

Do changes in debt maturity composition affect the term structure? Some Canadian evidence

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This paper examines the time series behaviour of the term structure of interest rates in the Government of Canada bond market to determine whether changes in the maturity composition of Canada debt have had a significant effect on bond yields. Methodologically, a reduced form model is estimated which incorporates exogenous information about both the level of domestic interest rates and the shape of the US term structure. The empirical results indicate that the null hypothesis of no debt management effects in the Government of Canada bond market is rejected for monthly data.

I. INTRODUCTION

An important issue in debt management is the extent to which changes in the maturity structure of aggregate debt affect the term structure of bond yields. Various studies using a number of different methodologies have examined this issue and produced conflicting results. Recent attempts to address the issue have generally relied on structural models of the bond market to test for so-called 'supply effects'. In contrast to these studies, this study employs reduced form techniques (Hannan *et al.*, 1980; Poskitt and Tremayne 1984) to test for a relationship between domestic supplies of Government of Canada marketable debt and the term structure of Government of Canada bond yields. This approach is useful because the null hypothesis of no supply effects can be tested using reduced-form estimates with only minimal identifying restrictions imposed.

In the following, section II provides an outline of previous studies on supply effects as well as a brief institutional background on the Government of Canada debt market. Section III describes the estimation methodology, outlining the derivation of the autoregressive form of the ARMAX model. Section IV provides the estimation results. Finally, section V outlines the important results contained in the paper. An Appendix describing the data set is also included.

II. PREVIOUS STUDIES AND INSTITUTIONAL BACKGROUND

In recent years, structural models have come to dominate the study of debt management effects on the term structure. For Canada, previous structural studies include Christofides

(1975), Christofides *et al.* (1976), Masson (1978) and Owens (1980) who found modest yet significant supply effects and Boothe *et al.* (1985) and Boothe (1987) who found insignificant or no supply effects. For the United States, Friedman and Roley (1980) and Roley (1981, 1982) report significant supply effects. The results from structural studies stand in contrast to results from reduced form estimation. Since the influential work of Modigliani and Sutch (1967), reduced form procedures have often produced little support for activist debt management in the US.¹ Similar results have been presented for Canada (Dobson, 1973, 1976; Pesando 1975). In support of activist debt management, Capie *et al.* (1986) have recently found significant impact on yields from the 1932 Conversion Loan in the UK.

Typically, structural studies specify a set of asset demand and supply functions, including short and long bonds amongst the assets. Bond demand equations are then estimated as a function of rates (and other variables). The hypothesis of supply effects is tested by examining the significance of the coefficient on the relevant bond yield. However, while structural studies have been useful in exploring the disaggregated relationships in debt markets, these studies have a number of drawbacks for testing the specific hypothesis of supply effects. Most significantly, the disaggregated nature of many of the structural models requires that identifying restrictions be imposed which are not directly needed to test the supply effects hypothesis. This makes for more complicated joint hypotheses than are required. In addition, if the identifying restrictions unnecessary to the supply effects hypothesis are incorrect, tests for supply effects based on structural models may be biased.²

Much as with structural studies, reduced form results can be subjected to a number of qualifications and criticisms. Significantly, the problem of misspecifying the reduced form equation requires attention. Following Modigliani and Sutch, previous reduced form studies for Canada have modelled the behaviour of the Canadian long rates with a distributed lag on short rates. This approach presents at least two potential misspecification problems. Firstly, in a small open economy the domestic term structure is dominated by the world (i.e. the US) term structure (Beenstock and Longbottom, 1981; Boothe, 1987). Hence, in order to examine the behaviour of the Canadian term structure, it may be necessary to introduce US bond yields as exogenous variables. Secondly, if long rates are modelled as a distributed lag on short rates, the level of interest rates is left undetermined. Hence, a variable for interest rate levels may be necessary if the shape of yield curve is not independent of the level of rates.

The current study aims to address the potential problem of misspecifying the reduced form equation with respect to both the level of rates and the influence of the US term structure. Specifically, US term structure variables are included as exogenous variables in

¹The issue of supply effects is intimately related to the distinction between the expectations hypothesis and the preferred habitat hypothesis in the term structure literature. Both the pure expectations and the liquidity premium-augmented expectations hypotheses implicitly maintain that debt management effects are insignificant (assuming that the term premia are not influenced by changing debt maturity composition.) See, for example, the discussion following Mankiw (1986). However, if investors do adhere to preferred habits, then it is likely that debt management could make use of this behaviour through alterations in the maturity composition of the debt.

²This source of bias could also make structural results sensitive to 'slight' variations in specification. In addition, estimation of structural models often requires the use of judgemental statistical procedures, e.g. system instrumental variables techniques where there is an implied search procedure needed to select either the instruments or the reduced form variables. As a result, it is not always clear what role 'data mining' has played in structural empirical results. This is reflected in a number of the structural studies where 'bizarre' results are reported which are inconsistent with the theoretical predictions of the underlying model (e.g. Masson, 1978).

the reduced form estimations. To account for the level of interest rates, contemporaneous and lagged realizations of the Canadian TBill rate are also included as exogenous variables. This specification does not imply that the bond market and the TBill market are independent but, rather, that bond rates are recursively determined after the TBill yields.³ In other words, bond market variables will, in general, be correlated with the TBill rate but the causality is assumed to run from the TBill market to the bond market so that the bond market can treat TBill yields as exogenous. This implies that TBill rates act as an 'anchor' for the term structure, determining the yield curve's location at any point in time. As a result, *the lag structure for TBills is restricted to be of the same order as that for Government of Canada bond yields*. By introducing information about both the level of domestic interest rates and the shape of the foreign, US yield curve, the reduced form estimates of the Government of Canada yield curve's shape should be more efficient.

In examining variations in the term structure over a times series (1972–84), this study differs from many of the previous studies of supply effects which have concentrated on major debt management events (e.g. the Canadian Conversion Loan (1958), Operation Twist in the US, or the Stock Conversion Loan (1932) in the UK). This raises the issue of debt stock variability. If there is little variability in the debt stock over time then statistical inference is complicated by the multicollinearity of the debt variables and the constant. Relative to the US data, one advantage of Canadian data is the greater amount of variation in the debt programme over time. In the current sample, average maturity of Government of Canada domestic marketable debt outstanding (including TBills) decreased from four years six months in 1972, to a low of three years nine months in 1975, increasing from there to seven years two months in 1979 finally ending up at five years seven months in 1984. Over the same period in the US, the average maturity of outstanding marketable Treasury debt went from three years three months in 1972 to a low of two years seven months in 1976, then rising slowly to reach a high of four years six months at the end of 1984.

A number of possible factors contributed to the variation in the maturity composition of Government of Canada marketable bonds over the period. For example, a portion of the variation in the maturity composition was a result of changes in the market's demand for bonds in the various maturity categories, i.e. due to 'tailoring' of maturities to meet the demands of the market. Tailoring was due to technical features of the 'final allotment' method used to market domestic Government of Canada bonds. Specifically, in a typical primary distribution a given bond issue would feature a stated total amount to be issued and a number of available bond maturities (or 'tranches'). Subject to some restrictions, securities dealers would be able to drawdown in any of the available bond maturities the amount of the issue they have been allocated – typical bond offerings included a short bond (two to four years), a medium term bond (five to seven years) and a long bond (twenty plus years).⁴ Restrictions were placed on the percentage of their allotment that dealers could drawdown in the long bond tranche. Early in the period when debt managers were attempting to develop the long end of the market, these restrictions were in the 40–50% range. However,

³This approach to short rate/long rate interaction is supported in a number of theoretical studies (e.g. McCafferty, 1986).

⁴Allotments to dealers were based on objective performance criteria. These criteria were largely based on past performance in marketing both Government of Canada bonds and TBills. For more on the primary distribution process see 'Government of Canada direct marketable bonds', *Bank of Canada Review* (March 1980).

later in the period when rates were historically high and were generally expected to fall, long bonds restrictions were tightened to the 15–25% range.

Variations in maturity composition also depended on debt management decisions regarding the financing split between marketable bonds, TBills and (non-marketable) Canada Savings Bonds (CSBs). Up to the mid-1970s, considerable reliance was placed on CSBs to meet financing requirements. However, increased Government borrowing demands combined with a decision to reduce reliance on CSBs led, in 1975, to a debt management shift to greater reliance on marketable securities. In the ensuing period, some effort was made on the part of the Government to encourage the use of long bonds to meet new financing requirements. For example, this is reflected in the restrictions on the dealer's allowable drawdown of long bonds which could be taken by dealers as an indicator of the Government's desired long bond drawdown. For the following four years, this initiative met with considerable success as the average maturity of Canada debt increased considerably. However, in 1979–80 increasingly volatile interest and exchange rates produced market pressures which led to a considerable reduction in the public's demand for long bonds. In addition, the high and generally falling level of nominal rates during the early to mid-1980s led the Government to pursue a policy of a debt maturity reduction through expanded use of TBills as a financing source. During this period, the strategy of debt maturity reduction conflicted with the public's increased demand for long bonds. As a result, restrictions on the maximum allowable drawdown of long bonds were tightened giving less scope for market determination of the maturity composition of the debt.

III. THE ESTIMATION METHODOLOGY

Briefly, an ARMAX model is a vector ARMA model with exogenous variables included in the reduced form estimation procedure. Exogenous variables are defined as predetermined (right hand side) variables other than lagged endogenous variables (and the constant). Hence, the autoregressive form of an ARMAX model is the conventional VAR model with exogenous variables included as regressors. The VAR methodology begins with the assumption of a jointly covariance stationary set of time series x_t , where x_t is a column vector containing k variables, the x_{it} observations ($i = 1, \dots, k$), at time t . In the present context, the x_t vector contains subvectors of Government of Canada bond yields, Government of Canada bond stocks, Government of Canada Treasury Bill rates and US Treasury bond yields. Assuming joint covariance stationarity, the multivariate version of Wold's decomposition theorem states that the x series can be represented (Hannan, 1970) as the sum of a deterministic component (which may vary through time), and a moving average of white noise errors. Without loss of generality, x_t can be centred about its (possibly time varying) mean and the moving average representation may be inverted to yield a vector autoregressive process in which x_t is a linear function of lagged realizations of x_t and a vector of contemporaneous v_t shocks. Contemporaneous correlation amongst the x_t 's is reflected in the off-diagonal elements of the variance–covariance matrix (V) of the v shocks.

In conventional econometric terms, the VAR model is a reduced form model where the only predetermined variables are the lagged endogenous variables. Extending the VAR framework, it is possible to incorporate additional information regarding the exogeneity of variables leading to more efficient reduced form estimates. Following Hannan *et al.* (1980), the ARMAX model incorporates such information by decomposing the x_t vector into $x_t = (x_{1t}, x_{2t})$ where x_{1t} are the 'true' endogenous variables and the x_{2t} are the 'true'

exogenous variables. With this in mind, the vector moving average specification given in Equation 1 corresponds to the following structural system which underlies the ARMAX model:⁵

$$\begin{bmatrix} \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ 0 & A_{22}(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = A(L)e_t \quad (1)$$

$E[e_t] = 0$, $E[e_t e_{t-n}] = 0$ for all n not equal to zero, $E[ee'] = Z$, where Z is diagonal and non-singular, where Equation 1 is the 'orthogonalized moving-average representation' of the x process since the e innovations are orthogonal by construction. $A(L)$ is a polynomial in the lag operator L and, by convention, the A_0 matrix is assumed to be triangular to ensure identification.⁶ Contemporaneous and lagged correlations across variables are solely a function of $A(L)$. Using Equation 1, the reduced form equation for the \mathbf{x}_{1t} follows:⁷

$$\mathbf{x}_{1t} = D(L)\mathbf{x}_{2t} + B(L)\mathbf{x}_{1,t-1} + \mathbf{v}_{1t} \quad (2)$$

By construction, the \mathbf{x}_{2t} variables are independent of the residuals for the \mathbf{x}_{1t} regression.

As in the conventional VAR model, contemporaneous (or 'structural') relationships between the \mathbf{x}_{1t} variables are embedded in the estimate of the variance-covariance matrix V of the \mathbf{v}_{1t} 's (equal to $Z_{11}A_{11,0}A'_{11,0}$) and are not identified. An important feature of the VAR representation, as with any reduced-form representation, is that while estimation of the model provides an estimate of the V matrix it does not (without the imposition of identifying restrictions) provide information regarding the ordering imposed contemporaneously (i.e. on the $A_{11,0}$ matrix). The system can be estimated using OLS which is consistent and efficient given the assumptions made in specifying the model.⁸ For present purposes, the

⁵This system is derived from the more general form:

$$\begin{bmatrix} W_{11}(L) & W_{12}(L) \\ 0 & W_{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{bmatrix} = \begin{bmatrix} W_{11}^*(L) & 0 \\ 0 & W_{22}^*(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

Inverting the matrix associated with the x 's gives Equation 1. Exogeneity of \mathbf{x}_{2t} is represented by the zero restrictions imposed on the left hand side matrix of polynomial lag operators.

⁶The lag operator L^i is defined as implying a lag of i length. The lag process $A(L)$ is the polynomial sequence $A_0 - A_1L - A_2L^2 - \dots - A_qL^q$, where q is non-negative and A_i is a $k \times k$ matrix of coefficients. Further, the imposition of identification restrictions on $A(L)$ is not necessary to specifying the model. Rather, identification restrictions are necessary for interpreting the model's results, Hannan (1969).

⁷Solving for \mathbf{x}_{1t} and inverting the moving average process yields:

$$\begin{aligned} \mathbf{x}_{1t} &= A_{12}(L)A_{22}(L)^{-1}\mathbf{x}_{2t} + A_{11}(L)e_{1t} \\ &= A_{12}(L)A_{22}(L)^{-1}\mathbf{x}_{2t} + (I - A^*(L))A_{11,0}e_{1t} \\ &= (I - A^*(L))^{-1}A_{12}(L)A_{22}(L)^{-1}\mathbf{x}_{2t} + B(L)\mathbf{x}_{1,t-1} + A_{11,0}e_{1t} \\ &= D(L)\mathbf{x}_{2t} + B(L)\mathbf{x}_{1,t-1} + \mathbf{v}_{1t} \end{aligned}$$

where $A_{11,0}$ is the appropriately dimensioned part of A_0 corresponding to e_{1t} and $A^*(L)$ is defined as $(A_{11,1}L + A_{11,2}L^2 + \dots)A_{11,0}^{-1}$, and $B(L)$ is defined as $I - (I - A^*(L))^{-1}$ from the inversion in Equation 2 above.

⁸Comparing the modified VAR modelling procedures with the more familiar Box-Jenkins approach (e.g. Tiao and Box (1981)), the identification stage of the Box-Jenkins based approach must be altered since, with exogenous variables in the model, it is no longer generally necessary (or possible) to test for the stationarity of the endogenous variables prior to model estimation. This is because stationarity of the endogenous variables is required to hold only *after* the influence of the exogenous variables has been removed. Hence, if the VAR model includes exogenous variables, stationarity tests are generally conducted on the residuals of the fitted model.

endogenous variables are assumed to be Government of Canada bond yields and Government of Canada bond stocks. The remaining variables, the x_{2t} , include the specified exogenous variables – the Government of Canada TBill rate and US bond yields.⁹

The model in Equation 2 has an immediate connection to the Granger method of causality testing.¹⁰ To illustrate how Granger's approach may be utilized to test the hypothesis of supply effects in the bond market, decompose the x_{1t} vector into $x_{1t}' = (r_t', b_t')$ where r_t is the vector of bond yields and b_t is the vector of bond stocks. Equation 2 can now be expressed as:

$$\begin{bmatrix} r_t \\ b_t \end{bmatrix} = B(L) \begin{bmatrix} r_{t-1} \\ b_{t-1} \end{bmatrix} + D(L)x_{2t} + \begin{bmatrix} v_t^r \\ v_t^b \end{bmatrix}, \quad E[v_t v_t'] = V \quad (2')$$

$$B(L) = \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{bmatrix} \quad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

The Granger procedure for testing causality from stocks to rates is to perform an OLS regression of current r on a prespecified number of lagged r 's and lagged b 's and test for the significance of the coefficients on the lagged b 's, i.e. test the hypothesis that $B_{12}(L) = 0$.¹¹ If $B_{12}(L) = 0$, b is found not to Granger cause r . Similarly, the condition for r not Granger causing b is $B_{21}(L) = 0$. If exogenous variables are included in the regressions, Granger causality from b to r must be interpreted as b causing r *controlling for the effect of the exogenous variables on r* . In this case, the necessary (but not sufficient) condition for zero contemporaneous causality between r and b will be $V_{12} = 0$. In other words, the residuals from the VAR regressions are computed and examined for significant cross-equation relationships. If significant relationships are observed (V_{12} not equal 0) then the null hypothesis of no instantaneous causality between r and b can be rejected. However, the direction of causality will be undetermined.¹²

⁹These exogenous variables must obey certain technical restrictions. Sufficient conditions for deriving the asymptotic properties of the ARMAX model require that (Hannan *et al.*, 1980; Poskitt and Tremayne, 1984): (i) the predetermined variables must be independent of the error process (the innovations); (ii) the predetermined variables must have covariance functions which are well-defined and jointly stationary in the limit; and (iii) the means of the exogenous variables are well-defined in the limit.

¹⁰A variable z is said to Granger (or Weiner–Granger) cause a variable y if the projection of y on an information set which is exclusive of past z has a prediction error which is significantly greater than the projection of y on the information set which includes past z . This definition has been operationalized in a number of different ways, e.g. Feige and Pierce (1979), Geweke *et al.* (1983), Cooley and LeRoy (1985) and Pierce and Haugh (1977) for further description of these causality tests. Much of the discussion of causality is conducted in a bivariate context. However, Skoog (1976) examines the trivariate and higher order forms of causality testing and finds that the interpretation of causality test results is somewhat altered in these cases.

¹¹An important drawback to causality tests such as Granger's arises if the lag length is not appropriately specified. If the lag structure is incorrect then the test statistics are not valid under the null hypothesis. In the VAR context, technical conditions for truncation of the lag polynomial need to be satisfied. Another drawback arises because Granger (and contemporaneous) causality provides a necessary but not a sufficient zero restriction on the 'structural' equation coefficients (Jacobs *et al.* 1979; Cooley and LeRoy, 1985).

¹²Where exogenous variables are included in the model, *contemporaneous* values of the exogenous variables must be included to avoid potential bias in the estimation of V .

IV. ESTIMATION

Identification

The order of the model (2') is determined using the sequential testing procedure suggested by Jenkins and Alavi (1981) and Poskitt and Tremayne (1984); the model's lag structure is overfitted and a systems version of the likelihood ratio test is used to determine the true lag specification. Specifically, to test for lag specification the model is estimated both with $k + 1$ lags and k lags and the null hypothesis of k lags is tested by comparing the adjusted log ratio of the determinants of the variance/covariance matrices of the residuals using the likelihood ratio test proposed by Sims (1980). This procedure is also used to test for the significance of exogenous reduced form variables. The test statistic is defined as:

$$\text{LRD} = (T - k) [\log|\mathbf{V}'| - \log|\mathbf{V}|]$$

where $|\mathbf{V}'|$ and $|\mathbf{V}|$ are the determinants of the variance/covariance matrices of the restricted and unrestricted models respectively, T is the sample size and k is the number of estimated coefficients per equation. LRD is distributed asymptotically chi-squared with r degrees of freedom, where r is the number of total system restrictions, under the null hypothesis. Once the order of the model is established, the estimated coefficients are examined to test for supply effects. The residual variance-covariance matrix is also examined to determine whether or not the necessary condition for a lack of contemporaneous causality (zero cross correlations between the residuals of the debt supply and rate equations) is satisfied.¹³

A number of different definitions for the debt supply variables can be used. For example, the three bond maturity classes could be defined in terms of levels. However, because the nominal stock of debt has grown substantially over time due to increases in financing requirements and inflation, the use of debt stock levels risks biasing the results towards acceptance of the null hypothesis of no supply effects. A more accurate definition is to use the share of the maturity class in the total of publicly held domestic marketable debt, i.e. total debt is defined as the sum of the stock of debt outstanding for the three bond maturity classes and TBills. As well as avoiding the problems associated with nominal values, use of shares has the added advantage of allowing the indirect introduction of the effect of TBill financing (through its affect on the total stock of debt outstanding). By construction, the shares will not sum to one because of the use of TBills in total debt. If there are supply effects, variations in the bond shares will impact bond yields as the share of debt between maturity classes changes.

Table 1 illustrates the results of the preliminary search for the optimal lag length and exogenous variable specification.¹⁴ The initial system estimated (Model 1) is a six equation

¹³Following Sims (1980), the correction factor k has been included because the corrected statistic gives small sample results closer to the asymptotic distribution. While there are a multitude of proposed methods for testing the order of a VAR model (e.g. Lutkepohl, 1986), the selection of one method over another depends on the final purposes for the selected time series model. Because the objective here is primarily to provide a parsimonious representation of the data in the sample, LRD has desirable properties compared to other methods. When two test procedures designed for forecasting purposes, Aikake's final prediction error criteria and Schwarz's method, were tried in the current study, the general ordering of the models was unchanged, except that Model 2 was preferred for the full sample, i.e. the model with a greater number of parameters was selected.

¹⁴The results presented here are a subset of results from a much larger study. Hence, some of the paring of the data has not been reported for brevity.

model with a five month lag length for the Canadian bond yields and debt shares plus an additional lag at the twelfth month. In the mnemonics of Table 1, this lag structure for the endogenous variables is indicated 1-5, 12 (0-5, 12 for TBills because the contemporaneous value of TBills is also included). The lag length for the foreign rates was restricted to be two months or less. The six equation model was re-estimated under a number of different restrictions on both the lag length of the endogenous variables and the specification of the exogenous variables. For example, to test for the significance of foreign rates, no US rates are included in Model 6. To test for an annual 'seasonal', the twelfth lag is omitted in Model 5.

Table 1 also presents the chi-squared statistics and marginal significance levels of the log-determinant-based likelihood ratio (LRD) tests. The chi-squared statistic tests the null

Table 1. *Model specification tests*

Model	r	b	Lag length	
			r^b	r^*
1	1-5, 12	1-5, 12	0-5, 12	0-2
2	1-4, 12	1-4, 12	0-4, 12	0-2
3	1-3, 12	1-3, 12	0-3, 12	0-2
4	1-2, 12	1-2, 12	0-2, 12	0-2
5	1-3	1-3	0-3	0-2
6	1-3, 12	1-3, 12	0-3, 12	-
7	1-3, 12	1-3, 12	0-3, 12	0-1
8	1-3, 12	1-3, 12	0-3, 12	0-3

Likelihood ratio tests

Sample: 1972i to 1984xii

Hypothesis	Chi-squared statistic**	Significance level
2 preferred to 1	31.71 (42)	0.88*
3 preferred to 2	41.41 (42)	0.50*
4 preferred to 3	64.57 (42)	0.01
5 preferred to 3	58.00 (42)	0.05
6 preferred to 3	141.96 (54)	0.00
7 preferred to 3	41.47 (18)	0.00
3 preferred to 8	24.19 (18)	0.15*

r = bond yields, b = debt shares, r^b = TBill yield, r^* = US bond yields.

* Accepted at the 90% level of confidence. A significance level of $p\%$ indicates the probability that the test statistic is equal to or greater than the calculated value under the null hypothesis.

** Degrees of freedom, given in brackets, determined by the total number of system restrictions imposed.

hypothesis that the restrictions imposed are valid. As indicated in Table 1, for the full sample, the hypothesis of valid restrictions on the fifth and fourth lags could not be rejected at the 90% confidence level while restricting the lag length to order two was rejected. Based on a lag length of order three (Model 3), tests for specification of the exogenous variables and seasonality were conducted. For the US term structure variables, the null hypothesis of no US effects was strongly rejected by the data indicating that the world term structure of interest rates is an important determinant of the term structure in a small open economy. Regarding seasonality, the likelihood ratio test produced mixed results for the twelfth lag on the Canadian variables. The small seasonal effect observed is likely due to the shocks to shares produced by the annual CSB sales programme.

Regarding residual diagnostics, because the model treats the Canadian TBill rate and the foreign bond yields as exogenous variables, it is not possible to test for stationarity prior to fitting the model. Stationarity is tested indirectly by examining the residuals of the fitted model. The portmanteau Q-statistics for the preferred models and a number of other models are given in Table 2. For both preferred models, there was no evidence of non-stationarity in the residuals, i.e. the significance level of the Q-statistic was small enough to fail to reject the null hypothesis of stationary residuals.¹⁵ This was confirmed by simulating the response of the endogenous variables to exogenous shocks. In both the full and subsample cases the preferred model was stable. In addition, it was observed that when the TBill rate was omitted as an exogenous variable significant autocorrelations among the rate equations emerged, suggesting that while the premiums of bond yields over the TBill yield are stationary, the raw yields are not. This result confirms the specification of the yield curve model as being anchored by the TBill rate as opposed to a model that explains the term structure in isolation.

Table 2. *Q*-statistics*

Sample: 1972-i to 1984-xii						
Model**	Equation					
	r^s	r^m	r^l	b^s	b^m	b^l
3 (33)	41.3 (15.1)	24.3 (86.4)	33.5 (44.2)	30.1 (61.2)	31.6 (53.8)	39.9 (19.0)
4 (33)	41.4 (14.8)	26.7 (77.3)	33.1 (46.1)	41.9 (13.7)	34.1 (41.4)	43.0 (11.5)
2 (33)	44.9 (8.1)	34.8 (38.0)	44.4 (8.9)	31.3 (55.0)	33.9 (42.4)	37.2 (28.2)

r^s = short bond yield, r^m = medium yield, r^l = long yield, b^s = short debt share, b^m = medium share, b^l = long share.

* Q-statistics as defined Ljung and Box (1978). Marginal significance in brackets.

** Number in brackets is degrees of freedom. Degrees of freedom are determined by $\min\{T/2, 3(T^{1/2})\}$.

¹⁵The Q-statistic tests significance of the sum of a prespecified number of residual autocorrelations. The tests reported here incorporate the changes recommended by Ljung and Box to correct for small sample problems.

Granger and contemporaneous Causality

Testing for supply effects in the Government of Canada bond market involves evaluating the conditions for contemporaneous and Granger causality between bond yields and the debt shares. Two types of supply effects are considered: the first is that a change in aggregate maturity composition of the debt will affect bond yields; the second is that a change in the share of debt within a specific maturity class will affect bond yields. In other words, one type of supply effect arises when the total maturity composition changes; the other supply effect arises when the share for a specific maturity class changes. In both cases, testing for *contemporaneous* causality involves examining a submatrix of the correlation matrix of the residuals for the six estimated equations of the preferred model. Regarding the effect of the total maturity composition, contemporaneous causality is examined by testing the hypothesis that the cross correlations between all shares and the chosen bond yield are zero. Contemporaneous causality between the share in a specific maturity class and specific bond yields is examined by testing whether specific cross correlations are zero. Contemporaneous causality between the share for a specific maturity class and all rates is examined by testing the hypothesis that cross correlations between the chosen share and all rates are zero.¹⁶

As Table 3 indicates, for the full sample, the null hypothesis of no maturity composition effects (i.e. the row chi-squared tests) is rejected for all bond yields. In addition, the hypothesis of zero contemporaneous causality between bond yields and short and medium maturity shares also cannot be rejected at the 90% level of confidence. However, with regard to the individual cross correlations, the correlation between the errors of the long share equation and the short- and medium-yield equations indicate the hypothesis of a zero cross correlation is rejected. For the test of contemporaneous causality between the long-share and the term structure (the column chi-squared test) the hypothesis of zero contemporaneous causality is also rejected. The implication of these results is that for the full sample there does appear to be some contemporaneous feedback between innovations to both the long maturity share of outstanding debt and the term structure of Government of Canada

Table 3. *Cross correlations between stock and rate equation errors*

Sample: 1972-i to 1984-xii				
Rates/shares	Model 3			Chi ² (3)
	Short	Medium	Long	
short	-0.024	-0.026	0.206*	6.16
medium	-0.073	0.018	0.170*	4.87
long	-0.082	0.076	0.062	2.30
Chi ² (3)	1.03	0.91	10.60*	

* Significantly different from zero at the 95% level of confidence for the individual correlations and the 90% level for the row and column sums of the squared correlations (the chi-squared tests).

¹⁶The test for total share change on a specific rate and for maturity specific share change on all rates is based on Quenouille's (1947) chi-squared test. This test is based on the asymptotic properties of the correlation coefficient, i.e. $T^{1/2}$ times r is $N(0, 1)$. As is well known, this statistic is not well behaved in small samples. However, because the number of observations used in this study is well over one hundred, the large sample properties of the test statistic are assumed to hold.

bond yields. Subject to the caveat that the tests performed cannot identify a specific direction of causality, the *positive* sign of the significant correlations may indicate the lengthening of the maturity composition of the debt by increasing the long maturity share corresponds with a contemporaneous *flattening* of the yield curve.

To test for *Granger* causality of the share in a specific maturity class on rates, F tests are constructed to test whether all the coefficients for the lag structure associated with a share variable are zero in the yield equations. The results of these tests are contained in Table 4. For the full sample, examination of the F tests indicates that the coefficient-lag-structure for bond shares does not have a significant impact on bond rates. Changes in the share of debt in a specific maturity class do not appear to Granger cause any bond yields. The F tests indicate that changes in a specific share over time only Granger cause that specific share while bond yields tend only to Granger cause other bond yields. (The barely significant effect of the long share on the medium share is most likely an artifact of the maturity crossing problem.) While the yield results are not surprising, the share results are interesting because they indicate that there was little interaction between shares over time. This indicates that there were, at best, only minimal debt management effects on share behaviour—in the Granger sense.

Table 4. F tests for the significance of the lagged coefficients

Equation	Sample: 1971-i to 1984-xii					
	Model 3					
	Variable					
	r^s	r^m	r^l	b^s	b^m	b^l
r^s	9.84*	1.98*	1.39	1.37	1.26	0.47
r^m	5.11*	3.80*	2.68*	0.47	0.61	1.02
r^l	2.25*	3.63*	3.70*	0.27	0.31	1.07
b^s	1.12	0.65	0.67	170.80*	0.25	0.80
b^m	1.08	0.72	0.76	0.92	46.84*	0.47
b^l	1.58	1.18	0.63	0.71	2.70*	54.35*

*Significant at the 90% confidence level.

Conventionally, a supply effect arises because a relative decrease in the debt share in a specific maturity class increases rates comparably for other maturity classes. Higher rates have to be offered in order to encourage investors to hold the increased supply in the other maturity classes. Hence, the predicted sign for supply effects is *negative* for shares and rates not for the same maturity class (positive for shares and rates in the same maturity class). Given the contemporaneous and Granger causality results, limited evidence was found for conventional (negative) supply effects based on the full sample behaviour. Similarly, if dealers were determining the maturity composition, the predicted contemporaneous causality sign would also be negative, i.e. when rates are 'high' and falling dealers would want to lengthen term and, if rates are 'low' and rising, dealers would want to shorten term. Hence, dealer determination of the maturity composition would be consistent with a negative cross correlation between bond yields and shares.

Evidence was presented indicating *positive* contemporaneous causality of the long share with medium and short rates. This form of supply effect could originate from activist debt management motivated by interest cost minimization. Specifically, if rates are high and are

expected to fall, interest cost minimization would dictate that funding requirements be shifted into the short end. Similarly, if rates are low and expected to rise, the tendency would be to shift funding requirements into the long end. Given that there is a tendency towards yield curve inversion at the peak of an interest rate cycle and yield curve steepness at interest rate cycle troughs, part of the effect of changes in rate levels on debt shares would be picked up by bond yields and not by the exogenous TBill rate. The absence of any contemporaneous feedback between bond yields and short and medium shares can be attributed to other, extraneous (i.e. non-interest-rate) factors affecting those shares. For example, because the long share was the primary 'new money' financing vehicle, short and medium shares had to absorb the impact of shocks arising from the CSB programme, shortfalls in tax receipts and foreign exchange losses. The insignificance of a contemporaneous relationship between long shares and rates could be due to the lesser amount of interest-rate-cycle-related volatility in long rates.

Examining the evolution of yields and shares over the sample reveals the nature of the impact of debt management. In the early part of the sample the yield curve evolved from steeply sloped in early 1975 to flat to slightly inverted at the end of 1979 – the level of rates rising from around 6% to 12% for short maturity bonds over the period. Debt share behaviour during this period was largely driven by a dramatic shift into long bond financing. This shift was encouraged by debt managers both through moral suasion and the stated 'restrictions' on long bonds which, in practice, acted as indicators of how much of an issue the Government expected the dealers to take in long bonds. Consequently, the maturity composition of the debt was lengthening considerably at a time when the yield curve was flattening. Similarly, in the latter half of the sample rates were falling and the Government reduced term by discouraging drawdowns of long bonds. If expectations are based on a distributed lag on past rates, the persistence of rate trends during the sample would be consistent with the observed contemporaneous causality results.

V. SUMMARY

Using time series methodology, this paper examined the hypothesis that changes in maturity structure of outstanding Government of Canada marketable domestic debt affected the term structure of Government of Canada bond yields. Based on monthly data for the 1972–84 period, the empirical results found some evidence to support the hypothesis that there was feedback between debt management actions and changes in the maturity composition of the debt. The feedback was confined to positive contemporaneous effects of long shares with short and medium rates. Given the distribution method used to market Government of Canada bonds, the positive effect was likely due to the level and direction of rates affecting share determination decisions rather than the other way around. Evidence was also presented that the US term structure was highly significant in determining the Canadian term structure, although the US term structure variables had a shorter lag structure than that of Canadian rates.

APPENDIX

Data for the Government of Canada interest rates and bond stocks were derived from the data base described by Boothe and Reid (1986). The Government of Canada bond market is

aggregated into three maturity classes, shorts (S, zero to six years to maturity), mediums (M, seven to twelve years to maturity) and longs (L, thirteen years and over to maturity). Estimates for the representative Government of Canada bond yields for these maturity classes were derived from an estimated par yield curve at the three year (S), ten year (M) and twenty year (L) maturity points. The Government of Canada TBill yield is the average yield at tender for the three month TBill. US bond yield data are DRI's constant maturity Treasury yields at the five, ten and twenty years to maturity points. All data series are used in their log level form. Regarding data frequency, some authors have hypothesized that data frequency should be less than a quarter (e.g. monthly or weekly) in order to adequately identify supply effects in reduced form models. The current sample is monthly, from January 1972 to December 1984.

Theoretically, if debt stocks are discontinuous, correct aggregation of bond stock data into maturity groupings requires the solution to an errors in variables problem produced by the discrete crossing of issues between maturity classes (Boothe, 1987; Friedman and Roley, 1980; Roley, 1982). The method employed here to handle the maturity crossing problem is to utilize the distinction between active and passive stocks. The passive stock is equal to that stock which would have resulted in the absence of any Government or Bank of Canada purchases or sales (including new issues) between the current and last observation. The active change to bond stocks is equal to the change in publicly held bonds due to new issues or open market sales or purchases. The quantity of par value bonds in the i th maturity class equals the passive stock plus the active change to that maturity class. The total stock in each maturity class is proxied by the sum of the active change to the stock plus a three month moving average of the passive stock (where the moving average begins at the current month and ends two months prior). In this fashion the crossover from one maturity class to another is smoothed, but the variations in the total stock due to changes in the amounts held by the Bank of Canada and/or the Government or due to new issues are unaffected.

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