{F(0,N)} + \frac{\text{FPS}}{F(0,N)}\frac{dF(0,N)}{F(0,N)}\right]\right\}$$
$$= di* + (1 + i*)\frac{d(\text{FPS})}{F(0,N)} - (i - i*)\frac{dF(0,N)}{F(0,N)} \quad \text{(A.2)}$$

Examining Equation A.2, it can be seen that a hedge that offsets changes in the value of the cash Canadian T-bill position involves combining a U.S. interest rate futures position and an appropriately tailed currency futures spread.

B. Minimum-Variance Hedge Ratios

Case 1: Unconstrained Currency and Interest Futures Positions

Assume that second-order terms can be ignored, then:

$$E(\pi) = Q\{i*'(0) - E[i*(1)'']\} + A\{E[\text{FPS}(1)] - \text{FPS}(0)\} + B\{eu(0) - E[eu(1)]\}$$

$$\text{var}(\pi) = E[\pi - E(\pi)]^2$$
$$= Q^2\sigma_*^2 + A^2\sigma_f^2 + B^2\sigma_e^2 + 2[QA\sigma_{*f} - (QB\sigma_{*e} + AB\sigma_{fe})]$$

Differentiating var (π) with respect to A and B gives two first-order conditions that can be solved to get:

$$A^* = \frac{B\sigma_{fe} - Q\sigma_{*if}}{\sigma_f^2}$$

$$B^* = \frac{Q\sigma_{*e} + A\sigma_{fe}}{\sigma_e^2}$$

Solving for A:

$$A^* = \frac{Q}{1-R^2}\left(\frac{\sigma_{*e}}{\sigma_e^2}\frac{\sigma_{fe}}{\sigma_f^2} - \frac{\sigma_{*f}}{\sigma_f^2}\right)$$

where

$$R^2 = \frac{\sigma_{fe}^2}{\sigma_e^2\,\sigma_f^2}$$

i.e., the squared correlation between e and f.

Rearranging A^* gives:

$$A^* = Q\,[(\sigma_{*e}\,\sigma_{fe} - \sigma_{*f}\sigma_e^2)/\,(\sigma_e^2\,\sigma_f^2 - \sigma_{ef}^2)] \qquad \text{(A.3)}$$

In words, the minimum-variance hedge ratio for the currency spread position is equivalent to the currency spread regression coefficient in a regression of Canadian T-bill rates on Euro rates and currency spreads. Solving for B gives a similar result.

Case 2

In order to transform the hedge ratio problem into its more familiar bivariate regression interpretation, one of the two hedge positions can be constrained—thereby reducing the solution to a single first-order condition. This can be done by setting the exchange rate adjusted value of the Euro position to be equal to the value of the Canadian T-bill position or by setting the value of the currency futures positions equal to the implied CIP values, i.e., set $B = Q/F(0,T)$ or set $A = Q/F(0,T)$. If $B = Q/F(0,T)$, then differentiating var (π) with respect to the single choice variable A gives:

$$A^{**} = Q\left(\frac{1}{F(0,T)}\frac{\sigma_{fe}}{\sigma_f^2} - \frac{\sigma_{*f}}{{}^{2}_{f}}\right)$$

A similar result holds for B^* if A is constrained.

In this form, the hedge ratio expression involves bivariate regression coefficients. However, the variances and covariances are based on the *conditional* distribution and, as a result, imply a different regression specification than in the unconditional case (i.e., Case 1).

ibliography

Bell, D., and W. Krasker. "Estimating Hedge Ratios." *Financial Management* (Summer 1986), pp. 34-39.

Benninga, S., R. Eldor, and I. Zilcha. "The Optimal Hedge Ratio in Unbiased Futures Markets." *Journal of Futures Markets* 4(1984), pp. 155-59.

Cecchetti, S., R. Cumby, and S. Figlewski. "Estimation of the Optimal Futures Hedge." Salomon Bros. CSFM Working Paper No. 402, December 1986.

Ederington, L. "The Hedging Performance of the New Futures Markets." *Journal of Finance* 34(1979), pp. 157-70.

Engle, R. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica* 50(July 1982), pp. 987-1008.

Gemmill, G. "Optimal Hedging on Futures Markets for Commodity-Exporting Nations." *European Economic Review* 27(1985), pp. 243-61.

Hill, J., and T. Schneeweis. "A Note on the Hedging Effectiveness of Foreign Currency Futures." *Journal of Futures Markets* 1(1981), pp. 659-64.

Kaen, F., B. Helms, and G. Booth. "The Integration of Eurodollar and U.S. Money Market Interest Rates in the Futures Markets." *Weltwirtschaftliches Archiv* 119(1983), pp. 601-15.

Kreicher, L. "Eurodollar Arbitrage." Federal Reserve Bank of New York *Quarterly Review* 7(Summer 1982), pp. 10-20.

Krol, R. "The Term Structure of Eurodollar Interest Rates and Its Relationship to the U.S. Treasury Bill Market." *Journal of International Money and Finance* 6(1987), pp. 339-54.

Poitras, G. "The Pricing Performance of the When-Issued Market for Government of Canada Treasury Bills." *Canadian Journal of Administrative Science* (forthcoming).

Stigum, M. *Money Market Calculations.* Homewood, Il: Dow Jones-Irwin, 1981.

Toevs, A., and D. Jacob. "Futures and Alternative Hedge Ratio Methodologies." *Journal of Portfolio Management* 13(Spring 1986), pp. 60-70.

Wolff, C. "Autoregressive Conditional Heteroscedasticity: A Comparison of ARCH and Random Coefficient Models." CRISP Working Paper No. 209, July 1987.