Is Benter Better? A Cautionary Note on Maximizing Convexity

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In the literature on bond portfolio management, the concept of convexity has been gaining popularity. The policy prescription associated with this concept calls for maximizing a bond portfolio's convexity while setting the portfolio's duration equal to the investor's horizon.[1] "Convexity is good" and "the benter the better" have become the catch phrases of this approach.[2]

This note questions the appropriateness of convexity maximization. The assumption implicit in the prescription is that parallel shifts are the only type of change possible in the structure of interest rates. As the recent experience with inverted yield curves has served to emphasize, however, the interest rate structure may also twist. It is argued here that maximizing convexity is tantamount to maximizing exposure to such twists.

The duration of a bond portfolio, DUR, is a weighted average of the different points in time when the portfolio will generate cash flows. The weight applied to each point in time is the present (discounted) value of the cash flow generated at that time, expressed as a fraction of the total value of the portfolio. Let t_j denote a point in time when the portfolio generates a cash flow whose present value equals PV_j. Assume that there are m of these points in time, and let I_0 refer to the present market value of the portfolio. Then:

$$DUR = \sum_{j=1}^{m} t_j W_j, \qquad (1)$$

where $W_j = PV_j/I_0$.

Whereas a portfolio's duration is associated with this first derivative of the portfolio's value with respect to the interest rate, the portfolio's convexity is associated with the second derivative. Convexity, CON, is the weighted average of the *squared* values of the same points in time when the portfolio generates cash flows. Thus:

$$CON = \sum_{j=1}^{m} t_j^2 W_j. \qquad (2)$$

The policy prescription expressed in the above-cited literature entails choosing a portfolio that maximizes Equation (2), subject to the constraint that the portfolio be immunized (i.e., that the duration from Equation (1) equal the investor's planning horizon, H). If only parallel shifts in the interest rate structure are possible, the value of a maximum-convexity, immunized portfolio rises more sharply and drops more gradually than that of an immunized portfolio that exhibits less convexity. This is the rationale for "the benter the better."

Unfortunately, parallel shifts are not the only type of change possible. Fong and Vasicek have developed a theorem on risk minimization for immunized portfolios.[3] They have shown that an upper bound on the percentage reduction in the horizon value of an immunized bond portfolio is given by the following expression:

$$\frac{1}{2} KM^2 \cdot 100\%, \qquad (3)$$

where K represents the maximum change in the slope of the interest rate structure and M^2 is given by the following formula:

$$M^2 = \sum_{j=1}^{m} (t_j - H)^2 W_j. \qquad (4)$$

M^2 is thus a weighted average variance of the points in time when the portfolio generates cash flows around the investor's horizon date.

The weights employed in Equation (4) are identical to those found in the duration and convexity measures. Whereas K is beyond the investor's control, the investor can control M^2. K may be viewed as the maximum possible twist in the structure of interest rates and M^2 as the investor's exposure to such twists. The rationale for the Fong-Vasicek prescription—minimizing M^2 for immunized portfolios—is obvious.

How are M^2, convexity and duration related? By expanding and simplifying Equation (4), it is easy to show that:

$$M^2 = CON - DUR. \qquad (5)$$

The danger of maximizing convexity is now clear. For immunized portfolios, maximizing convexity is the same as maximizing M^2. Thus maximum convexity is tantamount to maximum exposure to twists in the interest rate structure.

The Fong-Vasicek theorem yields merely an upper bound on the percentage reduction in the portfolio's horizon value. There is thus no guarantee that, in a specific situation, a portfolio with maximum convexity or M^2 will be most adversely affected by a twist. It is nevertheless dangerous to single-mindedly maximize the convexity of an immunized portfolio, given the possibility of twists in the interest rate structure.

1. Footnotes appear at end of article.

To summarize, convexity should be viewed with caution. While maximizing convexity is definitely appropriate if only parallel shifts in the interest rate structure are possible, this course of action is questionable in the presence of twists. In fact, maximizing convexity may result in an immunized portfolio that would be most adversely affected by such a twist.

Footnotes

1. A lucid discussion of this approach is found in M. L. Dunetz and J. M. Mahoney, "Using Duration and Convexity in the Analysis of Callable Bonds," *Financial Analysts Journal*, May/June 1988.
2. See D. Duffie, *Futures Markets* (Englewood Cliffs, NJ: Prentice-Hall, 1989), pp. 251–267, and B. J. Grantier, "Convexity and Bond Performance: The Benter the Better," *Financial Analysts Journal*, November/December 1988.
3. See H. G. Fong and O. Vasicek, "The Tradeoff Between Return and Risk in Immunized Portfolios," *Financial Analysts Journal*, September/October 1983, and "A Risk Minimizing Strategy for Portfolio Immunization," *Journal of Finance*, December 1984.

Wolf footnotes concluded from page 73.

contract expiration). If a money market instrument other than a Treasury bill is purchased for delivery (say, a Eurocertificate of deposit), a blended rate is earned between expiration dates (that is, a blend of the Eurocertificate and Treasury bill rates). It is this blended rate that must be considered as the actual forward money market yield in Equation (2).

12. This is a direct result of Equation (2) and the definition of FRHP.