Convexity and Bond Performance: The Benter the Better

by Bruce J. Grantier, Vice President, Toronto Investment Management

With the increased use of duration in fixed income management, a related concept—convexity—has attracted some interest. This note discusses convexity and suggests a practical application to the evaluation of fixed income securities with embedded options.

Duration and Convexity

Convexity is defined as the change in a bond's duration for a given change in its yield. The duration measure used here is modified duration, as opposed to Macaulay duration. Macaulay duration is the weighted average time to receipt of all cash flow payments, where the weights are the present values of the payments themselves. Modified duration is the change in price for a given change in yield at a point on a bond's price/yield curve. The two are similar mathematically:

$$\text{Modified Duration} = \text{Macaulay Duration}/(1 + i/2),$$

where i is the interest rate. Thus, at an interest rate of zero, the two definitions are equal.

Macaulay duration is used as a measure of term in asset/liability matching. Modified duration is used as a measure of price sensitivity to yield changes.

The slope of a price/yield curve at any point represents the change in price for a given change in yield. Most straight bonds (ones with no embedded options) have a convex price/yield curve. The convexity arises from the fact that, as rates change, the weights given to the timing of cash flows change disproportionately to the change in rates. As rates rise, nearer coupons have a greater influence than distant coupons. The converse holds when rates fall.

Figure A compares the performances of two bonds with different convexities but the same yield and the same duration at a point. Bond A has greater convexity than Bond B, because Bond A's duration changes more for a given change in interest rates.

The above relationship illustrates the key concept of this note and the practical value of the concept of convexity: Bonds with high convexity are more desirable than bonds with low convexity, assuming equal duration and yield. Put simply, a high-convexity bond's duration will increase more as rates drop, and the bond will outperform a bond with lower convexity. The same is true when rates rise. As duration drops more for the high-convexity bond, it becomes more defensive, outperforming a lower-convexity bond. The superior performance characteristics of higher-convexity bonds is exemplified in the phrase: "The benter the better."

Of course, to be desirable, the bend in the price/yield curve must be convex. Bonds with a concave price/yield curve are said to have "negative convexity." Concavity may have been a better term, but who ever remembers the difference between convex and concave?

Properties of Convexity

The properties of convexity are generally the same as those of duration. Other things being equal, convexity is increased by lower coupon, lower yield level and longer term. Convexity also depends on the timing of the coupon stream, as noted. For example, a barbell portfolio (50 per cent cash and 50 per cent longs) with the same duration as a 100 per cent mid-term portfolio has higher convexity than the mid-term portfolio.

The main limitation of convexity as a practical tool for bond management is that, for straight bonds, it has only a small impact over most normal yield changes. For example, a Canada 10 per cent of 2008, given a yield change from 10 to 9 per cent, will go from 8.82 to 8.37 in duration, a difference of 0.45. This 45 cents per $100 is the difference in actual price compared with the price one would have estimated ignoring the bond's convexity.

The practical value of convexity stems from its application to bonds with special features or embedded options, such as callable bonds, extendible/retractables, sinking fund bonds and purchase fund bonds. The following examples illustrate the application of convexity to such bonds.

Some Examples

Figure B illustrates a **straight bond**. Here duration increases with lower rates, and convexity is positive. The essential characteristic of convexity is seen by looking at the actual price performance of the bond versus its expected performance, given a change in yield. With the bond at par (a 10 per cent yield), the expected price change for a small change in yield is given by the duration (the slope) of the price/yield curve. For larger changes in yield, the actual price change diverges from the expected price change. A bond with positive convexity will experience a larger price increase than expected for a given drop in yield. High quality (low yield), low coupon and long term contribute to convexity.

Figure C illustrates a **callable bond**. In this case, the embedded call option creates negative convexity around par. The bond trades to its long date below par and to its short date above par. The market assigns this characteristic a premium yield. That is,

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Figure A Two Bonds with Same Duration and Yield at a Point but Different Convexities

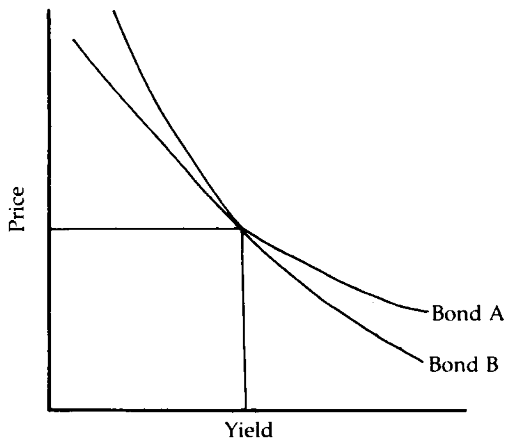
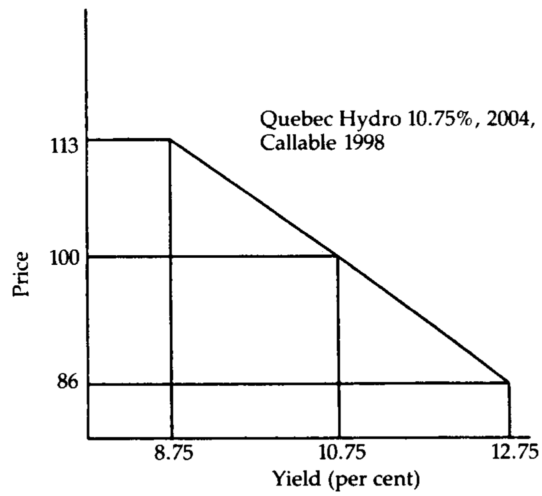


Figure C Callable Bond



the call feature means the bond trades at a higher yield than would a straight Quebec Hydro 10.75 per cent of 2004.

This bond was chosen because its price/yield curve has a high degree of negative convexity; it is very concave. A callable issue such as the Quebec Hydro 11.25 of 2008, callable in 2003, would have less negative convexity. This is not to say that one is better than the other; one must take the yield premium into account, and thus the cost of the call option.

Figure D illustrates an **extendible/retractable bond.** The price/yield curve now is very convex. Again, this is a desirable characteristic, but one to which the market, as with the callable bond, assigns a yield premium. In this case, the option in favor of the bondholder results in a lower yield than would be placed on a bond with the same long date but no optional short date.

From the convexity point of view, extendible/retractables and callables have a point of inflection on the price/yield curve that alters dramatically the slope at that point. Below par, this bond trades to its 1993 maturity, above par, to its 2003 maturity. It thus exhibits high convexity.

Figure E illustrates a **sinking fund bond.** Typically, its indenture allows the issuer to redeem bonds at par for sinking fund purposes according to a schedule of requirements. When the bonds are below par, the sinking fund simply buys them in the market, which is cheaper than redeeming them at par. When they

Figure B Straight Bond

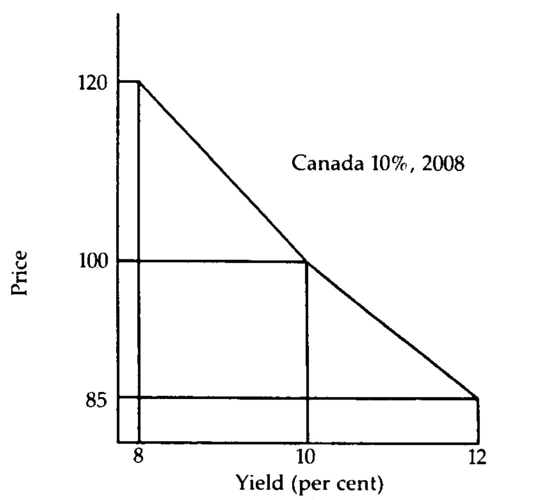


Figure D Extendible/Retractable

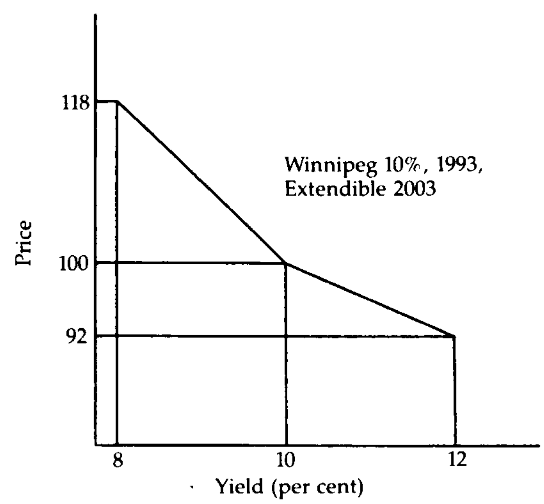


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Figure E Sinking Fund

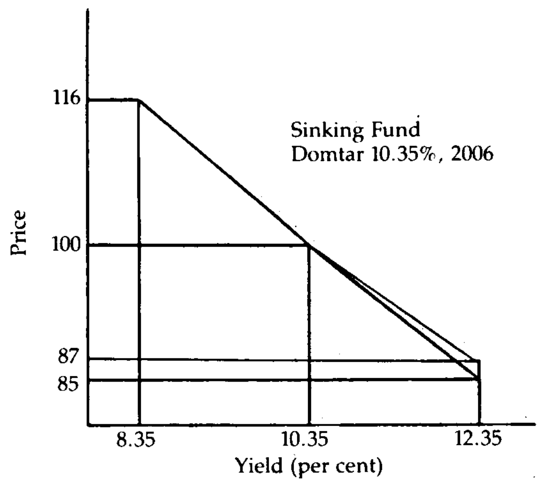
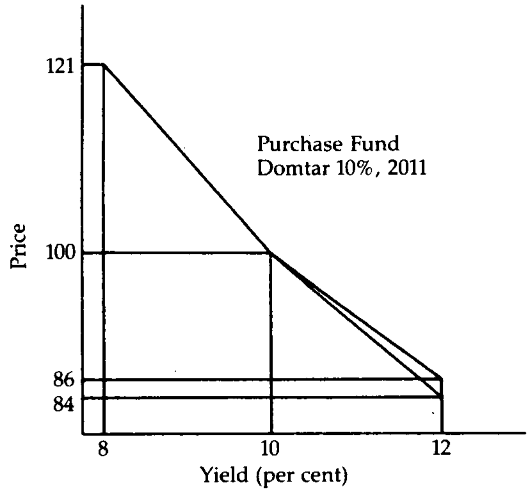


Figure F Purchase Fund



are trading above par, the sinking fund calls them at par to meet the sinking fund requirements.

From a convexity point of view, the sinking fund bond has less convexity than a straight bond of the same duration. Thus, above par, the bond trades to its average term (that is, its term counting redemptions for sinking fund purposes). At par, there is a point of inflection. Below par, the bond trades more like a straight bond—that is, to its final maturity. The sinking fund's demand for the bond raises the bond's price, however, with the upper limit being that which would assume the maximum annual redemptions for sinking fund purposes.

This discussion is, of course, general. The convexity of a specific issue would depend on other aspects, such as the bond's distribution, its discount or premium, and the requirements of the sinking fund.

Figure F illustrates a **purchase fund bond**. It is similar to the sinking fund bond below par. The extra demand for the bond causes it to trade at higher prices, the extreme being the case where the requirements are called at par every year and the bond trades to its average term. Above par, however, the purchase fund does not have the right to call bonds—the key difference between sinking funds and purchase funds. The bond simply trades to its long date and exhibits high convexity.

The bond shown—the Domtar 10 per cent due 2011—has an 85 per cent purchase fund. As such, the bond has good sinking fund protection below par and trades to its long date without fear of any sinking fund call above par. Furthermore, this bond trades only 5 to 10 basis points less in yield than the Domtar 10.35 per cent illustrated in Figure E, which has a 68 per cent sinking fund and much less convexity. The 5 to 10 basis points in yield given up to own the purchase fund instead of the sinking fund is clearly worth giving up for the more convex bond.

Table I Prices for Different Yield Changes

	−200 BP	*Par*	*+200 BP*
Straight	120	100	85
Callable	113	100	86
Extendible/Retractable	118	100	92
Sinking Fund	116	100	85–87
Purchase Fund	121	100	84–86

Table I summarizes the examples for 200-basis-point moves, up and down, from the bond's yield at par. One can see the negative convexity of callables, especially as compared with the extendible/retractables. The advantage of purchase funds over sinking funds is also apparent. Finally, we see the high-convexity characteristics of a long, high-quality bond. While the examples given are not entirely valid, because they do not use bonds of equal duration at par, they do serve to illustrate adequately the differences in convexity.

In summary, the convexity of straight bonds depends on coupon, term and yield. Callables and extendible/retractables have opposite properties, and their convexity depends greatly on the optimal maturity date. Purchase funds have higher convexity—and are thus more desirable—than sinking funds.

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