6. Risk Management: Hedging and Diversification

6.1 Transactions Hedging and Optimal Hedging

Risk Management Objectives

As discussed in Chapter 2, Merton (1993), Tufano (1996) and others state that corporate risk management can be achieved through diversification, hedging and insurance. In some situations, such as Tufano's gold mining sample, the firms involved have little opportunity to exploit diversification opportunities to manage business risk. In other cases, such as globally diversified investment funds, diversification is an integral part of risk management. Situations vary and the identification of an optimal risk management strategy depends on the objective function specified. Though it is difficult to formulate general rules, the results in Chapter 2 do provide useful guidance. Difficult does not mean impossible and this chapter focuses implementing corporate risk management through diversification and hedging with derivatives. The viability of a given technique will depend on the risk management philosophy adopted by the firm and the empirical characteristics of the firm's risk profile.

Drawing from the analysis in Sec. 2.1, it is possible to decompose the optimal solution for either the hedging or diversification problems into a speculative component and a risk minimizing component. Factors determining the speculative part can be significantly different than the elements of the risk minimization problem. Faced with an inability to forecast key variables, the firm is reduced to seeking risk management solutions aimed at minimizing the variability of a target variable, such as direct transactions cash flow, cash flow from firm operations, firm earnings before interest and taxes (EBIT) and so on, e.g., Culp et al. (1998). This theoretical observation is consistent with the rationales expressed by numerous non-financial and financial firms for using derivatives. Yet, even when the speculative component does not affect the optimal solution, i.e., the firm is seeking only to minimize the variance of firm profit by eliminating risks which can be managed with derivative securities, there is still considerable range of risk management solutions which could be selected, depending on the empirical characteristics of the firm's operations.

To see this, consider the problem of a financial institution managing interest rate risk. The risk management objective often specified in this situation is to have the firm to choose the composition of assets and liabilities such that the duration of surplus equal to zero, e.g., Reitano (1991, 1992, 1996). In theory, the best method to achieve this objective is to use "cash flow matching", to match each liability cash flow with an asset which has the same cash flow. Except in special cases, cash flow matching is not possible, e.g., due to the lack of assets with cash flows that match liabilities and the large number of transactions and planning periods involved. The resulting risk management rules which have been developed seek to aggregate the various transactions and planning periods, e.g., for a zero surplus fund, set the duration of assets equal to the duration of liabilities, e.g., Bierwag (1987). This situation can be contrasted with that of, say, a hog producer seeking to lock in the price which will be received for animals nearing the end of the feed cycle. In this case, there is only one transaction and one planning period. For these situations, using cash flow matching to implement the risk management strategy would be feasible and, possibly, desirable.

In what follows, hedging techniques will be illustrated using two approaches. The *transactions hedging* approach emphasizes the trading mechanics involved in *fully hedging* a specific transaction. A cash position is identified and the appropriate forward position is described and determined. It is conventional to have spot and derivative positions that have little or no basis risk though, as will be seen, this does not have to be the case.

Various examples of transactions hedges are provided in the following discussion. The transactions hedging approach does not address what the optimal size of the hedge position needs to be for a specific cash position. This is addressed in the optimal hedging approach. Transactions hedging involves cash flow matching, hence it is desirable to have only a small number of transactions to manage. The transaction hedging approach would be optimal in situations where risk management objective is to minimize the variance of firm cash flow, i.e., where the firm has no view on expected prices and has no ability or desire to speculate. In addition, the optimality of transaction hedging requires that the be little or no basis risk be associated with the derivative securities being used.

While transactions hedging takes the optimal hedge ratio to be one, optimal hedging requires estimating the hedge ratio from empirical data. The transactions hedging approach is the basis for arguments involving the relative benefits of hedging vs. no hedging, such as appear in the free lunch argument (Perold and Schulman 1988). The optimal hedging approach avoids these questions, starting from the premise that full hedging and no hedging are only two of a theoretically infinite number of possible hedge ratios. In a sense, a transaction hedge is a special case of an optimal hedge, applicable under certain specific situations. However, in general, transactions hedges will not be optimal. For example, there may be considerable basis risk associated with the hedging instruments. In addition, transactions hedging does not admit the possible use of derivatives as an essential component of a business plan, e.g., as evidenced in numerous hedge funds. The downside risks of using a transaction hedge approach when an optimal hedge is needed are evident in the MGRM case. Business risks are speculative. Integrating derivative securities into a program to optimally manage business risks is the ultimate goal.

The relevance of these two different approaches to hedging extends well beyond academic pedagogy, as illustrated by the MGRM saga. This firm was in the business of intermediating the long term forward market for oil. Faced with liquidity constraints on available contract maturities, MGRM employed a rolling stack hedge that featured a concentration of nearby contracts being used to hedge a deferred spot commitment. The hedging decision was modelled using the transaction hedging approach. Mello and Parsons (1995, p.19) draw the following conclusion about the MGRM strategy: MGRM had to

> incorporate the fact of cash flow variability across months into a decision about whether and how much to hedge. When the time pattern of cash flows matters — as it typically does for corporations looking to hedge — a smaller size hedge may be preferable to a one-for-one rolling stack or other strategy that actually increases initial cash flow variability.

In effect, MGRM used a transaction hedging approach when the situation was theoretically better suited to using an optimal hedge due to the substantial amount of basis risk facing MGRM. However, how an optimal hedge would have been estimated in the MGRM case is not obvious.

The airline industry provides another illustration of the contrast between an optimal hedge and a transactions hedge. Based on information contained in annual reports, this industry is characterized by a range of risk management activities using derivative securities. Some firms, e.g., Thai Airways, engage in only limited use of derivatives, e.g., for hedging specific FX or jet fuel transactions. Other firms, e.g., Singapore Airlines, have a sophisticated program cover the range of market risks. Though only about 10% of the cost of goods sold, the most volatile cost component for airlines is jet fuel prices. A transactions hedge of this commodity price risk would fully hedge the quantity of jet fuel to be purchased over the planning horizon. An optimal hedge would attempt to estimate the impact of changes in jet fuel prices on changes in net cash flows. Industry practice among firms using sophisticated derivative strategies indicates a hedge ratio of approximately one half, about half of near term jet fuel purchases are hedged.

Price Risk vs. Basis Risk

In Chapter 2, hedgers were taken to be traders who are using the derivatives market to cover a cash position. While there are some legal and regulatory interpretation problems with this broad definition of hedgers, analytically there is little problem using this approach. The analytical difficulties arise in handling interaction of the two basic components of hedge design: risk management, which is inherent in hedging, and speculation, which is required in order to design optimal trades. To see how speculation affects the problem, consider the unhedged grain elevator case. As a random variable, π will have a conditional mean and variance that can be used to assess the risk (volatility) of the unhedged position (see Appendix I).

Figure 6.1 **Profit Function for an Unhedged Grain Elevator**

DATE	Cash Position	Futures Position
$t=0$	Buy units of grain at $S(0)$ for storage in grain elevator	None
$t=1$	Q_A units are sold at $S(1)$ and loaded for shipment	None

Again ignoring carrying costs, the profit function can be specified:

$$\pi(1) = \{S(1) - S(0)\}\ Q_A$$

Application of the definition for conditional variance (where the conditioning notation has been dropped for convenience), $var[\cdot] = E\{(\pi - E[\pi])^2\}$ provides the result that the conditional variance is :

$$var[\pi] = Q_A^{\ 2}\ var[S(1)]$$

In the case of unhedged cash positions, risk depends on the size of the position and the volatility of the cash price. Despite some stylized textbook treatments to the contrary, hedging does not typically eliminate all the risk of cash price fluctuations. To see this, recall the profit function for the ***one-to-one*** grain elevator hedge from Sec. 2.1:

$$\pi = Q\ \{F(0,T) - S(0)\} - \{F(1,T) - S(1)\}$$

It was remarked that the profitability of the hedged position depends on the change in the basis. This discussion can now be extended to observe that the conditional variance for the hedged position is:

$$var[\pi] = Q^2\ var[F(1,T) - S(1)]$$

In other words, ***a transaction hedge substitutes basis risk for price risk.***

Leuthold et al. (1989) on Types of Hedges

Borrowing liberally from Working (1962), Leuthold et al. (1989, p.145-6) provide a partial taxonomy of hedges in which two of the most important types, optimal hedges and natural hedges, are not identified.

Carrying-charge Hedge: A carrying-charge hedge associates with the storage of a commodity. A merchant purchases and stores the commodity, and hedges it to profit from storage. The merchant seeks to profit from changes in the price relationship (basis), rather than price level changes.

Operational Hedge: An operational hedge facilitates merchandising or processing operations. A merchant hedges to establish the price of an input or output, usually holding it for only a short time, often ignoring changes in the basis. The hedge protects the merchant against rapid change in price while a product is being processed or transported. Typical examples of an operational hedge include a flour miller, buying wheat futures to offset a forward sales contract of flour to a baker, or a shipper, exporter or importer selling futures against a cash purchase. These hedges act as temporary substitutes and are liquidated as soon as the trader takes a corresponding cash position.

Selective Hedge: The trader decides whether to hedge or not according to price expectations. The holder of the commodity hedges if prices are expected to fall, and does not hedge if prices are expected to rise. Selective hedging introduces an additional speculative element to hedging as traders hedge only under certain price expectations. This common hedging procedure is often done to prevent large losses, and it can relate to optimal hedging …

Anticipatory Hedge: An anticipatory hedge is usually not matched or offset by an equivalent stock of goods or merchandising commitment. The anticipatory hedge serves as a temporary substitute for merchandising to be done later, that is, an expected future cash purchase or sale. The anticipatory hedge involves either the purchase of futures contracts against raw material requirements, or the sale of contracts by producers in advannce of the completion of production. For example, flour millers and soybean processors may buy futures contracts in anticipation of subsequent purchases of wheat and soybeans, respectively. Livestock feeders may sell live cattle and live hog futures long before the animals are ready for market. Similarly, grain farmers forward sell their crops before harvest ...

Cross Hedge: To cross hedge is to assume a futures position opposite an existing cash position, but in a different commodity. Typically, there is no active futures contract in the commodity corresponding to the cash position, so the trader must select a related commodity for hedging. To be effective, the prices and commodity values of the cash commodity and futures contract must have a fairly high positive correlation. Examples include hedging corporate bonds in the Treasury bond market, grain sorghum in corn, and boneless beef in live cattle.

The importance of understanding basis behaviour for designing effective hedges is apparent from the variance of the one-to-one profit function. Basis information can be used to make various adjustments to improve hedging performance. For example, a refiner seeking to hedge the price of purchasing copper scrap using the high grade copper contract will want to know the basis relationship between the grade of refined copper being produced from the scrap as well as the approximate amount of refined copper that can be produced per unit of scrap. With this information, appropriate adjustments can be made to Q. When will the hedge completely eliminate price risk? This will occur when $F(1,T) = S(1)$. Accomplishing this result requires a delivery hedge where the commodity being hedged is the same as the commodity specified in the futures contract and $T = 1$. In this case, $F(1,1) = S(1)$ and the futures position is satisfied by delivery of the commodity. This result is typically easier to achieve with the use of ***forward*** contracts that can be tailored to match the size, grade, location and other factors that can impact the basis relationship.

Transaction Hedging Example: Issuing Commercial Paper

Figure 6.2: **Perfect Transactions Hedge for a Commercial Paper Issuer** (circa 1981)

DATE	***Cash Position***	***Basis***	***Futures Position***
t= 4/13	Decide to issue \$20 million in 3 month commercial paper on June 3 and want to lock in current rate of 15.25%	1.93%	Short \$20 June (3 month) Tbill contracts at 13.32%
t= 6/3	Issue is offered and sold at 17.50%	1.93%	Close out position with Long contract at 15.57%

Basis variation in a transactions hedge can originate from three sources: ***cross-hedging***, where the cash instrument being hedged is an imperfect match to the deliverable commodity; ***dollar equivalency***, where the implied market value of the futures position differs from that of the cash commodity; and, ***cost of carry*** *considerations*, which include the 'time decay' (to $F(T,T) = S(T)$) built into the cost of carry relationship that determines the futures price. To see the role of these factors in basis variation consider the trading profile for a *perfect transactions hedge*, involving a corporation seeking to lock-in the cost of funds on a future issue of \$20 million of 3 month commercial paper. This hedge was perfect because the cash and futures positions moved up and down in unison. The commercial paper issuer was able to fully lock in the borrowing rate that prevailed when the originally hedging decision was made. Given that this trade involved a cross hedge, i.e., a commercial paper position is being hedged with Tbills, this is an unlikely outcome.

A more likely result for the cross hedging situation would be the following example of an actual hedge. For this hedge, the issue cost will be 15.44%, not 15.25% as in the perfect hedge. Hence, while the hedge was not perfect, the bulk of the considerable reduction in interest rates that took place was offset. The effectiveness of the hedge will depend on basis behaviour of the cash and futures commodities. While for some hedges, e.g., corporate bonds hedged with Tbonds, Iowa corn hedged with Chicago corn, the basis variation will be small relative to potential changes in cash prices, in other cases, e.g., tungsten hedged with platinum, kerosene hedged with crude oil, basis variation could be greater than the change in cash prices. This chapter examines some techniques for determining the hedge position in the face of cross hedging considerations.

Compared to the vagaries of cross hedging, basis variation arising from dollar equivalency is straight forward. *"Dollar equivalency refers to the process of structuring the hedge in such a way that one gets equivalent dollar movement in the two (cash and futures) positions even if the per-unit dollar changes in the two are not equal"* (Powers and Vogel, p.183) For some commodities, such as the grains and metals, dollar equivalency can be handled by *tailing* the hedge. Tailing is discussed in Chapter 3. For financial futures the problem is somewhat more complicated. In this case, dollar equivalency requires adjusting the number of futures contracts such that a basis point change in the futures yield will produce the same dollar value change as a basis point change in the cash position. For example, Appendix 3 derives the values for an 01 (basis point) for \$1 million par value of 3 and 6 month Tbills as \$25 and \$50, respectively. Hence, to hedge a 6 month Tbill position with the 3 month IMM Tbill contract, 2 \$1 million par value futures contracts would be required to hedge, on a dollar equivalent basis, the equivalent par value in 6 month Tbills. Given this, it does not follow that the dollar equivalent hedge ratios will be "optimal". This point will be addressed later in this chapter.

Figure 6.3: **Transactions Hedge Profit for a Commercial Paper Issuer** (circa 1981)

DATE	***Cash Position***	***Basis***	***Futures Position***
t= 4/13	Decide to issue $20 million in commercial paper on June 3 and want to lock in current rate of 18.75%	2.67%	Short $20 June Tbill contracts at 16.08%
t= 6/3	Issue is offered and sold at 17.50%	2.12%	Close out position with Long contract at 15.38%
Change:	-1.25	+ .55	-.70

The final component of basis variation to consider is the cost of carry related issues. More precisely, this is concerned with deviations of futures from cash-and-carry arbitrage conditions during the life of the hedge. In its simplest form, this systematically occurs with the implied "time decay" in the hedge. Depending on the commodity, associated factors would include a combination of fluctuations in carry costs, expectations, term structure effects, distortions arising from delivery and the time to maturity of the futures contract. These factors can impose some limitations on hedge design. For example, because of the impact that delivery considerations have on futures prices during the delivery month of the contract, the nearby contract selected should not enter the delivery month during the anticipated life of the hedge (unless "rolling the hedge forward" is incorporated as part of the hedging strategy or futures delivery is the objective). Ignoring delivery effects, a number of studies have shown that there is reasonably close correspondence between movements in cash and futures prices for US financial futures, e.g., Cecchetti, et al. (1988).

Given the various issues associated with basis variation, an essential feature of hedge design involves the incorporation of cash and futures price expectations into the hedge decision, i.e., at some point, hedge design has to deal with the speculative aspect of hedging. Armed with accurate price forecasts, it is clear that *transactions hedges will not always produce the most profitable outcomes*. This is because the futures position in an effective hedge will only be profitable when the cash price moves adversely. In cases where the cash price moves favourably, the futures position in the hedge portfolio will lose money and the hedger would have been better off (made a higher profit) not hedging. To see

Figure 6.4 **Actual Transactions Hedge Profit for a Commercial Paper Issuer**

DATE	***Cash Position***	***Basis***	***Futures Position***
t= 4/13	Decide to issue $20 million in commercial paper on June 3 and want to lock in current rate of 15.25%	1.93%	Short $20 June Tbill contracts at 13.32%
t= 6/3	Issue is offered and sold at 17.50%	2.12%	Close out position with Long contract at 15.38%
Change:	+ 2.25	-.19	-2.06

this, consider the commercial paper issuer again given in Figure 6.3. In this case, if the hedger had chosen not to hedge, then the issue rate would have been 17.50% instead of the 18.20% that is associated with the hedged offering. Hence, to be optimal, in the sense of maximizing profit, the hedger must build in expectations of future price changes and, as a result, engage in speculation.

Transaction Hedging: Using a Eurodollar Futures Strip Hedge

The considerable discussion and debate surrounding the MGRM rolling stack hedging strategy illustrates the potential complexity that a transaction hedge can assume. Examples illustrating the MGRM stack hedge abound, e.g., Edwards and Canter (1995), Culp and Miller (1994), Mello and Parsons (1995). Faced with massive position sizes to hedge, MGRM could not pursue a maturity and quantity matching hedging strategy, the strip hedge. Due to the substantial liquidity in nearby deliveries, MGRM was able to implement a rolling stack hedge. Because the MGRM cash position was not marked to market, such a hedging strategy can incur substantial variation margin cost, in addition to the usual basis risk. In a rolling stack hedge, the rollover can also add mark-to-market basis charges to the variation margin costs arising from spot price changes. The MGRM case illustrates that such charges can be enormous and constitute a serious complication when designing a hedging program. Conversely, it is also possible for the rolling stack hedge to generate enormous positive cash flows from variation margin. Such is the element of speculation inherent in the underlying business plan.

The application of the rolling stack hedging strategy is not confined to oil complex contracts. Dubofsky and Miller (2001) provide an excellent example comparing a stack hedge with a strip hedge for hedging a multi-period interest rate cash flow. The example proceeds as follows: suppose that on July 28, 1999, a bank plans to make a one-year fixed-rate loan for $50 million beginning on September 13,1999, with loan payments to be made in quarterly installments. The fixed rate is 7.50%. The bank plans to fund the loan by using quarterly borrowing in the spot Eurodollar market. This funding takes place in 47 days. To execute a strip hedge for this borrowing, the bank would sell 200 Eurodollar futures contracts, 50 in each of the four available, different delivery months. To execute a stack hedge, the bank would initially sell 200 futures contracts in the nearby delivery month, rolling the position forward into the next available nearby as expiration arrives.

Since the bank will borrow $50 million in September and approximately every 90 days three times thereafter, the bank initiates a strip hedge by selling 50 Eurodollar futures contracts in each of four delivery months: September, December, March, and June. The short positions in these 200 futures contracts are all entered on July 28th. Thus, a strip hedge can be thought of as a portfolio of single-period hedges. In the strip hedge, the hedger has hedged borrowing costs for four successive quarters. Table 7-1 contains data for the strip hedge example, with rates and prices that are known on July 28, 1999 appearing in *italics*. In this example, the firm has transformed the uncertain borrowing stream int a new and known loan rate over four successive quarters.[1] As an example of how to read the table, "S 50 @ 94.555" means go short 50 contracts at a futures price of 94.555, and "L 50 @ 94.35" means go long 50 contracts at a futures price of 94.35.

Table 6.1 **Strip Hedge Using Eurodollar Futures**

Date	Spot LIBOR	Sept. 1999	Dec. 1999	March 2000	June 2000
7/28/1999	*5.3125%*	*S 50 @ 94.555*	*S 50 @ 94.19*	*S 50 @ 94.185*	*S 50 @ 93.95*
9/13/1999	5.65%	L 50 @ 94.35			
12/13/1999	5.85%		L 50 @ 94.15		
3/13/2000	6.00%			L 50 @ 94.00	
6/19/2000	5.90%				L 50 @ 94.10
Futures LIBOR rate: (on 7/28/99)		**5.445%**	**5.81%**	**5.815%**	**6.05%**

Gain (Loss) in Euro$ Futures: $25,625 $5,000 $23,125 ($18,750)*.

Quarter	Firm's Borrowing Rate	Quarterly Interest Expense	Gain (Loss) on Futures Positions	Net Interest Expense	Effective Borrowing Rate
Sep 99 - Dec 99	5.650%	706,250*.	25,625	680,625	**5.445%**
Dec 99- Mar 00	5.850%	731,250	5,000	726,250	**5.810%**
Mar 00 - Jun 00	6.000%	750,000	23,125	726,875	**5.815%**
Jun 00 - Sep00	5.900%	737,500	(18,750)	756,250	**6.050%**
				Average:	**5.78%**

.* The $18,750 loss occurs because the futures price rose from 93.95 to 94.10. This is a loss of 15 ticks. With each tick worth $25 and the bank short 50 futures contracts: (-15 ticks)($25/tick)(50) contracts = -$18,750.
** ($50 million)(0.0565)(90/360) = $706,250.

In this example, the effective borrowing rate for each quarter is the implied futures rate at the time the hedge is placed, just as it is in the one cash flow case. Thus, the strip hedge can be viewed as a portfolio of one-period hedges. However, as Mello and Parsons (1995, p18) point out: "… even when constructing a hedge using a strip of futures, it is important to think about the cash flow effects of the maturity structure. If the hedge is constructed using OTC instruments, then there is greater flexibility in designing the terms: it may be possible to negotiate a deviation from the mark-to-market rules necessary for exchange-traded instruments so that payments on the hedge instruments better match payments under the delivery contracts." This speaks to both basis variation and variation margin cost. The futures example in Table 6.1 disguises certain elements involved in practical execution.

In terms of the strip hedge example in Table 6.1, the basis variation is avoided by assuming that spot LIBOR = futures LIBOR on each date, so that the firm goes to the capital market and actually borrows $50 million for another 3 months at the spot rate prevailing at simultaneously the same time as the futures contract expires. In this example,.this is evident from the four diagonal futures prices (L 50 @ 94.XX) equaling 100-spot LIBOR on these four dates. This can be assured only if the firm borrows using each of the futures contracts. In addition, the example does not take any account of variation margin. Gains or losses on futures are calculated at delivery. Observing that interest rates swaps can be priced using Eurodollar strips, this discussion can be extended to provide insight into motivations for using interest rate swaps.

Transactions Hedging: Using a Eurodollar Futures Stack Hedge

A stack hedge is sometimes referred to as a rolling hedge or a rolling stack hedge. In this transaction, the hedger would sell 200 September 1999 futures contacts on July 28, 1999. Instead of having 50 contracts spread over the four contract delivery dates, the 200 nearby contracts is substituted for the protection provided by 50 futures contracts for four quarters. If the firm only sold 50 September futures contracts, then the firm would only be hedging the first quarter's borrowings. Thus, the firm "stacks" the hedges for subsequent quarters in one nearby delivery date. In this example, this is the September futures contract. Because the Eurodollar futures contract is cash settled, the firm will hold the 200 September futures contracts until just prior to the expiration on September 13, 1991, at which time the firm will closes out the Sept contracts and sells 150 December futures contracts, holding these contracts until just prior to the expiration on December 13, 1999 at which time the firm sells 100 March delivery contracts. This process continues for the March, where 50 June contracts are rolled forward. Finally, the June futures contracts are closed out just prior to expiration. Due to possible basis risk, the end result is a less certain locking in of the quarterly borrowing rate over one-year than in the strip hedge.

Table 6.2 contains data for a stack hedge example comparable to the example in Table 6.1. The only difference between the Tables is that the futures prices at the time of the hedge placement differ with the futures prices at the time the hedge is lifted being the same. This implies that the spot LIBOR at the expiration of the futures contracts is the same in both examples.

Table 6.2 **Stack Hedge Using Eurodollar Futures**

Date	Spot LIBOR	Sept. 1999	Dec. 1999	March 2000	June 2000
7/28/1999	*5.3125%*	*S200@94.555*			
9/13/1999	5.65%	L200@94.350	S150@93.985		
12/13/1999	5.85%		L150@94.150	S100@94.145	
3/13/2000	6.00%			L100@94.000	S50@93.765
6/19/2000	5.90%				L50@94.100
Futures LIBOR rate**:**		**5.445%** (on 7/28/99)	**6.015%** (on 9/13/99)	**5.855%** (on 12/13/99)	**6.235%** (on 3/13/00)
Gain (Loss) in ED Futures:		$102,500	($61,875)	$36,250*	($41,875)

Quarter	Firm's Borrowing Rate	Quarterly Interest Expense	Gain (Loss) on Futures Positions	Net Interest Expense	Effective Borrowing Rate
**Sep 99-Dec 99	5.650%	706,250	25,625	680,625	**5.445%**
Dec 99-Mar 00	5.850%	731,250	(20,625)	726,250	**5.810%**
From Sep/Dec:			25,625		
Mar 00- Jun 00	6.000%	750,000	18,125	726,875	**5.815%**
From Sep/Dec:			25,625		
From Dec/Mar:			(20,625)		
Jun 00- Sep 00	5.900%	737,500	(41,875)	756,250	**6.050%**
From Sep/Dec:			25,625		
From Dec/Mar:			(20,625)		
From Mar/Jun:			18,125		
				Average:	**5.78%**

* The $36,250 gain occurs because the futures price fell from 94.145 to 94. This is a profit of 14.5 ticks, worth $25/tick. The bank was short 100 futures contracts. 14.5 ticks X $25/tick X 100 contracts = $36,250.
** The futures price on 9/13/99 is 94.35. It follows that:100-94.35 = 5.65; (0.0565/4)X $50 million = $706,250; (680.625/50.000.000) (4) = 5.445%

To calculate the effective borrowing rate in the stack hedge, as in the lower half of Table 6.2, gains and losses from previous quarters' futures trading must be allocated to the current quarter. For example, one-fourth of the $102,500 gain from the Eurodollar futures hedge in place during the Sep 99/Dec 99 quarter must be allocated to the Dec 99/Mar 00 quarter. The net interest expense during this quarter will reflect a $25,625 ED futures gain from the previous quarter as well as a $20,625 ED futures loss in the current quarter (one-third of$61,875). Casual inspection of Tables 6.1 and 6.2 reveals an average effective borrowing rate of 5.78% for both transactions. This is comparable to the perfect hedge of a single cash flow from Figure 6.2. In Table 7-1: the September futures rate is 0.132% above the spot LIBOR on July 28th; on July 28th, the December futures rate is 0.365% higher than the September futures rate; the March futures rate is 0.005% higher than the December futures rate; and, the June futures rate is 0.235% higher than the March futures rate. This can be compared with the basis in Table 7-2: on July 28th, the September futures rate is also 0.132% higher than the spot LIBOR rate; on September 13, 1999, the December futures rate is 0.365% higher than the spot LIBOR rate; on December 13, 1999, the March futures rate is 0.005% higher than the spot LIBOR rate; and, on March 13,2000, the June futures rate is 0.235% higher than the spot LIBOR rate.

Choosing Between a Strip Hedge and a Stack Hedge

Movements in the futures yield curve determine the relative effectiveness of a stack hedge versus a strip hedge. When the futures term structure shifts in this parallel fashion, a strip hedge or a stack hedge will result in the same effective borrowing rate. Other changes in the futures term structure will favour one hedge over the other. For most futures contracts, the advantage of the stack hedge is that the hedger is always trading the most liquid futures contract. Casual inspection of the volume and open interest for Eurodollar futures, readily available on the exchange website or in the financial press, reveals considerable liquidity out to five years. Volumes can exceed 1,000 contracts in contracts over twenty expirations in the future. In contrast, the futures contract volume and open interest for various commodities, e.g., Tbills, stock indexes, currencies, is concentrated in the nearby delivery date. Hence, in some commodities hedgers have a choice to use a strip or stack hedge, while in others only a stack hedge is available.

What is the rationale for using a stack hedge when contract liquidity permits the use of strip hedges? For the risk manager will to take a view on future price movements, the risks inherent in stack hedging can be a blessing. In particular, stack hedges in liquid markets can offer the hedger the opportunity to take advantage of relative mispricings when switching to later delivery months. More precisely, if the hedger sees mispricing between two deferred futures contracts, the hedger can roll the hedge into (sell) the relatively higher-priced futures contract. An important disadvantage of stack hedges is the higher transactions costs compared to a strip hedge. The importance of transactions costs in strip hedging has inspired the CME to introduce, starting in 1994, trading in packs and bundles of Eurodollar futures. A Eurodollar bundle is the simultaneous sale or purchase of one contract each of a series of consecutive Eurodollar futures contracts. Bundles are a convenient way to construct a Eurodollar strip position because all contracts are entered concurrently. That is, the trader does not have to construct the strip contract by contract, eliminating execution risk and reducing transactions costs.

In practice, the relative performance of stack hedges vs. strip hedges will depend on the uncertain movement of the yield curve. Table 6.3 presents an example in which the futures term structure slopes subsequently upward. As discussed in chapter 4, the change in the futures term structure is due to changes in the difference between the implied carry cost and the carry return, in effect a change in the short term Eurodollar yield curve. In Table 6.2, on July 28th, the September futures rate is still 0.132% higher than the spot LIBOR rate. However, on September 13, 1999, the December futures rate is now 0.23% higher than the spot LIBOR rate. On December 13, 1999, the March futures rate is now 0.33% higher than the spot LIBOR rate, and on March 13, 2000, the June futures rate is now 0.43% higher than the spot LIBOR rate. As a result, the average effective borrowing rate for the stack hedge is 5.89% in this case. Under these conditions, the firm would have been better off, in terms of interest costs, with a strip hedge instead of a stack hedge. However, if the futures term structure slopes subsequently downward, a stack hedge would have outperformed a strip hedge.

Table 6.3 **A Stack Hedge with a Futures Term Structure Slope that Subsequently Increases**

Date	Spot LIBOR	Sept. 1999	Dec. 1999	March 2000	June 2000
7/28/1999	*5.3125%*	*S200@94.555*			
9/13/1999	5.65%	L200@94.350	S150@94.12		
12/13/1999	5.85%		L150@94.15	S100@93.82	

3/13/2000	6.00%			L100@94.00	S50@93.57
6/19/2000	5.90%				L50@94.10
Futures LIBOR rate:		**5.445%**	**5.88%**	**6.18%**	**6.43%**
Premium (Discount) to Spot LIBOR at Hedge:		0.1325% (5.445-5.3125)	0.230% (5.88%-5.65%)(	0.330% 6.18%-5.85%)	0.430% (6.43%-6%)
Gain (Loss) in ED Futures:		$102,500	($11,250)($45,000)($66,250)		

Quarter	Firm's Borrowing Rate	Quarterly Interest Expense	Gain (Loss) on Futures Positions	Net Interest Expense	Effective Borrowing Rate
Sep 99-Dec 99	5.650%	706,250	25,625	680,625	**5.445%**
Dec 99-Mar 00	5.850%	731,250	(3.750)	709,375	**5.675%**
From Sep/Dec:			25,625		
Mar 00- Jun 00	6.000%	750,000	(22,500)	750,625	**6.005%**
From Sep/Dec:			25,625		
From Dec/Mar:			(3,750)		
Jun 00- Sep 00	5.900%	737,500	(66,250)	804,375	**6.435%**
From Sep/Dec:			25,625		
From Dec/Mar:			(3,750)		
From Mar/Jun:			(22,500)		
				Average:	**5.89%**

The decision to choose a stack hedge or a strip hedge is an excellent illustration of the rich texture of decisions involved in implementing risk management decisions. Even when transactions hedging is employed, there are decisions involving business risks that have to be identified. Recognizing the speculative component in the risk management decision is an essential element in the development of an effective business plan. An essential limitation of the transaction approach is the failure allow for the size of the hedge position to vary significantly from the size of the cash position. For some hedging situations this approach is more than adequate. However, in other situations, the transactions approach does not capture observed behaviour. For example, airlines seeking a hedge against oil prices do not typically take a hedge position that is equal in size to expected fuel usage. Rather, hedge ratios around one half are often observed. Ultimately, there are many possible uses for derivative securities. Whether a transactions approach or an optimal hedging approach is used to structure derivative decisions depends on the specifics of the situation.

Multivariate Optimal Hedge Ratio Estimation

Section 2.3 dealt with the mean-variance optimal univariate hedge ratio, i.e., solutions for the optimal univariate hedge ratio associated with the expected utility function: $EU = E[\pi] - b\ var[\pi]$. The problem is univariate because a single cash position is being hedged using a single derivative security. The mean-variance solution was composed of two parts: the minimum variance hedge ratio and the optimal speculative position. While the minimum variance component depends on the ratio of statistical parameters, the speculative component depends on the hedger's risk attitudes as reflected in b. Hedgers who are "less risk averse" will have lower b (*ceteris paribus*) and, as a result, will be more willing to take speculative positions in the form of over or under hedges. In addition, because the futures price variance enters in the numerator of the "speculative" term, as the *perceived* volatility increases the hedger will be less willing to take positions over or under the minimum variance hedge. Because variances as well as expectations are conditional on the information available on the hedge date, the subjective probability assessments of the hedger, the less capable or willing the hedger is to make forecasts, the less important is the speculative component of the hedge.

In some hedging situations, it is possible to reformulate the hedging problem to have not one but many different futures contracts involved in the hedge. One example would be hedging a portfolio of foreign assets denominated in a number of different currencies. In this case, currency futures for all the relevant currencies could be used to construct a hedge. Another example is a commodity, such as tungsten or titanium, where there is no traded futures contract available. A hedge could be constructed by using an appropriately weighted combination of a number of different futures contracts. Yet another example would be a portfolio of mortgages, for which there is not a precise traded futures contract. Instead a hedge could be constructed using a number of different futures contracts. Sec. 5.4 discusses a sophisticated example where a currency spread and an interest rate future are used to hedge a foreign interest rate.

The objective of increasing the number of different future contracts used in the hedge is, ultimately, to improve hedge performance. At some point, this will be a fruitless exercise because there would be so many futures contracts to monitor and transactions costs would increase accordingly. The obvious question is: how to optimally construct multivariate futures hedges? Theoretically, it would be most appropriate to develop an equilibrium model explaining the relationship between the commodity being hedged and the commodities underlying the futures contracts being used to construct the hedge. The approach used here is to assume that such a model has been specified and to work with the general profit function for a multivariate hedge position.

Two problems will be considered, the minimum variance hedge and the mean-variance optimal speculative solution. These two solutions will then be combined to produce the mean-variance optimal multivariate hedge solution. As with the univariate hedge solutions, it is assumed that there is one random variable to be hedged, the price risk of the cash position. The size of the cash position is fixed and the relevant components of perfect markets are adopted. Given this, consider the general profit function for a hedge involving k futures contracts and a fixed cash position:

$$\pi(1) = \overline{Q_s}\,(S_1 - S_0)$$

$$- Q_1\,(F_1(1,T) - F_1(0,T)) - Q_2\,(F_2(1,T) - F_2(0,T)) - \ldots\ldots - Q_k\,(F_k(1,T) - F_k(0,T))$$

where the bar on Q_s indicates that the size of the cash position is not a choice variable. Like the previous univariate hedge where it was assumed that the cash position was long and that the futures position was short when $Q > 0$, this formulation also permits futures to be short or long with the $Q_i > 0$ solution denoting a short futures position matched with a long cash position and $Q_i < 0$ denoting a long futures position matched with a long cash position.

The key step in transforming the general profit function into a form for suitable for ***estimating*** a minimum variance solution is to reformulate the problem in the form:

$$y = X\,\beta + u$$

where $y = Q_s\,\Delta S$, $X = (1, \Delta F_1, \Delta F_2, \ldots, \Delta F_k)$, $\beta = (\alpha, Q_1, Q_2, \ldots, Q_k)$ and u is an equation error that captures the unexplained variation in y not accounted for by $X\beta$ and α is the equation constant term. It is possible to normalize this formulation by dividing through by Q_s, in which case β would refer to the hedge ratios Q_i/Q_s. The general formulation with Q_s on the lhs permits the possibility of using income as the random variable to be hedged. It is also useful for deriving the mean-variance optimal solution.

Using $y = X\beta + u$ to specify the problem permits the OLS solution ($\hat{\beta}$) to be immediately specified as:

$$\hat{\beta} = (X^T X)^{-1}\, X^T y$$

In a time series estimation framework, this is an OLS regression estimator where y is a $Tx1$ vector containing ΔS_t and X is a $T\,x\,(k+1)$ matrix containing k columns of $Tx1$ vectors of ΔF_{it} together with a $Tx1$ column of ones to represent the constant term. (This specification for y assumes that Q_s has been used to normalize and that β represent hedge ratios.) For a long cash position, $\beta_i > 0$ indicates the fraction of the cash position that will be hedged with a short position in commodity future i. If $\beta_i < 0$, then this indicates the fraction of the cash position that will be hedged with a long position in commodity future i. Extensions of this solution to GLS, ARCH and 2SLS follow naturally.

Transforming the solution for the multivariate minimum variance hedge ratio estimator to an equilibrium framework involves taking expectations at the appropriate point to produce the result that:

$$(X^T X)^{-1} = \mathbf{\Sigma}^{-1} \qquad X^T y = \{\sigma_{1,s}\ ,\ \sigma_{2,s}\ ,\ ....\ ,\sigma_{k,s}\}^T\ \overline{Q_s} = cov[F,s]\ \overline{Q_s}$$

where Σ is the variance-covariance matrix of the ΔF_i:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^{\ 2} & \sigma_{1,2} & ... & \sigma_{1,k} \\ \sigma_{2,1} & \sigma_2^{\ 2} & ... & ... \\ ... & ... & ... & \sigma_{k-1,k} \\ \sigma_{k,1} & ... & \sigma_{k,k-1} & \sigma_k^{\ 2} \end{bmatrix}$$

and the individual σ_{ij} refer to variances and covariances for changes in the relevant futures prices. It follows that:

$$Q_{mv} = \frac{Q^*}{\overline{Q_s}} = \mathbf{\Sigma}^{-1}\ cov[F,s]$$

This result is identical to the estimation solution, with the proviso that the equilibrium solution involves conditional population parameters while the estimated solution involves estimates of the population parameters determined using T observations of data on the ΔF_i and ΔS.

As an example of the multivariate solution, consider the case of a minimum variance hedge using two futures contracts. In this case:

$$\mathbf{\Sigma}^{-1} = \frac{1}{\det}\begin{bmatrix} \sigma_2^{\ 2} & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^{\ 2} \end{bmatrix} \qquad X^T y = \begin{bmatrix} \sigma_{s,1} \\ \sigma_{s,2} \end{bmatrix} \overline{Q_s}$$

$$\det = \sigma_1^{\ 2}\sigma_2^{\ 2} - \sigma_{12}^{\ \ 2}$$

The conventional result follows:

$$\frac{\hat{Q}_1}{\overline{Q_s}} = \frac{\sigma_2^{\ 2}\sigma_{s,1} - \sigma_{1,2}\sigma_{s,2}}{\sigma_1^{\ 2}\sigma_2^{\ 2} - \sigma_{1,2}^{\ \ 2}} \qquad \frac{\hat{Q}_2}{\overline{Q_s}} = \frac{\sigma_1^{\ 2}\sigma_{s,2} - \sigma_{1,2}\sigma_{s,1}}{\sigma_1^{\ 2}\sigma_2^{\ 2} - \sigma_{1,2}^{\ \ 2}}$$

These results are intuitively the same as those given previously, with the proviso that the sign of β has positive coefficients reflecting short hedge positions combined with long cash positions and negative coefficients representing long futures combined with a long cash position.

As in the univariate hedge case, the optimal mean-variance solution is a combination of the minimum variance solution and the mean-variance optimal speculative solution. To derive the optimal speculative solution, observe that the objective is to maximize expected utility which, in this case, is specified using the mean and variance of speculative profit:

$$L = E[\pi] - b\ var[\pi] = Q_1\ E[\Delta F_1] + Q_2\ E[\Delta F_2] + ... + Q_k\ E[\Delta F_k] - b\{Q^T \Sigma Q]$$

$$= Q^T \begin{bmatrix} E[\Delta F_1] \\ E[\Delta F_2] \\ . \\ . \\ . \end{bmatrix} - b\{Q^T \Sigma Q\}$$

Differentiating the objective with respect to the choice variables Q^T produces the solution:

$$Q^* = \begin{bmatrix} Q_1{}^* \\ Q_2{}^* \\ . \\ . \\ . \end{bmatrix} = \frac{1}{2b}\ \Sigma^{-1} \begin{bmatrix} E[\Delta F_1] \\ E[\Delta F_2] \\ . \\ . \\ . \end{bmatrix}$$

where * denotes an optimum value.

Solving the general mean-variance optimal speculative solution for the case of two futures positions gives:

$$\begin{bmatrix} Q_1{}^* \\ Q_2{}^* \end{bmatrix} = \frac{1}{2b\ \det} \begin{bmatrix} \sigma_2{}^2 & -\sigma_{1,2} \\ -\sigma_{1,2} & \sigma_1{}^2 \end{bmatrix} \begin{bmatrix} E[\Delta F_1] \\ E[\Delta F_2] \end{bmatrix}$$

where det is the same as that given in the minimum variance hedge example given above. Solving for the specific case of Q_1*:

$$Q_1{}^* = \frac{\left[\sigma_2{}^2\ E[\Delta F_1] - \sigma_{1,2}\ E[\Delta F_2]\right]}{2b\ (\sigma_1{}^2 \sigma_2{}^2 - \sigma_{1,2}{}^2)}$$

The result for Q_2* follows appropriately.

This result has much the same implications as in the univariate case. As $b \to \infty$ the speculator's risk aversion becomes so high that the optimal speculative solution goes to zero and as $b \to 0$ the speculator's risk aversion approaches risk neutrality and the size of speculative position is very sensitive to even small expected changes in futures prices. Similarly, the speculator's ability to forecast price changes, as reflected in the size of the elements in Σ, also affects the solution. In the univariate case the result was that if the speculator has limited ability to forecast the next price change then the variance of the forecast ($\sigma_f{}^2$) will be large and the difference between current and expected prices will have to be large in order to induce a significant speculative position. The multivariate case has similar intuition but adjusts this result to account for the covariances between the various contracts involved in the position as well as the variances of the forecasts for the prices of other contracts involved in the position.

Having derived the minimum variance and optimal speculative solutions, it is now possible to derive the mean-variance optimal solution for a multivariate hedge. The objective function for this problem is:

$$EU[\pi_{mh}] = \overline{Q_s}\ E[\Delta S] - Q_1\ E[\Delta F_1] - Q_2\ E[\Delta F_2] - .... - Q_k\ E[\Delta F_k] - b\{var[\pi_{mh}]\}$$

where, using the definitions from the previous derivation of the equilibrium minimum variance solution:

$$var[\pi_{mh}] = \overline{Q_s}^2\ \sigma_s^2 + Q^T\ \Sigma\ Q - 2\{\overline{Q_s}\ Q_1\ \sigma_{s,1} + \overline{Q_s}\ Q_2\ \sigma_{s,2} + ... + \overline{Q_s}\ Q_k\ \sigma_{s,k}\}$$

The first order condition for futures position 1 gives:

$$\frac{\partial EU}{\partial Q_1} = -\{E[\Delta F_1]\} - 2b\ \{Q_1^*\ \sigma_1^2 + Q_2\ \sigma_{1,2} + \ldots + Q_k\ \sigma_{1,k} - \overline{Q_s}\ \sigma_{s,1}\} = 0$$

where, as before, the * indicates an optimum value. Solving Q_1* involves determining the solution of the other *k-1* positions.

Proceeding with to determine all k derivatives and expressing the solution in matrix form:

$$\Sigma\ \frac{Q^*}{\overline{Q_s}} = -\frac{E[\Delta F]}{2b\ \overline{Q_s}} + cov[F,s]$$

Inverting Σ gives the solution:

$$Q^* = -\Sigma^{-1}\ \frac{E[\Delta F]}{2b\ \overline{Q_s}} + \Sigma^{-1}\ cov[F,s] = Q_{mv} - \frac{Q_{os}}{\overline{Q_s}}$$

where Q_{mv} is the minimum variance solution and Q_{os} is the mean-variance optimal speculative solution. This verifies the generalization of the univariate result: the mean-variance optimal hedge ratio is determined by combining the minimum variance hedge ratio and the mean-variance optimal speculative solution.

Again considering the example where two futures contracts are being used to hedge a fixed cash position, the optimal mean-variance solution can now be specified as:

$$\frac{Q_1^*}{\overline{Q_s}} = \frac{\sigma_2^2\ \sigma_{s,1} - \sigma_{1,2}\ \sigma_{s,2}}{\sigma_2^2\ \sigma_1^2 - \sigma_{1,2}^2} - \frac{\sigma_2^2\ E[\Delta F_1] - \sigma_{1,2}\ E[\Delta F_2]}{2b\ \overline{Q_s}\ (\sigma_1^2\ \sigma_2^2 - \sigma_{1,2}^2)}$$

From this it follows that if the different futures contracts involved in the hedge have price changes that are uncorrelated ($\sigma_{1,2} = 0$), then the individual hedge ratios will be equal to the univariate solutions. The dependence of the optimal solution on the utility parameter b casts doubt on the ***general*** validity of the numerous empirical studies that have taken the minimum variance solution to be the optimal solution.

The two period structure of the profit function leads to a number of other qualifications about the generality of the optimal mean-variance solution. Extending to a multi-period framework raises the possibility of readjusting the hedge position over time, producing a different hedge ratio at each hedge readjustment date. In addition, the solution to an inter-temporal, multi-period optimization problem will not necessarily produce the decomposition of the hedge ratio into the minimum variance and optimal speculative solution (Heaney and Poitras 1991). Among other reasons, this is because as prices evolve over the life of the hedge, risk propensities will affect the desire to adjust to observed data.

Optimal Hedge Ratios for Different Utility Functions

Regarding the minimum variance component of the optimal hedge, it has long been recognized that OLS is optimal for only a restricted set of expected utility functions. Recent work on hedge ratio estimation has demonstrated that the "optimal" futures hedge ratio does depend on the objective function selected. For example, significantly different empirical estimates of the hedge ratio for Tbonds have been obtained from log and minimum variance expected utility functions (Cecchetti, et al. 1988). However, available evidence to date has been restricted to comparisons of a small number of specific objective functions. With this in mind, it is possible to formulate further solutions to the general expected utility problem underlying the hedge ratio optimization. Two types of solutions are considered. Firstly, under the assumption of bivariate normality of the spot and futures returns, a general relationship between the OLS estimate and the hedge ratio implied by a general expected utility function will be specified, and a number of specific expected utility functions examined as illustrations. Secondly, admitting riskless lending and borrowing in the wealth dynamics, the hedge ratio can be shown to be independent of preferences, i.e., depending solely on parameters of the joint distribution of the return generating processes. These results are derived for the traditional, single period "myopic" objective function.

While there are a number of roughly equivalent specifications of the hedger's optimization problem, following Heaney and Poitras (1991) and Cecchetti et al. (1988) another approach to the relevant hedger's problem can be expressed as maximizing the expected utility of terminal wealth for a hedged portfolio with wealth (W) determined by:

$$W_{t+1} = W_t\,(1 + R_s(t+1) - h_t R_f(t+1)) \qquad (6.1)$$

where: h_t is defined to be the ratio of the values (price times quantity) of the spot and futures positions at time t,[2] $R_s(t+1)$ and $R_f(t+1)$ are the t to $t+1$ returns to holding spot and futures and $t \in [0 \ldots . T]$, i.e., $R_A(t+1) = (A_{t+1} - A_t)/A_t$ where A is either the spot or futures price at t and $t+1$. By construction, the selection of (6.1) to specify the wealth dynamics restricts the problem in order to derive implementable solutions. In particular, (6.1) assumes a single period decision framework with no potential for variation in the quantity of the spot commodity being held.

Another significant feature of (6.1) is the absence of portfolio theoretic considerations. In particular, by not incorporating lending and borrowing into the specification of the profit function, the size of the cash position has been fixed. In effect, the resulting hedging optimization assumes away the portfolio decision by having the hedger fully invested in the spot commodity. If lending and borrowing is admitted, considerations of leveraging to buy the spot commodity and short-selling the spot to invest in the riskless asset enter the hedger's decision process. Theoretically, this is translated into a change in the underlying wealth dynamics to:

$$W_{t+1} = W_t\,(1 + x_t R_s(t+1) + (1-x_t) r(t+1) - H_t R_f(t+1)) \qquad (6.2)$$

where: x is the fraction of total wealth invested in the spot commodity, H is the value (price times quantity) of the hedge position divided by initial wealth (*not the value of the spot position*) and r is the riskless rate. In turn, (6.2) can be used as the argument in the hedger's optimization problem. In practice, the primary advantage of using (6.1) over (6.2) is analytical simplification: the optimal hedge ratio requires specification of only a joint probability distribution and a utility function. The addition of lending and borrowing results in the introduction of an additional choice variable.

Given this, the conventional hedge ratio optimization problem can be generalized to admit any type of well-behaved utility function, Using (6.1):

$$\max_{h_t} \; E\,\{\, U[\pi(t+1)\,]\,\} \quad \Rightarrow \quad \max_{h_t} \int_{R_f}\int_{R_s} U[\; W_t(1 \; + R_s(t+1) \; - \; h_t \; R_f(t+1))\;]\; dP_{sf}(t) \qquad (6.3)$$

where the conditional expectation $E[\cdot]$ has been formally defined using an appropriately specified conditioning information set. In this form, the joint conditional probability distribution (measure) associated with the expectation is P_{sf}, the profit function is either $\pi(t+1) = W_t\,(R_s(t+1) - h_t R_f(t+1))$ or $\pi(t+1) = W_t\,(x_t R_s(t+1) + (1-x_t) r(t+1) - H_t R_f(t+1))$.[3] In practice, there are restrictions on the types of commodities for which the *EU* optimization using (6.1) is the appropriate hedging problem. For example, because no allowance has been taken of unexpected variation in quantity of the spot commodity, the hedged portfolio would not fully capture the wealth dynamics associated with many harvestable crops. However, (3.3) would be appropriate in the case of financial futures, such as Tbonds, money market securities and currencies.

In terms of solutions, under some strong distributional assumptions, the minimum variance solution leads to an optimal hedge ratio that equals the slope coefficient in an ordinary least squares (OLS) regression of spot on futures prices. By construction, OLS depends fundamentally on the selection of joint probability distributions that are constant over time. This assumption results in equality of conditional and unconditional parameters.

When time variation in the joint probability distributions is permitted, e.g., due to ARCH errors, the decision problem can be more complicated. In this case, the specification of the optimal hedging problem typically takes on a more complicated form and has to be solved using some dynamic optimization procedure, e.g., dynamic programming, which takes account of the state variable time paths. The resulting solutions are potentially intractable and difficult to interpret. However, in the special case of *log utility* (Cecchetti, et al. 1988), the dynamic solution will reduce to a sequence of one-period solutions (Samuelson 1969). This important simplification permits the introduction of certain types of temporal variation in the conditional variances and covariances without significantly complicating the solution.

In addition to complications arising from non-constant distributional parameters, when the structure of the optimal hedging problem is altered by the introduction of riskless lending and borrowing, variation in the size of the spot position means that the hedge ratio cannot be determined by choosing the relative size of the futures position. There are now two choice variables, the fraction of initial wealth invested in the riskless asset and the size of the hedged position. Again, while there has been explicit recognition of riskless lending and borrowing, analysis has been restricted to special cases, particularly mean-variance (Bond and Thompson 1986, Turvey and Baker 1989). In certain special cases (e.g., Poitras 1989a), the resulting optimal hedge ratio has been shown to be independent of hedger risk preferences, depending solely on the parameters of the (un)conditional joint distribution of returns.

This Section provides two Propositions corresponding to the two different formulations of the "myopic" optimal hedge ratio problem, where myopia is a direct consequence of the single period specification of the optimization problem. The first formulation is based on the conventional approach that omits lending and borrowing from the portfolio decision, where (6.1) is the basis of the objective function. The second approach uses (6.2) thereby admitting portfolios that allow riskless lending and borrowing. In this analysis, "myopia" dictates that future time paths of the conditioning variables are ignored; the trade is initiated at time t and profits are taken at $t+1$. This permits use of the unconditional distributions. Given this, Proposition 6.1 extends the conventional constant distributional parameter solution to include a general expected utility function. It is shown that the optimal hedge ratio can be decomposed into the OLS-based hedge ratio (h_{OLS}) and a utility function dependent term. Proposition 6.2 incorporates riskless lending and borrowing to determine a market equilibrium hedge ratio that is shown to be independent of the utility function selected.

Conventionally, h_{OLS} has been the foundation of empirical estimation of hedge ratios. Hence, it is important to know the relationship between specific solutions to (6.3) and the OLS estimate. More precisely, the linkage between (6.3) and the minimum variance hedge ratio is given by the following Propositions (proofs for Proposition 6.1 and 6.2 are provided in Heaney and Poitras 1991):

Proposition 6.1: **Optimal Myopic Hedge Ratio**

Under the assumption of constant parameter bivariate normality of $R_s(t)$ and $R_f(t)$, the generalized optimal hedge ratio can be specified as:

$$h^* = h_{OLS} + \frac{E[R_f]}{var[R_f]} \frac{E[U'(\cdot)]}{W_t \, E[U''(\cdot)]}$$

where: $E[\cdot]$ is the (un)conditional expectation taken with respect to the joint density; U' and U'' are the first and second derivatives of the selected utility function with respect to π; $var[R_f]$ is the (un)conditional variance of R_f; h_{OLS} is from a regression of spot on futures prices.

In words, Proposition 6.1 demonstrates that, for myopic agents, the optimal hedge ratio can always be decomposed into a sum of the OLS-based hedge ratio and an additional term that is fully determined by statistical parameters and the risk aversion propensity of the selected utility function.

The primary upshot of Proposition 6.1 is that in addition to h_{OLS} consideration must be given to the risk aversion adjusted, variance-deflated expected return on the futures position. When the expected return is non-zero, the properties of the particular expected utility function assumed, i.e., the inverse of the coefficient of relative risk aversion, takes on importance. Examining the affect of the statistical parameters, an important *general* corollary follows: when the current futures price is an unbiased predictor of the distant futures price ($E[R_f] = 0$), h_{OLS} is optimal. Hence, results that apply for specific utility functions (e.g., Benninga, et al. 1984, Poitras 1989) can be generalized to any type of admissible utility function, albeit under the restriction of bivariate normality. However, for many commodities, $E[R_f] = 0$ is not observed in which case the issue of selecting an appropriate utility function is raised.

To better illustrate, consider some specific examples. Because of the normality assumption, if utility is taken to be negative exponential, $U = -exp\{-\alpha W\}$, this is equivalent to assuming mean-variance expected utility, a case for which a solution has already been provided using a slightly different approach. (See Question #5 at the end of the Chapter.) The resulting optimal solution reflects the different approaches:

$$h^*_{MV} = h_{OLS} - \frac{E[R_f]}{\alpha \; W_t \; var[R_f]}$$

This form of solution also emerges for other methods of generating mean-variance expected utility, such as quadratic utility where $U = \pi - \frac{1}{2} b(\pi - E[\pi])^2$. In order to contrast constant absolute and relative aversion utility functions, consider the power utility function, $U = (\pi^p / p)$, where $p < 1$. In this case:

$$h^*_{pow} = h_{OLS} - \frac{E[R_f]}{var[R_f]} \frac{E[\pi^{p-1}]}{W_t \; (1 - p) \; E[\pi^{p-2}]}$$

For the important specific power utility case of log utility, $U = ln(\pi)$, the solution reduces to:

$$h^*_{\ln} = h_{OLS} - \frac{E[R_f]}{var[R_f]} \frac{E[\pi^{-1}]}{W_t \; E[\pi^{-2}]}$$

From these results it follows that a given optimal hedge ratio depends on parameters of both the conditional distribution and the expected utility function.

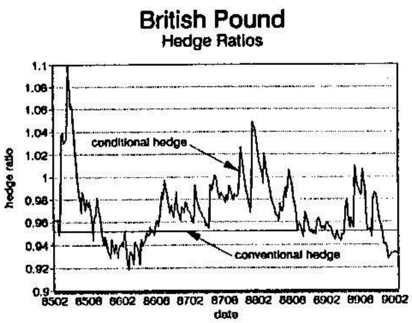
Significantly, Proposition 6.1 demonstrates that, ***when*** $\boldsymbol{E[R_f] \neq 0}$, it is not "optimal" to use OLS hedge ratios without making further assumptions about the return and profit generating processes and the form of expected utility. In practice, given specific distributions for the relevant processes, the h* of interest can be approximated with numerical methods (e.g., Cecchetti, et al. 1988). It is also possible to empirically model and estimate the conditional hedge ratio, e.g., Kroner and Sultan (1993). However, such applications typically result in a substantive increase in the complexity of the estimation problem. Unfortunately, it is not possible to establish, theoretically, whether there will be corresponding increases in the value of the resulting hedge ratio estimates. While there is related work on portfolio specification that would indicate there may be potential benefits (e.g., Grauer 1986), the value-added of more direct specification of the both the expected utility function and return generating processes is largely an unresolved empirical issue.

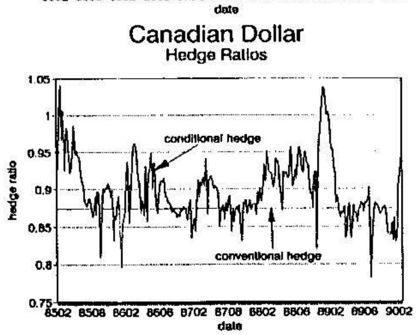
Proposition 6.2: **Quasi-Separation of the Hedge Ratio**

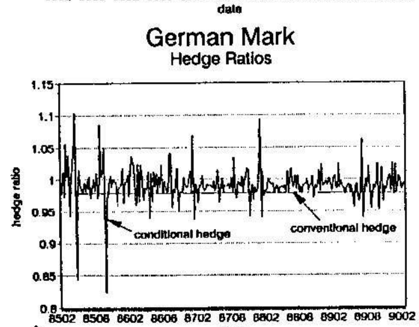
In the presence of riskless lending and borrowing, the myopic optimal hedge depends solely on expectations and other statistical parameters and is not affected by risk attitudes or initial wealth. Specifically the optimal hedge ratio is given by:

$$H^* = \frac{var[R_s]\,(\beta[F,S](E[R_s - r]) - E[R_f])}{var[R_f]\,((E[R_s - r]) - \beta[S,F]E[R_f])}$$

where: $\beta[S,F] = (cov[R_s,R_f]/var[R_f])$; $\beta[F,S] = (cov[R_s,R_f]/var[R_s])$.







Turning to the specification of the underlying optimization problem, within the myopic model the introduction of riskless lending and borrowing alters the objective function such that the wealth dynamics are now given by (6.2). Solution of the resulting optimization problem leads to Proposition 6.2. Because this result depends only on parameters of the joint (un)conditional distribution, the optimal hedge ratio with riskless borrowing and lending is independent of both the specification of the expected utility function and initial wealth.

While not immediately apparent, the relationship between Propositions 6.1 and 6.2 can be seen by evaluating h^* in Proposition 6.2 where $E[R_f] = 0$. In this case, $h^* = \beta[S,F] = h_{OLS}$. Significantly, as is the case without lending and borrowing, h_{OLS} is optimal when the current futures price is an unbiased predictor of the distant futures price. Hence, while it is not possible to provide a revealing closed form expression, the introduction of lending and borrowing into the hedger's optimization problem does not alter the general result that the optimal hedge ratio is decomposable into h_{OLS} and another term that depends on statistical parameters. However, admitting the ability to short-sell and leverage eliminates the need to consider the risk aversion properties of the selected utility function. The upshot is that, in practice, optimization problems based on (6.2) may produce more implementable solutions than those based on (6.1).

On a more critical level, as demonstrated by Cecchetti, et al. (1988), Poitras (1988), Sephton (1992), Kroner and Sultan (1993) and many others, hedge ratio estimation is complicated because both the means and the variance-covariance matrix of the relevant variables are not typically constant through time as required for ordinary least squares (OLS) regression.[4] In addition, because the minimum variance hedge ratio does not take the mean return on the hedge portfolio into account, regression-based hedge ratios will not necessarily be optimal for all types of hedger expected utility functions.

Theoretically correct estimation of hedge ratios suggests the use of sophisticated (e.g., ARCH) estimation procedures.[5] However, straight forward implement of ARCH assumes that only minimum variance solutions are relevant to the hedger. In an intertemporal model that permits changing means and variances, Heaney and

Poitras (1991) demonstrate that minimum variance techniques will only be valid for log utility. To date, there is little empirical evidence indicating that introducing more advanced estimation methods, such as ARCH techniques, substantially improves OLS-based hedge ratio estimates. On the contrary, Cecchetti et al. (p.21) present results for US Treasury bonds that the *profitability* of trading strategies based on OLS estimates are not substantially different from those based on hedge ratios derived from ARCH procedures. Similarly, while it is possible to incorporate information on the mean returns of the hedge portfolio into the estimation procedure, this also leads to considerably more complicated estimation procedures.

Figure 6.5 provides an example from Kroner and Sultan (1993) comparing OLS and ARCH hedge ratios for a number of currency hedges. The conventional hedge is the OLS estimate, which is constant over the sample. The conditional hedge is an ARCH estimate of the hedge ratio. The relevance of ARCH has been examined in various sources, e.g., Hsieh (1988). ARCH takes account of the conditional movement of volatility over the sample. Because a bivariate regression hedge ratio is determined as the ratio of a covariance between the dependent and independent variables divided by the variance of the independent variable, if these parameters change conditional as time evolves, the associated hedge ratio will also change. This is the basis for the conditional hedge that is reported in Figure 6.5. Unfortunately, evaluation of the relative performance of the ARCH and OLS hedge ratios is difficult. Following Leitch and Tanner (1991) comparing different hedge ratios using statistical criteria, e.g., by examining the minimum mean square forecast error, fails to account for the profitability of the hedge. To date, while there is a wealth of studies examining the estimation problem, there is still little direction on the most appropriate method of evaluating hedge performance.

Determining the Dynamic Hedge Ratio

Poitras and Heaney (1991) and others have extended the static framework to the dynamic case. This involves reformulation of the underlying the optimization problem (6.3). While of considerable practical interest, Propositions 6.1 and 6.2 are theoretically imprecise due to the assumption of myopia that allows expected utility of terminal wealth to be optimized without considering the entirety of lifetime consumption. This is an important theoretical development because it permits the future time paths of the conditioning variables to affect the optimization problem. As in the previous section, the results in this section feature a general expected utility function. In addition, conditional probability measures with state-dependent parameters are admitted. All the results depend on the assumption of conditional joint normality of the underlying returns. While it is possible to further generalize to include other types of joint distributions, this typically reduced the sharpness of the results.

The problem of maximizing the expected utility of lifetime consumption (not terminal wealth) can be achieved expediently by assuming additive separability of the utility function. Given this, at any time, t, the hedger's more general "intertemporal" optimization problem is:

$$J[W_t \, , \, X_t] \; = \max_{C_i \, , \; i=\{t,\ldots,T\}} E \left[\sum_{i\,=\,t}^{T} U[C_i] \; + \; D_T \; | \; X_t \right] \qquad (6.4)$$

where C_i is consumption in period i, and $D_T$is the bequest function for the terminal date T, e.g., Ingersoll (1987). The introduction of consumption into the intertemporal problem is dependent on using a different specification for the wealth dynamics. More precisely, W for the intertemporal case is:

$$W_{t+1} \; = \; (W_t \; - \; C_t) \left[1 \; + \; R_s(t{+}1) \; - \; h \; R_f(t{+}1) \right] \qquad (6.5)$$

In addition to incorporating consumption, the intertemporal problem involves conditional distributions that require the relevant state variables to be identified. While it is possible to be more general, for present purposes, the rate processes (R_s and R_f) are the only state variables considered. Given this, the general solution to the optimization problem must now incorporate compensation for the hedger's "nervousness" about future changes in the state variables.

Applying Bellman's dynamic programming principle (e.g., Malliaris and Brock, 1982), the dynamic generalization of Proposition 6.1 reveals the corresponding complications:

Proposition 6.3: **Optimal Intertemporal Hedge Ratio**

Under the assumption of conditional bivariate normality of $R_s(t)$ and $R_f(t)$, the generalized optimal hedge ratio using (6.4) and (6.5) can be specified as:

$$H_t^* = (1 + \gamma)\, h_{OLS}\big|_{x(t)=c} + \frac{E[R_f]}{var[R_f]}\,\frac{E[J_W(\cdot)]}{E[J_{WW}(\cdot)]'} + \frac{E[J_{Wf}]}{E[J_{WW}(\cdot)]'}$$

where:

$$\gamma = \frac{E[J_{Ws}]}{E[J_{WW}(\,\cdot\,)]'} \qquad E[J_{WW}(\,\cdot\,)]' = (W_t - C_t)\; E[J_{WW}(\,\cdot\,)]$$

and where c is the observed values on the state variables *(X)* up to and including t, and all expectations are taken conditionally on $X(t)$ with:

$$E\left[\frac{\partial J}{\partial W_{t+1}} \mid X(t)\right] \equiv J_W \qquad E\left[\frac{\partial^2 J}{\partial W_{t+1}{}^2} \mid X(t)\right] \equiv J_{WW}$$

$$E\left[\frac{\partial^2 J}{\partial W_{t+1}\,\partial R_{s,t+1}} \mid X(t)\right] \equiv J_{Ws} \qquad E\left[\frac{\partial^2 J}{\partial W_{t+1}\,\partial R_{f,t+1}} \mid X(t)\right] \equiv J_{Wf}$$

In addition to h_{ols} now being a conditional estimate, the role of preferences in the intertemporal optional hedge ratio is more complicated than in the myopic case. More significantly, h_{ols} is no longer generally optimal when $E[R_f] = 0$

Specifically, the additional preference-dependent terms arise from expected changes in the state variables affecting the marginal utility of wealth. In this situation, utility function selection takes on added importance.

Figure 6.6: **Profit Function for a Non-Cross Hedging Farmer**

DATE	**Cash Position**	**Futures Position**
t= 0	Plant with expectation of reaping $\hat{q}$ at $t = 0$ (based on price $S(0)$ and $f(K,L,Land)$	Short $\hat{q}$ at $F(0,t)$
t= 1	Reap $\tilde{q}(1)$ and sell at $S(1)$	Close out position with Long $\hat{q}$ at $F(1,T)$

The associated profit function can now be derived as:

$$\pi = \{\tilde{q}(1) - \hat{q}\}\, S(1) + \hat{q}\, \{S(1) - S(0) + F(0,T) - F(1,T)\}$$

The first term can be interpreted as the income gain or loss due to unexpected output changes. The final term is

For example, log utility posses the important simplifying property that $J_{ws} = J_{wf} = 0$, which allows the intertemporal solution to correspond directly to the myopic case, with the caveat of potential inequality of conditional and unconditional statistical parameters. Empirical estimation proceeds by assuming a specific form of conditional distribution, e.g., ARCH (Cecchetti et .al., 1988). Significantly, other important types of utility functions such as quadratic and power are not so well behaved. Estimation of intertemporal optimal hedge ratios for these types of functions can be problematic.[6]

Similar complications arise when riskless lending and borrowing is admitted. In this case, the wealth specification is

$$W_{t+1} = (W_t - C_t)\left[1 + (1 - x_t)\, r(t+1) + x_t\, R_s(t+1) - H_t\, R_f(t+1)\right]$$

In an intertemporal context this leads to the introduction of an additional conditioning (state) variable, r, which has to be taken into account. As in Proposition 6.3, the risk associated with the potential changes in the state variables must be compensated. This leads to an equilibrium condition for the generalized hedge ratio that has terms that involve $J_{ws,}$ $J_{wf,}$ and J_{wr} i.e., indirect utility function *(J)* terms appears. However, J_{ww} terms do not appear and, as a result, the hedge ratio does not depend on risk attitudes of the hedger. The exact expression for the optimal hedge ratio is quite complicated and not revealing and, as a result, is not given here though, as before, log utility provides an important simplification: because the $J_{ws,}$ $J_{wf,}$ and J_{wr}, terms are zero, the myopic results of Proposition 6.2 applies.

The Farmer's Hedging Problem

One of the limitations of the conventional optimal hedging model is that there is only one source of uncertainty that has to be hedged. For example, this is reflected in the assumption of a non-stochastic cash position. Another example occurs when hedging the domestic currency value of the return on a foreign stock, where both the exchange rate and the foreign stock price interact to determine the variable to be hedged. Hence, in a number of hedging situations, it is not practical to assume there is only source of uncertainty. This Section considers the case where both price and quantity to be hedged are random. This means that ***income*** and not price is the variable to be hedged. This problem is motivated by the stylized farming situation, at planting time the farmer must estimate the size of the future crop in order to determine the size of the futures hedge. More generally, this problem also applies to hedging situations where future inputs to the (uncertain) production process have to be estimated. For the farmer's problem, because Q_S is not fixed at $t = 0$, it must be estimated. At $t = 0$ the farmer plants with the objective of reaping $\hat{q}$ where $\hat{q}$ is generated by some production function, $f(K, L, Land)$, and offered for sale at some expected price, $E[S(1)]$. Even at this basic level, the complexities of the farmer's problem are apparent. To make the problem more manageable, a number of simplifying assumptions are required.

In the analysis of the minimum variance hedge ratio (see Sec. 2.1) it was shown that the relevant objective function was $EU = -var[\pi]$. This functional form can be compared to the mean-variance EU. Given the similarities in the mean-variance and minimum variance optimal hedge ratios, it is possible to rationalize the minimum variance approach by assuming that hedgers, in this case farmers, do not forecast spot prices. In effect, the price in the future is expected to be the same as it is today. This form of myopia permits use of the minimum variance hedge ratio by eliminating the need to consider expected price change. Another simplifying assumption is the requirement that the hedge position put on at t= 0 is held and not changed until harvest at $t = 1$. More formally, no dynamic hedge adjustment is permitted. Given these two assumptions, two situations can be considered: where $\hat{q} = Q_H = Q$, no cross hedging; and $\hat{q}$ and Q_H are allowed to differ, cross hedging. If farm output realized at $t = 1$ is taken to be $\tilde{q}$, then it is possible to define the profit function for the farmer doing a delivery hedge (where no cross hedge is involved).

Creating a new random variable $\tilde{q}(1)\, S(1) = Y(1)$, which effectively represents future farm income, it is possible to define an associated variance of profit function that can be used to solve for the minimum variance hedge ratio:

$$var[\pi] = \sigma_Y^2 + Q^2\, \sigma_F^2 - 2Q\, \sigma_{YF}$$

$$\frac{\partial var[\pi]}{\partial Q} = 2[Q\, \sigma_F^2 - \sigma_{YF}] = 0$$

$$Q^* = \frac{\sigma_{YF}}{\sigma_F^2}$$

While interesting, this form of the optimal hedge ratio has little practical value. Expanding the solution to allow for Q_H to differ from the expected size of the crop position does not substantially change the practicality of the solution. The primary analytical difficulty is the presence of a random variable, farm income, which is the product of two random variables. Only one of these variables, spot prices, is hedgable. With this in mind, it is possible to reconstruct the optimal problem into the form used in Proposition 6.1 and 6.2.

The basic model is discrete. Farmers have access to a variety of possible risk management instruments to hedge production decisions. The representative farmer plants a crop at time t and harvests it at time $t+1$. Both the price at harvest and the quantity harvested are unknown at time t, the date the relevant risk management and planting decisions are initiated. As conceived here, in addition to choosing the usage of hedging instrument(s), the farmer's optimization problem also involves choosing the amount of initial wealth to invest in crop production. Hence, the production decision is treated in a portfolio context. As a result, the costs associated with planting the given acreage are also determined. Starting from a given initial level of wealth, the farmer's objective is to maximize the value of terminal wealth assuming that the balance (possibly negative) of initial wealth that is not allocated to planting costs will earn (pay) the riskfree rate of interest.

Given this basic structure, it will initially be assumed that the only hedging instrument available is futures contracts. In this case, the underlying wealth dynamics can be specified:

$$W_{t+1} = A\, Y_{t+1} P_{t+1} + [W_t - C(A)]\,(1+r) + Q_f\,(f_{t+1} - f_t)$$

where: W_{t+1} is wealth at time $t+1$ and W_t is the known level of initial wealth; A is the number of acres planted; Y_{t+1} is the random yield per acre observed when the crop is harvested at $t+1$; P_{t+1} is the random spot price at $t+1$; $C(A)$ is the known cost function associated with planting the A acres; r is the riskfree interest rate; Q_f is the quantity of futures contracts sold (-) or bought (+); and f_{t+1} and f_t are the futures prices observed at $t+1$ and t respectively. Manipulation gives:

$$\begin{aligned} W_{t+1} &= W_t\,(x(1+R) + (1-x)(1+r) + HR_f) \\ &= W_t\,((1+r) + x(R-r) + HR_f) \\ &= W_t + \pi_{t+1} \end{aligned} \tag{6.6}$$

where: π_{t+1} is the profit realized at time $t+1$, x is $(C(A)/W_t)$ the fraction of initial wealth invested in the crop production, H is the value (f_t times Q_f) of the hedge position divided by initial wealth (*not the value of the spot position*), R_f is $(f_{t+1} - f_t)/f_t$ and $(1+R)$ is $[(A\ Y_{t+1}P_{t+1})/C(A)]$ one plus the rate of return on planting.

Given this, the farmer's optimal risk management decision problem is to choose x and H such that the expected utility of terminal wealth is maximized. The decision problem is modelled with a general expected utility function. However, in order to achieve analytically concise results, joint normality of R and R_f is invoked. This leads to the following:

Proposition 6.4: **The Crop Investment and Hedging Decision**

Assuming that the returns R and R_f are jointly normal random variables, and that the farmer chooses x and H so as to maximize the expected utility of terminal wealth given by (6.6) then:

$$\left\{\frac{H}{x}\right\}^* = \frac{\dfrac{E[R_f]}{\sigma^2_f} - \rho\dfrac{E[R]-r}{\sigma_R\sigma_f}}{\dfrac{E[R]-r}{\sigma^2_R} - \rho\dfrac{E[R_f]}{\sigma_R\sigma_f}}$$

where:

$$\rho = \frac{\sigma_{Rf}}{\sigma_R\sigma_f} \qquad \sigma_{Rf} = Cov(R,R_f) \qquad \sigma_f^2 = Var(R_f) \qquad \sigma_R^2 = Var(R)$$

and U is the farmer's utility function for wealth ($U' > 0$, $U'' < 0$).

Significantly, while the derivation of Proposition 6.4 reveals that the individual optimal solutions (denoted by *) to the farmer's risk management problem (x^*, H^*) depend on preferences, the ratio (H^*/x^*) *only involves statistical parameters*.

The portfolio-theoretic intuition behind the Proposition is as follows: the farmer faces two problems, one involving hedging, the other involving the scale of production. To determine the fraction of the crop to hedge, the farmer must solve a portfolio problem involving two risky "assets" with returns $(R - r)$ and R_f. From mean-variance portfolio theory, it is well known that if asset returns are jointly normal and riskless borrowing and lending is permitted then all investors, regardless of preferences, hold the same portfolio of risky assets. In addition, the ratio of any two assets in an optimal portfolio will be independent of risk preferences. Since the farmer's choice of the fraction of initial wealth to invest in crop production (x) is unconstrained, as long as returns are independent of the scale of production-- and the other assumptions relevant to Proposition 6.4 are satisfied-- (H^*/x^*) will not involve preferences.

When used to analyze (H^*/x^*), the practical implication of Proposition 6.4 is that the fraction of the investment in crop production to be hedged $(Q_f f_t/C(A))$ is independent of the size of the crop. Further, when the futures price is unbiased $(E[R_f] = 0)$, only joint normality of R and R_f is required to motivate ordinary least squares as the optimal hedge ratio estimation technique. Though similar types of conditions have been derived for related problems, e.g., Benninga, et al. (1984), Heaney and Poitras (1991), this result has not been recognized as applying to the farmer's hedging problem with both price and production uncertainty. On balance, Proposition 6.4 is significant because it establishes the connection between the results of portfolio theory and the farmer's hedging problem providing a motivation for the use of regression analysis to estimate the farmer's optimal hedge ratio.

6.2 Currency Hedging for International Activities

Transaction Hedging Example: Managing Currency Exposure

The final transaction hedging example being considered involves hedging currency exposure. As discussed previously, a fundamental difficulty of textbook discussions of hedging involves the assumption that a single transaction is involved. While this may work for the grain elevator or the hog farmer, assessing interest rate exposure for a financial institution or currency risk for a multinational firm is much more complicated. For example, financial institutions such as pension funds have a systemic exposure to changes in the level and shape of the term structure of interest rates due to a structural imbalance between the duration of liabilities and assets. Even if derivatives with sufficiently distant expiration dates were available, it is often not feasible to hedge each individual transaction due to the large number of implied trades. A similar problem faces financial institutions such as chartered banks that also have a large number of interest and exchange rate sensitive transactions. Despite this, there are numerous practical situations where interest rate and exchange rate hedges are treated as individual transactions. These simplified cases require the specifics of the hedge to be known at the time the hedge is initiated. Illustration of these cases are the objective of this Section.

Predictable foreign currency cash flows permit the use of a range of conventional hedging instruments associated with foreign asset/liability management to be applied. The procedures for implementing some of these techniques, such as currency swaps and options, will be discussed in later sections. The textbook use of a long currency futures hedge to cover a foreign currency cash flow is illustrated in Figure 6.7 from Chance (1998, p.557) where a US dealer is making a purchase in pounds for sale in dollars. The hedge requires that the amount of the cash flow and the payment date be known with certainty. The anticipated transaction is approximately fully hedged. The position is long because futures contracts are measured in US direct terms (US$/£) and profit will be positive if the (US$/£) increases over time. The hedger expects the pound to strengthen. In the example, the hedger was correct and the pound increased from $1.278 to $1.4375. The hedge was profitable and was able to more than offset the loss on the purchase of British sport cars. This was a best case scenario. It was also possible for the pound to weaken and for the hedge to lose money.

Figure 6.7 **A Long Transactions Hedge with Foreign Currency Futures**

Scenario: On July 1, an American auto dealer enters into a contract to purchase 20 British sports cars with payment to be made in British pounds on November 1. Each car will cost 35,000 pounds. The dealer is concerned that the pound will strengthen over the next few months causing the cars to cost more in dollars.

Date	Spot Market	Futures Market
July 1	The current exchange rate	December pound contract is at $1,278.

	is \$1.3190 per pound. The forward rate of the pound is \$1.3060. Forward cost of 20 cars: 20(35,000)(\$1.3060) = \$914,200.	Price per contract = 62,500 (\$1.278) = \$79,8875. The appropriate number of contracts is $\frac{20(35,000)}{62,500} = 11.2$ **Buy 11 contracts**
November 1	The spot rate is \$1.442 . Buy the 700,000 pounds to purchase 20 cars. Cost in dollars: 700,000(\$1.442) = \$1,009,400.	December pound contract is at \$1.4375. Price per contract: 62,500(\$1.4375) = \$89,843.75 **Sell 11 contracts**

Analysis:
The cars ended up costing \$1,009,400 - \$914,200 = \$95,200 more.

The profit on the futures transaction is

11 (\$89,843.75)	(sale price of futures)
<u>-11(\$79,875.00)</u>	(purchase price of futures)
\$109,656.25	(profit on futures)

The profit on the futures more than offsets the higher cost of the cars, leaving a net gain of \$109,656.25 - \$95,200 = \$14,456.25. The dealer effectively paid \$1,009,400 - \$109,656.25 = \$899,743.75 for the 20 cars.

Figure 6.8 illustrates the use of a short currency futures hedge. In this example, a transfer of funds from pounds into dollars at a future date is involved. Again, the details of the transaction are known with certainty when the hedge is initiated. The risk in this transaction is that the pound will weaken, the (\$US/£) will fall, and the number of dollars that can be purchased will be less in the future than at the time the hedge is being considered. Once again, a full hedge is established and the hedger is correct about the direction of the exchange rate change. The hedge was able to more than offset the loss on the spot position. Why did the futures gain and spot loss not fully cancel out in the full hedge? This has to do with the basis. On June 29, the date the hedge was initiated, the basis was F(0,T) - S(0) = \$1.375 - \$1.357 = .018. On Sept. 28, the date of the transfer and the end of the hedge, the basis was \$1.238 - \$1.2375 = .005. The difference of .013 times the size of the cash position, adjusted for the rounding error incorporated in the number futures contracts (394.76 vs. 395) used in the hedge, produces the appropriate excess value earned by the hedge. The bulk of the basis change is due to the impact of covered interest parity on the reducing maturity of the futures contract being used in the hedge.

Figure 6.8 **A Short Transactions Hedge with Foreign Currency Forwards**

***Scenario*:** On June 29, a multinational firm with a British subsidiary decides it will need to transfer 10 million pounds from an account in London to an account with a New York bank. Transfer will be made on September 28. The firm is concerned that over the next two months the pound will

Date	**Spot Market**	**Futures Market**
June 29	The current exchange rate is $1.362 per pound. The forward rate of the pound is $1.357. Forward value of funds: 10,000,000($1.357) = $13,570,000.	Sell pounds forward for delivery on September 28 at $1.357.
September 28	The spot rate is $1.2375.	Deliver pounds and receive 10,000,000($1.357) = $13,570,000.

Analysis:
The pounds end up worth $13,570,000 - $12,375,000 = $1,195,000 less but are delivered on the forward contract for $13,5
risk. Had the transaction not been done, the firm would have converted the pounds at the spot rate of $1.2375.

In the short and long currency futures hedges, expected profit on the hedge was achieved because the hedger was correctly able to predict the expected change in the spot exchange rate. For many types of international transactions, such as bidding on contracts denominated in foreign currency, other sources of risk also enter the hedging problem. For example, bidding on a construction contract where the contract payments and, possibly, the construction expenses are denominated in foreign currency involves the additional risk that the bid will be unsuccessful. If futures are used to hedge expected change in the currency between the time the bid is submitted and the time of the payments and expenses, and the bid is ***unsuccessful***, then the firm will face significant potential losses. For some firms, this would present a significant barrier to bidding on foreign contracts. Because the outcome of the bidding process is a contingency, it is decidedly more efficient to use options than futures to hedge the currency risk. Some of the advantages of options over futures in theses types of hedging situations are listed in Figure 6.7.

Figure 6.9 **Comparison of a Foreign Currency Forward Hedge and an Option Hedge for a Firm Undertaking a Hedge without Knowing whether a Bid will be Successful**

Assumption: A firm is bidding on a construction project in foreign country. There is a significant lead time between the time the bid is submitted and the results are announced. If the bid is successful, payment will be made in a fixed number of British pounds. The bulk of the firm's cash flows and balance sheet items are denominated in US dollars. The firm is seeking to design a transactions hedge which will protect the firm from changes in the £/US$ exchange rates in the period between the time the bid is submitted and the results are announced. The table below indicates the outcome from the forward or option contract but does not consider the profit from the construction itself.

Outcome of Bid	**No Hedge**	**Short Forward Hedge**	**Option Hedge (Buy Put)**
Successful:			

Pound increases	Gain on pound	Gain on pound reduced by hedge Small profit or loss	Put expires Premium lost
Pound decreases	Loss on pound	Loss on pound reduced by hedge Small profit or loss	Loss on pound reduced by exercise of put Small profit or loss
Unsuccessful:			
Pound increases	No effect	Potentially large loss on pound	Put expires Premium lost
Pound decreases	No effect	Potentially large gain on pound	Potentially large gain on pound by exercise of put

When is Hedging Foreign Assets Effective?

Textbook presentations of currency hedging indicate that a hedged foreign asset position will have a less variable return (cash flow) than an unhedged foreign asset. This intuition is, unfortunately, not always correct. To see this, consider the calculation for the return on a domestic asset, R:

$$R = (P_1 - P_0 + Div.)/P_0 = [(P_1 - P_0)/P_0] + [Div./P_0]$$
$$= [P_1/P_0] + [Div./P_0] - 1 = \text{Capital Gain (Loss)} + \text{Dividend (or Coupon) Yield:}$$

The return calculation can be contrasted with the domestic currency return on a foreign asset. Showing the distinction requires some notation for: the domestic currency return on a foreign asset position, $R_\$$; the return on a foreign asset denominated in foreign currency terms is R_f; e is the growth rate of the currency, *div* is the single dividend that is known to be paid at $t= 1$, P is the asset price in foreign currency terms and S is the spot exchange rate:

$$1 + R_\$ = 1 + \frac{[P_1 + Div_1]S_1 - P_0 S_0}{P_0 S_0} = \frac{P_1 + Div_1}{P_0}\frac{S_1}{S_0} = [1 + R_f][1 + e]$$

In effect, the stock return can be decomposed into the returns associated with local factors, R_f, and currency changes, e.

In order to evaluate the behaviour of hedged portfolios, an appropriate specification of the hedging rule is required. A naive specification is to assume all currency risk is "fully hedged". In practical applications, this requires determining at $t= 0$ a hedge amount. To see how this is done, consider the following:

$$1 + R_H = 1 + \frac{[P_1 + div_1]F(0,1) - P_0 S_0}{P_0 S_0}$$

$$= \frac{P_1 + div_1}{P_0}\frac{F(0,1)}{S_0} = [1 + R_f][1 + fp]$$

where R_H is the return on the fully hedged foreign asset position. The fully hedged US dollar return can then be calculated from the observed domestic currency return and the forward premium (*fp*). Hence, the hedged US dollar return on, say, a Japanese government bond will be higher than the Japanese yen denominated return by the amount of the forward premium, where:

$$fp = \frac{F(0,1)}{S_0} = \left[\frac{1 + r(0,1)}{1 + r*(0,1)}\right]$$

This condition follows from covered interest arbitrage.

It is now possible to demonstrate that hedging foreign assets will not necessarily reduce the variability of the asset' s cash flow, e.g., Benari (1991). This requires a direct comparision of $[R_\$]$ and $var[R_H]$. For expedience, it is useful to use the approximation $ln\{1+ r\} = r$. Given this, then $var[R_\$] = var[R_f] + var[e] + 2\ cov[R_f, e]$. Assuming that the hedge size can be completely determined at $t= 0$, then application of the log approximation gives: $var[R_H] = var[R_f]$. This follows because the forward premium is known at $t = 0$. Given this, what is now required is the conditions for which $var[R_\$] < var[R_H]$ or, in words, the variance of the hedged return is larger than the unhedged return. Using $\rho\ \sigma_f\ \sigma_e = cov[R_f, e]$ gives a result that can be referred to as Benari's condition:

$$\rho < -\frac{\sigma_e}{2\sigma_f}$$

As stated, if Benari's condition is satisfied, then the hedged return will be more volatile, i.e., if the observed conditional correlation between the return on the foreign asset and the foreign exchange rate is sufficiently negative, then hedging can increase risk.

While it might seem intuitively unlikely for a hedged asset position to have a more volatile cash flow than an unhedged position, Tables 6.4 and 6.5, adapted from Madura and Tucker (1992), confirm that Benari's condition may hold in practice. Table 6.4 provides the relevant hedged and unhedged returns for three monthly sample periods: a weak dollar from Jan. 1985-July 1987; a strong dollar period from 1981-1984; and, a Aug. 1987-Nov.1988 sample impacted by the October 1987 market crash. The benchmark portfolio is an equally weighted collection of the market indices for the seven countries listed. Significantly, Madura and Tucker find that the results for the crash period indicate: "during the period surrounding the 1987 world stock market crash, international portfolios that were hedged with forward contracts exhibited more total risk than those that were not hedged" (Madura and Tucker 1992).

Tables 6.4 and 6.5 **Hedging and the Crash of 1987**

TABLE 7.4
Intertemporal Comparison
of Hedged and Unhedged Portfolios

MONTHLY RETURNS / EQUAL WEIGHTS

Subperiod	Unhedged Portfolios: Mean Return	Unhedged Portfolios: Standard Deviation	Unhedged Portfolios: Coefficient of Variation	Hedged Portfolios: Mean Return	Hedged Portfolios: Standard Deviation	Hedged Portfolios: Coefficient of Variation
1. 1981–1984 (Strong $)	0.174%	3.924%	22.55	1.049%	3.052%	2.91
2. Jan. '85–Jul '87 (Weak $)	3.330	4.011	1.20	2.178	3.302	1.52
3. Aug. '87–Nov '88	0.133	6.313	47.46	−0.004	6.881	NA

TABLE 7.5
Market Risk Characteristics

Stock Market	Sub-Period	Variance of Stock Market Move-ments	Variance of Exchange Rate Move-ments	Covariance Between Stock Market and Exchange Rate Move-ments	Correlation Between Market and Exchange Rate Move-ments	Variance of Exchange Rate-Adjusted Stock Market Move-ments
Canada	1	31.69	1.53	3.10	0.45	39.43
	2	13.39	2.01	1.90	0.34	19.21
	3	47.05	1.96	1.90	0.19	52.81
France	1	38.81	11.28	1.20	0.05	52.50
	2	40.57	13.76	−2.50	−0.09	49.34
	3	101.60	11.69	−11.00	−0.32	91.30
Germany	1	17.64	10.95	0.76	0.05	30.11
	2	50.41	15.81	−3.20	−0.08	59.85
	3	86.67	13.91	−19.00	−0.54	62.58
Japan	1	20.79	14.13	4.70	0.25	44.33
	2	30.14	14.06	2.90	0.14	50.00
	3	38.31	20.61	−11.00	−0.36	36.92
Switzerland	1	10.30	12.60	1.80	0.15	26.50
	2	21.99	20.07	−7.30	−0.35	27.46
	3	68.39	17.72	−24.00	−0.70	38.11
United Kingdom	1	19.09	8.41	−0.30	−0.02	26.90
	2	21.71	16.16	5.10	0.27	48.07
	3	72.76	16.48	−12.00	−0.37	65.24
United States	1	14.13	--	--	--	14.13
	2	15.52	--	--	--	15.52
	3	49.98	--	--	--	49.98
Average Across Markets	1	21.77	9.82	1.88	0.16	33.41
	2	27.67	13.64	−0.52	0.04	38.49
	3	66.39	13.73	−12.52	−0.35	56.71

Table 6.6 and 6.7 **Decomposition of the Variance of the US$ and Yen Currency Return for Six Foreign Equity Indices**

Table 3 **Decomposition of the Variance of Bond/Stock Returns in U.S. Dollars[a] (Monthly Data: 1978.1–89.12)**

		Components of Var($R_{i\$}$)			
	Var($R_{i\$}$)	Var(R_i)	Var(e_i)	2 Cov(R_i, e_i)	Δ Var
Bonds					
Canada	15.29	10.82	1.72	2.67	0.08
France	16.48	2.82	12.74	0.60	0.32
Germany	21.53	2.59	13.84	4.91	0.19
Japan	24.70	3.03	15.13	6.09	0.45
Switzer.	21.16	1.14	17.64	2.34	0.04
U.K.	27.67	8.88	12.39	6.08	0.32
U.S.	10.24	10.24	0.00	0.00	0.00
Stocks					
Canada	37.70	30.58	1.72	5.37	0.03
France	59.75	43.03	12.74	3.75	0.23
Germany	43.82	29.27	13.84	0.00	0.71
Japan	41.47	19.45	15.13	5.83	1.06
Switzer.	34.81	20.07	17.64	−3.76	0.86
U.K.	40.96	29.27	12.39	−1.52	0.82
U.S.	21.16	21.16	0.00	0.00	0.00

[a] The variances are computed using monthly percentage returns.

Table 4 **Decomposition of the Variance of Bond/Stock Returns in Japanese Yen[a] (Monthly Data: 1978.1–89.12)**

		Components of Var($R_{i¥}$)			
	Var($R_{i¥}$)	Var(R_i)	Var(e_i)	2 Cov(R_i, e_i)	Δ Var
Bonds					
Canada	24.90	10.82	14.75	−1.01	0.34
France	10.43	2.82	8.58	−1.08	0.11
Germany	14.52	2.59	9.86	2.02	0.05
Japan	3.03	3.03	0.00	0.00	0.00
Switzer.	12.25	1.14	10.43	0.69	−0.01
U.K.	22.85	8.88	11.42	2.93	−0.38
U.S.	21.16	10.24	14.59	−3.91	0.24
Stocks					
Canada	51.27	30.58	14.75	5.96	−0.02
France	50.84	43.03	8.58	−0.38	−0.39
Germany	40.45	29.27	9.86	0.68	0.64
Japan	19.45	19.45	0.00	0.00	0.00
Switzer.	32.04	20.07	10.43	1.16	0.38
U.K.	42.90	29.27	11.42	2.45	0.24
U.S.	37.58	21.16	14.59	2.11	−0.28

[a] The variances are computed using monthly percentage returns.

Table 6.5 contains information required for evaluation of Benari's condition. Examining, say, Japan in period 3 gives -.32 < -{√20.61}/2{√38.31}, which indicates the condition is not satisfied, but only just. Hence, hedging the currency risk of Japanese stocks would provide virtually no risk reduction benefit. Germany, Switzerland, France and the UK all satisfy the condition for the crash period, meaning that hedging these international stock groups during the crash would have increased volatility. In effect, in an international context, the risk reduction benefits of hedging depend on the correlation between stock market and exchange rate movements. The October 1987 crash was a stock market contagion triggered by an information event that had strong negative implications for the US dollar. This contagion spread from the US to international stock markets, which experienced severe stock market declines at the same time there was marginal strengthening of their currencies.

Tables 6.4 and 6.5 illustrate that Benari's condition is not satisfied in most periods. Currency hedging does typically reduce return volatility. This much is apparent from a casual inspection of the condition. When the correlation is positive, Benari's condition is automatically satisfied. Even if the correlation is negative, hedging is still risk reducing if the variability of the exchange rate is sufficiently greater than that of the foreign stock market. Because it is not possible for $\rho < -1$, it follows that if $\sigma_e > 2\sigma_f$ then the condition can not be satisfied. On balance, Benari's condition serves to emphasize the importance of selective hedging. Following the analysis of Sec. 2.1, selective hedging should be used unless the investor is: unable to forecast effectively; the exchange rate forecast indicates that $F_{0,1} = E[S]$; or, there is a high degree of investor risk aversion (*b* large).

Some of the issues raised by Benari's condition are quite subtle and require further elaboration. Benari's basic result is a special case of a more general result: when using one derivative to hedge the product of two random variables, even if the derivative has no basis risk with one of the variables, then an unhedged position could have less cash flow volatility than a hedged position. This can happen when the correlation between the two random variables is negative and the derivative contract is written on the variable that has the least volatility. It is possible to generalize this result further to where *n* derivatives are hedging *k* random variables $(n > k)$. Madura and Tucker use an *ex post* conditional empirical analysis to demonstrate that Benari's condition did apply during the year surrounding the Oct. 1987 stock market crash. All this raises the question: how can this result be used to facilitate *ex ante* hedging decisions?

The distinction between *ex ante* and *ex post* analysis is fundamental to the analysis of risk management tactics and strategies. It is one thing to show that, *ex post*, a particular strategy was optimal, it is quite another for that strategy to translate *ex post* into *ex ante* performance. This is a fundamental criticism of the Markowitz mean-variance portfolio optimization model when the investment universe includes both domestic and foreign assets. The *ex post* optimality does not seem to translate into *ex ante* performance, e.g., Jorion (1985), Eun and Resnick (1994), Wilcox (2000). In the absence of some method of accurately predicting changes in the relevant parameters, Benari's condition would have to rely on unconditional parameter estimates, obtained from a specific sample. Tables 6.6 and 6.7 report some relevant estimates from Eun and Resnick (1994) for seven countries using monthly from 1978:1 to 1989:12. Of particular interest, Eun and Resnick report the results using two different reference currencies, US dollars and Japanese yen. Both intermediate term bond returns and stock market index returns are examined.

The first column in Tables 6.6 and 6.7 give the total variance of the monthly asset return, where the return is denominated in the reference currency. The next four columns fully decompose this variance into its components. Recalling that $(1 + R_\$) = (1 + R_£)(1 + e) = 1 + R_£ + e + R_£ e$, the last column is associated with the cross product term that was ignored above by assuming that $R_\$ = R_£ + e$. The fourth column of Tables 6.6 and 6.7. measures the contribution of the cross product term. The insignificant numbers in this column confirms the validity of the approximation, at least for monthly returns. Whether this conclusion would change for quarterly or annual returns is another matter. This term is small because the cross product is small relative to the levels. Larger differencing intervals could produce some pairs of large returns that could alter this result for specific countries.

In colum three, Tables 6.6 and 6.7 directly address the possibility of negative correlations, which is necessary for Benari's condition to apply. Few negative values are observed. There were two cases where the negative values were large enough to be of interest, US$ bond returns with yen as the reference currency and Swiss stock returns with US$ as the reference currency. In both these cases, the volatility of exchange rates was larger than the volatility of local returns, another violation of a necessary condition. In total, there was only one of the twenty-four variances where two of the necessary conditions were satisfied, i.e., negative covariance and exchange rate volatility less than one half the local return volatility. That case was French stocks with yen as the reference currency. Yet, this case had the lowest covariance of the six negative covariances that were observed. Benari’s condition is far from being satisfied in the case.

The Optimal Hedge Ratio for a Single Foreign Asset

In deriving Benari's condition, the following assumption was used: the currency risk of the foreign asset could be fully hedged at $t=0$. In practice, it is not always possible to determine the terminal payoff on the foreign asset at $t=0$. In other words, the exact size of the foreign currency hedge will be indeterminate because the precise payoff on the foreign asset at $t=1$ will be unknown when the hedge is initially established. Though it may be possible to start the hedge at $t=0$ market value and sequentially increment the hedge at discrete intervals, such a strategy would be path dependent and would give an uncertain outcome. Another possible approach would be to set the size of the hedge position equal to the expected value of the position at the end of the hedge horizon. The success of this approach would depend on the accuracy of the estimate of future values.

Eun and Resnick (1994, p.147) examine this point in more detail. Consider the return on a hedged foreign asset position, where the size of the hedge position is determined by estimating the value of the position at the end of the investment horizon. Unexpected gains or losses are left uncovered, to be converted back at S_1. Letting R_{eh} be the return to a hedged foreign asset where the size of the hedge position is established by estimating the value of the position, it follows that:

$$1 + R_{eh} = (1 + E[R_f])(1 + fp) + (R_f - E[R_f])\, S_1$$

This result is derived from the payoff on the hedged position:

$$1 + R_{eh} = \frac{E[P_1 + Div_1]}{P_0\, S_0}\, F(0,1) + \frac{P_1 + Div_1 - E[P_1 + Div_1]}{P_0\, S_0}\, S_1$$

Manipulation produces:

$$R_{eh} = R_f + fp + e\, R_f + (fp - e)\, E[R_f] \cong R_f + e + (fp - e)\, E[R_f]$$

$$= R_\$ + (fp - e)\, E[R_f]$$

R_{eh} - $R_\$$ depends on *(fp - e)*. Hence, if forward exchange rates are unbiased predictors of future spot interest rates, then establishing the size of the hedge position by estimating the value of the asset at the end of the investment horizon will produce much the same result as for the stylized full hedging problem.

Figure 6.10 **Profit Profile for an Optimal Currency Futures Hedge**

Assume: One unit of the foreign asset is being purchased; hedge position is constructed using a contract that matures at the end of the investment horizon

Date	***Cash***	***Futures***
t= 0	Convert at S_0 and buy the foreign asset at P_0	Short $h\ P_0$ of the foreign currency at $F(0,T)$
t= 1	Sell the asset at P_1, receive dividend of Div_1 and convert back to domestic currency at S_1	Go long h P_0 at $F(1,T)$

The profit function for this trade can be now stated as:

$$\pi(1) = (P_1 + Div_1)\, S_1 - P_0\, S_0 + h\, P_0\, (F(0,T) - F(1,T))$$

The full hedging concept is useful in developing certain basic properties of currency hedges, such as the free lunch argument of Perold and Schulman (1988). However, the full (or transactions) hedge ignores the possibility that assuming a fully hedged position is consistent with the best method of determining the hedge position. Armed with this observation, it is possible to proceed to the more difficult question of determining the optimal currency hedge ratio for a portfolio containing a single foreign asset. To accomplish this, let h be the fraction of the value of the foreign asset position (P_0) that is being hedged. With the value of the hedge being determined as $h\ \{P_0\ F(0,T)\}$. Figure 6.10 gives the profit profile for the optimal hedge. Some presentations of the profit function, e.g., Glen and Jorion (1994), use forward contracts where delivery takes place at $t= 1$, using the proceeds from the foreign asset to settle the forward position.

Using R_{oh} for the return on the optimal hedged position, it follows that:

$$\frac{\pi(1)}{P_0\ S_0} = R_{oh} = \frac{(P_1 + Div_1)S_1}{P_0\ S_0} - 1 + h\, \frac{P_0\ \Delta F}{P_0\ S_0}$$

$$= (1 + R_f)\ (1 + e) - 1 - h\, \frac{\Delta F}{S_0} \cong R_f + e - h\frac{\Delta F}{S_0}$$

From this the variance can be determined and df is defined:

$$var[R_{oh}] = var[R_\$] + h^2\ var[\Delta F/S] - 2\ h\ cov[R_\$,\ \Delta F/S]$$

$$\equiv var[R_\$] + h^2\ var[df] - 2\ h\ cov[R_\$,\ df]$$

With this it is now possible to determine the optimal hedge ratio using the minimum variance solution, e.g., Eaker et al. (1993). The size of the hedge position (h^*) is the choice variable:

$$\frac{d\ var[R_{oh}]}{d\ h} = 2h\ var[df] - 2\ cov[R_\$,\ df] = 0 \quad \rightarrow \quad h^* = \frac{cov[R_\$,\ df]}{var[df]}$$

By observing that the covariance term can be further expanded as:

$$R_\$ \cong R_f + e \quad \rightarrow \quad cov[R_\$, df] = cov[R_f, e] - h\ cov[e, df] - h\ cov[R_f, df]$$

With this result the minimum variance hedge ratio can be expressed:

$$h^* = \frac{cov[e,\ df]}{var[df]} + \frac{cov[R_f,\ df]}{var[df]}$$

Eaker et al. (1993) provide selected empirical estimates for this form of the optimal currency hedge ratio.

Closer inspection of the minimum variance hedge ratio provides some useful information. Consider the term *cov[e,df]/var[df]:*

$$\frac{cov[e,\ df]}{var[df]} = \frac{cov[(S_1 - S_0)/S_0\ ,\ (F(1,1) - F(0,1))/S_0]}{var[F(1,1) - F(0,1)/S_0]}$$

Substituting the value of F from covered interest arbitrage reveals that this term will be close to one. If changes in the local asset return, R_f, are uncorrelated with changes in the forward exchange rate, an empirically plausible assumption, then the optimal currency hedge ratio for a single foreign asset will be close to one. Hence, the conditions under which full hedging is optimal may be empirically valid. Unfortunately, this relatively sharp result only applies to the restricted case of hedging a single foreign asset. Given this, it is natural to consider extending the analysis to allow for two assets, one domestic and one foreign.

Optimal Hedge Ratio for the Domestic/Foreign Portfolio[7]

The single foreign asset case is pedagogically interesting because it allows the general principles of optimal hedging for single transactions to be illustrated. Yet, in practice, the problem of optimal hedging for foreign assets is often considerably more complicated, if only because foreign assets are typically part of a larger portfolio that also contains domestic assets. Extending the portfolio to include a domestic asset and a foreign asset follows naturally. As in the single asset case, the problem is to determine the optimal size of the currency hedge, where optimal is defined using a minimum variance objective function. The reformulation requires the use of value weights, W_d and W_f, the fractions of the total market value of the portfolio invested in the domestic and foreign asset respectively. The budget constraint requires: $W_d + W_f = 1$.

Using the Eaker et al. (1993) approach, substitution of the budget constraint produces:

$$R_{p2} = (1 - W_f)\ R_d + W_f\ (R_f + e - h\ df)$$

The variance follows appropriately:

$$var[R_{p2}] = (1 - W_f)^2\ \sigma_d^2 + W_f^2\ (\sigma_f^2 + \sigma_e^2 + h^2\ \sigma_F^2)$$

$$+\ 2\left[(1 - W_f)\ W_f\ [\sigma_{fd} + \sigma_{ed} - h\ \sigma_{Fd}] + W_f^2\ (\sigma_{fe} - h\ \sigma_{Ff} - h\ \sigma_{Fe})\right]$$

where for ease of notation, $F = df$ has been used in the subscripts. The minimization problem produces the solution, e.g., Filatov and Rappoport (1992):

$$h^* = \frac{W_f}{1 - W_f}\frac{\sigma_{Fd}}{\sigma_F^2} + \frac{\sigma_{Ff} + \sigma_{Fe}}{\sigma_F^2} \cong 1 + \frac{W_f}{1 - W_f}\frac{\sigma_{Fd}}{\sigma_F^2} + \frac{\sigma_{Ff}}{\sigma_F^2}$$

This implies the following approach to determining the optimal currency hedge ratio. Determine (σ_{Fd}/σ_F^2) and (σ_{Ff}/σ_F^2) by regressing the foreign and domestic returns on $df = (\Delta F/S)$. Starting from a fully hedged position $(h = 1)$, increment the hedge by taking the values of the regression coefficients into account, adjusting the coefficient from the domestic return regression by the relative size of the domestic asset in the portfolio.

The domestic/foreign asset case immediately suggests the next step in the analysis of risk management decision problems involving foreign currency risk: solving for the optimal weight W_f*. More precisely, it is immediately possible to solve for the minimum variance solution. Following Filatov and Rappaport (1992), the following reformation of R_{p2} is possible:

$$R_{p2} \cong (1 - W_f)\, R_d + W_f\, (R_f + (1 - h)\, e + h\, fp)$$

It follows that:

$$(1 - W_f)^* = \frac{\sigma_f^2 - \sigma_{fd} + (1 - h)\left[(1 - h)\, \sigma_e^2 + 2\, \sigma_{fe} - \sigma_{ed}\right]}{\sigma_d^2 + \sigma_f^2 + (1 - h)^2 - 2\left[\sigma_{fd} + (1 - h)\, \sigma_{ed} - (1 - h)\, \sigma_{fe}\right]}$$

When the $h = 1$, the solution reduces to the minimum variance solution familiar from basic financial economics:

$$(1 - W_f)^{**} = \frac{\sigma_f^2 - \sigma_{fd}}{\sigma_d^2 + \sigma_f^2 - 2\, \sigma_{fd}}$$

Comparing this solution with the $h = 0$ case is useful for revealing the intuition behind the impact of hedging on diversified portfolios.

Figure 6.11 **An Example of the Diversification Benefits of a Domestic/Foreign Portfolio**

Question: You are considering purchasing two portfolios. One portfolio is composed 50/50 of two domestic assets each with $E[R] = .1$ and $\sigma = .15$ and with a .5 correlation between the asset returns. The other portfolio is also 50/50 and contains one of these domestic assets and a foreign asset. The foreign asset has $E[R_\$] = .1$ with $\sigma_£ = .15$ and $\sigma_e = .03$. The correlations between the foreign and domestic asset returns and between all the asset returns and the exchange rate are zero.

Solution: The portfolio variance for the domestic assets is just the conventional result. To get the portfolio variance when there is a foreign asset observ that the return on a foreign asset when the return in denominated in domestic currency $(R_\$)$ is given as: $R_\$ = (1 + R_£)(1 + e) - 1$. Taking logs and observing $\ln(1 + x)$ is approximately equal to x when x is sufficiently small produces the result:

$$var[R_\$] = \sigma_\$^2 = \sigma_£^2 + \sigma_e^2 + 2\, \sigma_{£,\, e}$$

Using the variance formula for two securities and doing appropriate substitutions, it follows that for a portfolio containing a foreign asset:

$$\sigma_p^2 = W_d^2\, \sigma_d^2 + W_\$^2\, \sigma_\$^2 + 2\, W_d\, W_\$\, \sigma_{d,\$}$$

$$= W_d^2\, \sigma_d^2 + W_\$^2\, \{\sigma_£^2 + \sigma_e^2 + 2\, \sigma_{£,e}\} + 2\, \{W_\$\, W_d\, (\sigma_{£,d} + \sigma_{e,d})\}$$

Evaluating the relevant formulas gives for the domestic portfolio $var[R_{dp}] = .016875$ ($\sigma_{dp} = .13$) and $var[R_{\$dp}] = .011475$ ($\sigma_{\$dp} = .107121$)

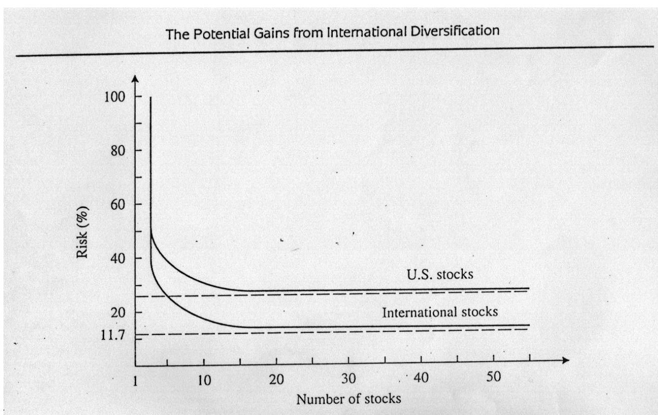
b) Due to the much lower correlation between domestic asset returns and foreign asset returns and the exchange rate (than with other domestic asset returns) including foreign assets enhances the diversification process considerably.

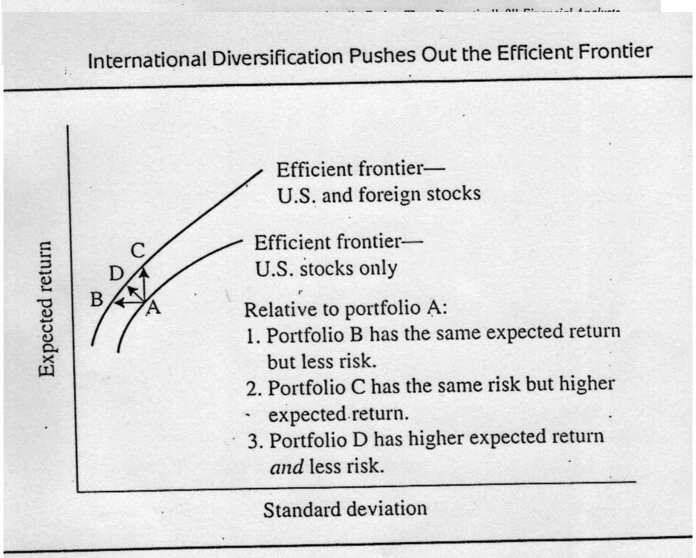
6.3 Mean Variance Analysis and Optimal International Diversification

Benefits of International Diversification

Going back to Grubel (1968) and Levy and Sarnat (1970), numerous writers have demonstrated the potential benefits to international diversification of investment portfolios, e.g., Solnik (1974), Lessard (1976), Grauer and Hakansson (1987), Burik and Ennis (1990), Fosburg and Madura (1991), Eaker, et.al. (1991), Jorion and (1999). What the early writers on this subject demonstrated was that the introduction of international securities into portfolios composed solely of domestic securities results in a significant outward movement in the *ex post* efficient frontier, as depicted in Graph 6.1. In effect, when international diversification is permitted it is possible to form portfolios with higher expected returns for the same level of risk. Another method of presenting this information is provided in Graph 6.2. It is possible to achieve a lower level of systematic risk than is achievable with domestic securities alone.

The diversification advantages of forming international portfolios arises due to low correlation of the returns for securities denominated in different currencies. By construction, it would seem that the greatest diversification benefits would require that the investor accept foreign exchange exposure. Upon closer inspection, many questions are raised by the results on the benefits of international diversification. For example, what are the implications of including hedged, as opposed to unhedged, foreign securities? Can currency risk be eliminated by fully hedging? How much diversification is necessary to achieve the bulk of the risk reduction benefits? What type of asset mix between the various countries and assets, i.e., stocks and bonds, is indicated? What are the implications of allowing short sales? Is a riskless asset permitted and, if so, is leveraging using the riskless asset permitted? Does the selection of the reference currency, which is used to denominate the return, matter? At present, there is only a limited amount of empirical information available to answer such questions.





In order to make a beginning toward addressing these types of questions, further information about the decomposition of the risk of foreign securities into the currency and local components is needed. Ignoring for the moment that the efficient frontier may contain other securities in addition to unhedged common stocks, consider the evidence provided by Fosberg and Madura (1990) for a sample of 67 non-US stocks, from France (7), Japan (24), Switzerland (7), UK (14) and West Germany (15). Using quarterly US$ returns over 1976-1989, the variance of equally weighted, unhedged, randomly selected portfolios is calculated for different numbers of stocks. The variance is calculated using the familiar Taylor series approximation that $ln\{1+R\} = R$, which reduces the portfolio return to $R_\$ = R_f + e$. The resulting variance relationship is $var[R_\$] = var[R_r] + var[e] + 2\,cov[R_f, e]$. The following results are given:

Portfolio	**Aver.**	**Aver.**	**var[$R_\$$] ÷**

Size	**var[$R_\$$]**	**var[R_f]**	**var[R_f]**
1	.1023	.0851	83.2%
2	.0525	.0425	81.0
4	.0373	.0280	75.1
6	.0200	.0144	72.0
8	.0154	.0103	66.9
10	.0164	.0111	67.7
12	.0139	.0085	61.2
16	.0101	.0059	58.4

The first average var column indicates that the benefits of diversification are substantial. The average variance of the portfolio with 16 stocks is less than 1/10 of the average variance of a portfolio with only one stock. Close to this amount of variance reduction can be achieved with only half that number of stocks. This rapid decrease in portfolio variance is consistent with the results of previous studies, e.g., Solnik (1974).

These diversification benefits arise from stock returns being not as highly correlated across countries when compared to correlations between domestic stocks. Evidence of this for stocks has been provided in various sources, at least since Ibbotson, et.al.(1982). An important practical implication is that only a relatively small number of stocks is required to achieve significant diversification. Recalling that the middle average var column captures the variance of the portfolio where stock prices are measured in their home currency, the final column provides an estimate of the total portfolio risk attributed solely to stock market movements. Subtracting this value from 100% gives the fraction of total portfolio risk originating from a combination of pure currency fluctuation and the covariance between local returns and the exchange rate. The evidence presented indicates that, for well diversified portfolios, at least 40% of the returns of unhedged portfolios can be attributed to currency related movements. As the stated results are based on unhedged common stock return, this raises practical questions about whether this result also applies to other asset specifications, such as foreign bond holdings. Answering such questions is facilitated by introducing the Markowitz model framework.

The Markowitz Model

The Markowitz model is a central paradigm of modern Finance. The essence of the model is captured in the following optimization problem, e.g., Elton and Gruber (1995), Luenberger ():[8]

$$\min_{\{W_i\}} \quad var[R_p] = \sum_{i=1}^{k} \sum_{j=1}^{k} W_i W_j \sigma_{i,j}$$

$$subject\ to: \quad E[R_p] = \sum_{i=1}^{k} W_i E[R_i] = \overline{c_n} \quad for \quad \overline{c_n} \in \{c_0, c_1, c_2, ...\} \quad where: \ c_0 = c_{mv}$$

$$and: \quad \sum_{i=1}^{k} W_i = 1$$

where: k is the number of risky assets available for investment; $E[R_i]$ is the (conditional) expected return on asset i; $E[R_p]$ is the expected return on the portfolio; and, $var[R_p]$ is the variance of portfolio return. In this model, the $\{W_i\}$ are the value weights, the fraction of the total value of the portfolio invested in each asset. For example, if there are four assets in the portfolio and the market value of each asset is equal, then the W_i for each asset is .25. It follows that the restriction requiring the sum of the weights equal to one is a budget constraint requiring that the sum of market values of all the assets held in the portfolio equals the total amount of money invested. The optimization problem is to determine the value weights for each asset that minimize the variance of the return on the portfolio, subject to a target level of expected return. Because there is range of possible expected returns that can be chosen, the solution to the optimization problem will be a set of portfolios, each with its own set of optimal weights.

The number of variations that have emerged from this basic model is staggering.[9] Initially, implementation of the model was impeded by the large number of parameters required to make the model operational. In addition to the k individual asset returns, $E[R_i]$, there are k variances, σ_i^2, and $\{k(k-1)\}/2$ covariances that have to be estimated from past data. Even if these parameters are available, the model is only capable of generating a set of mean-variance optimal portfolios, the ***efficient frontier***. Additional structure is needed to select a specific portfolio from the set of optimal portfolios. Sharpe and others handled this problem by introducing a riskless asset. This permits the investor to form portfolios that combine the riskless asset with an efficient frontier portfolio. In this fashion, the investor is able to achieve the same level of expected return as that generated by an efficient frontier portfolio, again with a lower level of risk. Effectively, the addition of a riskless asset transforms the investment opportunity set from a convex function, the efficient frontier, to a set of linear functions, the capital allocation lines.

By exploiting the properties of mean-variance expected utility, Sharpe was able to show that the efficient frontier portfolio that would be selected is that portfolio associated with the capital allocation line which is just tangent to the efficient frontier. This particular capital allocation line is identified as the ***capital market line***. This tangency portfolio can be determined by solving the following optimization problem, e.g., Eun and Resnick (1994):

$$\max_{\{W_i\}} \frac{E[R_p] - R_f}{\sigma_p} \quad \textit{subject to:} \quad \sum_{i=1}^{k} W_i = 1$$

On theoretical grounds, the tangency portfolio is the *ex ante* mean-variance-expected-utility optimizing risky portfolio.[10] Even though the precise combination of riskless asset and risky tangency portfolio for any given investor requires specification of the relevant parameters for the investor's mean variance expected utility function, the optimal risky portfolio has been determined.

Yet, despite the theoretical importance of the tangency portfolio, practical identification of the value weights for this portfolio requires *ex post* estimates of the relevant means, variances and covariances. Once the set of parameter estimates has been obtained and the optimality problem is solved, the resulting tangency portfolio will represent the optimal, *in-sample* portfolio. Whether this in-sample optimality translates into superior *out-of-sample* performance is an open question. The answer to this question becomes even more complex when foreign assets are admitted into the asset universe. In particular, the domestic currency return on a foreign asset depends on a combination two random variables: the return denominated in foreign currency terms; and, the change in the exchange rate. The correlation between foreign and domestic asset returns will tend to be lower than the correlations between domestic assets, making foreign assets excellent candidates for diversification. In addition, foreign assets can also provide the possibility of significantly higher returns than domestic assets.

The basic *ex post* returns and volatilities associated with hedged and unhedged international portfolios can be found in numerous sources, e.g., Burik and Ennis (1990). Unhedged common stocks are found to have a higher mean return, at the expense of higher total volatility. The "active" decision involved in hedging is captured when the dollar strengthens the hedge will be profitable, resulting in higher returns. However, in periods when the dollar is weakening, the hedge loses money, lowering return relative to an unhedged position. Consistent with the theoretical results of Sec. 2.3, this evidence confirms that the appropriate hedging decision should incorporate speculative factors, if possible, in order to improve hedge performance. Burik and Ennis also provide information on the composition (optimal value weights) of the portfolios that lie on the efficient frontier, i.e., the set of optimal portfolios that achieve the highest level of return for a given level of risk. For a wide range of expected return/standard deviation combinations, the bulk of the portfolio is composed of a short-term money market fund (STIF) and hedged foreign stocks. Only at high risk/return levels does portfolio composition shift to unhedged stocks. Unhedged foreign bonds are almost never held, with hedged foreign bonds playing a role over the intermediate risk/return combinations.

Criticism of Mean-Variance Portfolio Analysis

While the mean-variance rationalization for international diversification has considerable appeal, attempting to capture the gains, *out of sample*, has proved to be illusive. In practice, the use of ***ex post*** data to estimate the relevant ***ex ante*** parameters creates numerous problems, not the least of which is instability in both the mean and variance-covariance parameter estimates. This is especially the case where expected returns are of interest. As pointed out by Eaker, et.al. (1991): "The problem with including returns in the portfolio selection decision is that such portfolios generally perform poorly in out-of-sample tests." The first wave in the assault on the mean-variance approach can be credited to Jorion (1985, p.265), which describes the problems emphatically:

> Mean-variance analysis has serious shortcomings which are too often ignored ... Perhaps the most serious defect in the classical (portfolio) approach is the poor out-of-sample performance of the optimal portfolios. Performance measures always deteriorate substantially outside the sample period, and the supposedly optimal choice is sometimes dominated by a naive method....Another problem is the instability in the optimal portfolio: the proportions allocated to each asset are extremely sensitive to variations in expected returns, and adding a few observations may change the portfolio distribution completely. Also, optimal portfolios are not necessarily well diversified. Often a corner solution appears, where most of the investments are zero and large proportions are assigned to countries with relatively small capital markets and high average returns.

As it turns out, this attack is decidedly overstated. However, the basic point remains: *ex post* estimates of expected returns, based on arithmetic or weighted average estimators, are not reliable estimates of future returns. Relative to estimates of variances and covariances, Jorion (1985), Eun and Resnick (1988) and others demonstrate that estimates of expected returns are considerably more unstable over time.[11]

Empirically, the parameter instability problem has a number of implications. For example, *ex ante* results concerning the return on a given portfolio may vary significantly from sample to sample. Jorion (1985) examines the out-of-sample performance of the two *ex post* optimal portfolios identified by Grubel (1968) and Levy and Sarnat (1974), together with two "naive" portfolios, the equally weighted and market value weighted portfolios. As measured by the Sharpe ratio, Jorion found that over the next investment horizon, the *ex ante* performance of the two mean-variance efficient portfolios was inferior to the performance of the naive equally weighted portfolio. Jorion (1985) also provides evidence that, in estimating *ex post* returns, longer sampling windows, e.g., five years for monthly data, provides superior *ex ante* forecasting when compared with shorter sampling windows, e.g., 1 year of monthly data. The difficulty with longer sampling windows is that it takes a longer time interval for the estimates to reacte to changing market conditions.[12]

In addition to the length of the sampling window, the type of estimator also can have a significant impact on the *ex ante* results. In applications, means and variances are estimated from the most recent data available at the time the portfolio is rebalanced. Grauer and Hakansson (1992) provide data on the behavior of the mean-variance efficient portfolio over time, using quarterly rebalancing of the portfolio, based on the most recent 40 quarters of data. The investment universe includes US equities and bonds, seven non-US equities (Canada, Japan, UK, Switzerland, Netherlands, Germany and France). The simulations try four different methods of estimating the mean and provide results over a wide range of possible investor preferences. Their results indicate the importance of mean estimation to portfolio composition. When simple historical averages are used, the average portfolio contains 3 risky assets or less for the "typical" range of investor preferences. When the CAPM is used, eight to the maximum possible ten risky assets is common in the average portfolio.

In addition to the estimation method used to determine the relevant parameter inputs, the presence or absence of short-selling has been found to be fundamental in assessing the performance of mean-variance efficient portfolios. Even though the early studies implicitly assumed short selling was not permitted, at least since Jorion (1985) it has been recognized that odd results can be obtained when short selling is permitted. For example, Jorion reports results for the time series properties of the optimal weight on domestic assets in the *ex post* tangency portfolio. A considerable amount of short-selling is indicated at various times, as much as -2.4 times the total principal value of the portfolio at one point in 1978. For many types of investment situations, e.g., pension funds, life insurance companies, this amount of short selling would be unacceptable and unobtainable. Evidence on portfolio composition with short selling restrictions, e.g., Glen and Jorion (1993), indicates a dramatic narrowing of the number of assets held in the portfolio is likely, amplifying the concentration of a given portfolio in a small number of assets.

Eun and Resnick (1994)[13]

Almost a decade after Jorion (1985), issues surrounding the difficulties of diversifying internationally were examined in Eun and Resnick (1994) which provides a detailed examination of the *ex ante* performance of three mean-variance efficient portfolios derived from the Markowitz framework: the tangency portfolio, the minimum variance portfolio and the Bayes-Stein portfolio. Allowing short sales, Eun and Resnick (1994) consider an impressive number of variations on the basic optimization model. Three different types of asset groups are used to form portfolios: stocks only, bonds only and combinations of stocks and bonds. Both hedged and unhedged returns are examined. In addition, two different reference currencies are used to denominate returns, US and Japanese. Over the *ex post* sample period Japan was a high return country and the US was a low return country. The returns from seven countries are included: Canada, France, Germany, Japan, Switzerland, UK and the US. Results on the decomposition of var[$R_\$$] for each of these countries, over the sample, have been

Table 2 Ex-post Optimal International Portfolios: 1978:1–89.12

	U.S. Perspective			Japanese Perspective		
Market	Bond Portfolio	Stock Portfolio	Bond/Stock Portfolio	Bond Portfolio	Stock Portfolio	Bond/Stock Portfolio
Canada	0.0218	−0.0579	−0.0764/0.0009	−0.0255	−0.0956	−0.0499/−0.0079
France	0.4488	0.0509	0.2379/0.0245	0.2392	0.1165	0.1531/0.0145
Germany	0.0204	0.0550	0.1747/−0.0177	−0.1294	−0.0170	−0.0905/0.0139
Japan	0.2838	0.3945	−0.1278/0.3650	0.8064	0.6700	0.6049/0.1896
Switzer.	−0.4896	0.0006	−0.3988/0.0500	−0.0483	0.1252	−0.0301/0.0428
U.K.	0.0895	0.0770	−0.0055/0.0836	0.0319	0.1011	−0.0032/0.0349
U.S.	0.6254	0.4792	0.5145/0.1752	0.1257	0.0998	0.0930/0.0348
ME	1.06%	1.72%	1.59%	0.61%	1.55%	0.89%
SD	3.15%	4.19%	3.49%	1.51%	3.65%	1.68%
SHP	0.34	0.41	0.46	0.41	0.42	0.53
Domestic Strategy*						
ME	0.86%	1.34%	1.10%	0.61%	1.66%	1.14%
SD	3.20%	4.60%	3.20%	1.74%	4.41%	2.57%
SHP	0.27	0.29	0.34	0.35	0.38	0.44

* For the domestic strategy, the bond/stock portfolio consists of domestic bond and stock indices, with 50% weight each.

given in Sec. 6.2.

After providing summary statistics for the securities involved, Eun and Resnick report the *ex post* optimal weights derived for the *tangency portfolio* derived from a sample of monthly returns covering 1978-1989 (see Table 6.8). Results are reported for both US and Yen denominated returns. Observing that ME= mean, SD= standard deviation and SHP= Sharpe ratio, a number of interesting results emerge. For US investors, both the bond and stock portfolios reflect the full benefits of international diversification. There is a reduction in portfolio return volatilty ***and*** increase in expected returns. The combined stock/bond portfolio does exhibit an increased volatility but this is offset by the higher expected return. Examining the specific weights, a definite home country bias is observed with the positive weight on domestic asset being significantly larger than weights for other countries. There is also evidence of significant short selling, with the negative weight on Swiss being particularly noticeable, e.g., a short position equal to about half the principal value being indicated for the bond only portfolio.

Table 8 **Average Out-of-Sample Performance Results of the Ex Ante Investment Strategies: U.S.[a]**

		Bonds	Stocks	Bonds and Stocks
A. Unhedged				
CET	ME (%)	1.50	1.65	6.74
	SD (%)	6.48	4.98	21.10
	SHP	0.32	0.37	0.31
MVP	ME (%)	1.20	1.86	2.23
	SD (%)	2.61	4.38	4.71
	SHP	0.40	0.51	0.41
EQW	ME (%)	1.29	2.07	1.68
	SD (%)	3.07	4.40	3.08
	SHP	0.41	0.55	0.54
BST	ME (%)	1.33	1.80	3.77
	SD (%)	3.92	4.46	14.68
	SHP	0.37	0.48	0.35
US	ME (%)	0.91	1.32	1.11
	SD (%)	2.67	4.84	3.14
	SHP	0.31	0.33	0.36
B. Hedged				
CET	ME (%)	0.78	1.77	0.88
	SD (%)	1.09	4.78	1.27
	SHP	0.74	0.49	0.72
MVP	ME (%)	0.73	1.72	0.86
	SD (%)	0.90	4.39	1.12
	SHP	0.87	0.51	0.81
EQW	ME (%)	0.83	1.60	1.21
	SD (%)	1.37	4.24	2.37
	SHP	0.63	0.53	0.60
BST	ME (%)	0.75	1.79	0.90
	SD (%)	1.05	4.58	1.21
	SHP	0.75	0.51	0.79

[a] In each cell, the three numbers represent the average of 29 out-of-sample values.

The results for the *ex post* optimal portfolios using returns denominated in Japanese yen are also revealing. Again, there is a decided home country bias, only in this case Japanese investors desire Japanese assets. This can be explained by observing that investing in domestic assets allows domestic investors to avoid the exchange risk.[14] This evidence calls into question the hypothesis that an optimal diversified international portfolio will be the same, regardless of the currency of denomination. However, unlike the US case where including international assets resulted in an increase in portfolio expected returns, international diversification only produced a reduction in portfolio return volatility. For both the stock and stock/bond portfolios, Japanese investors had a reduction in portfolio return. The increase in the Sharpe ratio was due to the substantial reduction in return volatility.

While interesting, the *ex post* results only confirm what is available in a number of sources. Eun and Resnick (1994) proceed to examine the *ex ante* performance of the mean-variance optimal portfolios for the three years 1990-1992. The optimization exercise is conventional. Each month the relevant means, variance and covariances are estimated using the 60 most recent observations. The portfolio weights are determined and the portfolio is held for one month, at which point the parameters are re-estimated and the optimal portfolios determined. This continues for three years at which point the *ex ante* returns are calculated. The naive benchmarks selected to compare the performance are an equally weighted portfolio and the domestic-only portfolio. Two types of results are reported: the average portfolio weights and the *ex ante* performance results (see Tables 6.9 and 6.10). Both the results are revealing and require close inspection.

Examining the average weights for unhedged US portfolio reported by Eun and Resnick indicates disturbing results for the weights in the tangency portfolio. Several of the weights, particularly on the Swiss, French and

Table 11 Average Out-of-Sample Performance Results of the Ex-Ante Investment Strategies: Japan*

		Bonds	Stocks	Bonds and Stocks
A. Unhedged				
CET	ME (%)	0.92	1.34	1.15
	SD (%)	1.95	4.57	2.66
	SHP	0.53	0.35	0.46
MVP	ME (%)	0.65	1.53	0.93
	SD (%)	1.47	4.21	2.00
	SHP	0.53	0.47	0.51
EQW	ME (%)	0.37	1.18	0.78
	SD (%)	2.14	4.38	2.87
	SHP	0.17	0.37	0.32
BST	ME (%)	0.78	1.48	1.07
	SD (%)	1.59	4.21	2.13
	SHP	0.59	0.45	0.54
JA	ME (%)	0.75	2.18	1.46
	SD (%)	1.64	5.24	2.97
	SHP	0.59	0.44	0.52
B. Hedged				
CET	ME (%)	0.59	1.28	0.64
	SD (%)	1.28	5.83	3.81
	SHP	0.49	0.26	0.21
MVP	ME (%)	0.52	1.81	0.14
	SD (%)	0.89	5.30	4.76
	SHP	0.64	0.40	0.04
EQW	ME (%)	0.67	1.89	1.04
	SD (%)	1.33	5.40	3.24
	SHP	0.52	0.43	0.38
BST	ME (%)	0.04	1.45	0.44
	SD (%)	4.85	5.52	4.47
	SHP	0.06	0.30	0.14

* In each cell, the three numbers represent the average of 36 out-of-sample values.

German bonds in the stock/bond portfolio, indicate average weights well in excess of invested principal. The -4.75 **average** weight on Swiss bonds is almost frightening. The contrast with the minimum variance and Bayes-Stein portfolios is evident. For these portfolios, only the Swiss securities in the stock/bond Bayes-Stein portfolio have weights in excess of one. As expected, the minimum variance portfolio typically have the smallest weights. Evidence of a home country bias is apparent for all the mean-variance optimal portfolios. Turning the weights associated with the hedged portfolios reveals some dramatic changes. The largest weight for any asset is a .77 weighting on Swiss bonds in the stock/bond minimum variance case. The tangency portfolio weights now appear to be similar to the weights in the other two portfolios. The home country bias disappears to be replaced by a Swiss bias. This is strong evidence of the impact that hedging has on the results of international diversification. The average weights for the Japanese returns are somewhat confounding. The large weights on certain assets in the unhedged tangency case has disappeared, being replaced by a strong home country bias. No weights in excess of one are reported and there is not much difference between the weights for the different portfolios. Examining the hedged results again reveals the domestic bias being replaced by a Swiss bias, albeit not a dramatic a reduction in the home country weights as in the US case.

The final and most important results reported by Eun and Resnick are associated with the *ex ante* performance results. The monthly unhedged US returns in Table 6.9 indicate, for bonds, that the naive equally weighted portfolio had the best Sharpe ratio, followed closely by the minimum variance portfolio. The tangency portfolio, which financial theory suggests will have the best performance, outperforms only the domestic-only portfolio, though the tangency portfolio did report the highest return. There is a similar story for stocks, albeit with the tangency portfolio now reporting a mean return that is higher than only the domestic-only strategy. The results for the combined portfolio are dramatic. Despite having a mean value that is huge, the Sharpe ratio ranks the tangency portfolio dead last in performance. The hedged results repeat this general story. For bonds and stock/bonds, the minimum variance portfolio has the highest Sharpe ratio with the naive equally weighted portfolio have the best performance for stocks. The tangency portfolio is either last or second last in all cases. The weakness of the mean returns for the tangency portfolio indirectly indicate the difficulty of using *ex post* estimates to achieve *ex ante* results.

The *ex ante* Japanese results in Table 6.10 repeat this general story, with some provisos. For the unhedged results, the equally weighted naive portfolio, which had such good performance in the US case, now does poorly. The performance of the Bayes-Stein portfolio improves considerably, with minimum variance continuing a strong relative showing. In the stock case, for example, minimum variance has both a higher mean and lower volatility than the tangency portfolio. Compared to the US case, there is a strong performance by the domestic Japanese portfolios. The unhedged case is somewhat confounding. Despite having the highest Sharpe ratio for bonds and second highest for stock, the reported result for the minimum variance bond/stock portfolio gives the appearance of a typographical error. The upshot of all these results is that the Markowitz model, which holds so much promise from a theoretical standpoint, may have serious difficulties in delivering an *ex ante* performance consistent with the *ex post* predictions.

World Stock Market Performance, 1996
(ranked by 1996 % change in price indexes in US$)

Rank	Market	% change in price index	Rank	Market	% change in price index
1	Bangladesh *	196.0	45	Trinidad & Tobago *	10.9
2	Russia *	155.9	46	Italy	10.8
3	Venezuela *	131.8	47	Swaziland	9.9
4	Hungary *	94.6	48	Belgium	8.8
5	China *	89.4	49	Coto d'Ivoire *	5.3
6	Lithuania *	82.0	50	Colombia *	4.5
7	Poland *	71.1	51	Austria	3.2
8	Paraguay	59.9	52	Slovenia *	2.6
9	Zimbabwe *	59.6	53	Malta	1.2
10	Nigeria *	55.6	54	Switzerland	1.2
11	Iran	52.7	55	Peru *	0.7
12	Turkey *	42.4	56	Kyrgyz Republic	0.3
13	Jamaica *	39.8	57	Greece *	-1.0
14	Egypt *	38.8	58	Barbados	-3.1
15	Morocco *	38.4	59	India *	-3.5
16	Spain	36.5	60	Jordan *	-3.6
17	Costa Rica	36.3	61	Israel	-5.4
18	Taiwan, China *	36.1	62	Tunisia *	-6.5
19	Namibia	36.0	63	Mauritius *	-6.9
20	Sweden	35.4	64	Singapore	-8.0
21	Finland	31.7	65	Kenya *	-9.9
22	Brazil *	30.4	66	Sri Lanka *	-14.0
23	Hong Kong	28.9	67	Ecuador *	-15.2
24	Ireland	28.8	68	Japan	-16.0
25	Norway	26.8	69	Chile *	-17.2
26	Canada	26.4	70	South Africa *	-19.2
27	Oman	26.1	71	Botswana *	-20.3
28	Panama	24.9	72	Ghana *	-21.4
29	Portugal *	24.9	73	Pakistan *	-21.8
30	Netherlands	24.5	74	Thailand *	-38.1
31	U.K.	23.3	75	Korea *	-39.2
32	Malaysia *	22.9	76	Bulgaria *	-83.0
33	USA	21.4			
34	Denmark	20.0	**Regional Indexes**		
35	Czech Republic *	20.0	1	IFCG Latin America	16.0
36	France	19.4	2	IFCI Latin America	14.0
37	Philippines *	19.1	3	MSCI World	11.7
38	Argentina *	18.8	4	IFCI Asia	8.9
39	Indonesia *	18.0	5	IFCI Composite	7.0
40	Mexico *	16.2	6	IFCG Composite	5.8
41	Slovakia *	15.7	7	MSCI EAFE	4.4
42	New Zealand	13.5	8	IFCG Asia	4.2
43	Australia	13.4	9	FT/S&P, Europac	3.6
44	Germany	12.1	10	IFCI EMEA	-5.2

Notes: Markets marked by an asterisk indicate IFC Global Index as source; developed market returns are from MS

Recognizing the connection to the results of Sec. 2.1, the apparently poor *ex ante* performance of the tangency portfolio, relative to the minimum variance portfolio, can be attributed to the poor forecasting performance of the arithmetic averages in predicting actual returns. This poor *ex ante* performance is happening despite the use of foreign markets that have relatively high correlation with each other. Introducing emerging markets, almost certainly, would confound the Markowitz optimization procedure. An inspection of "World Stock Market Performance, 1996" in Table 6.11 reveals the extreme return distributions that occur when emerging markets are considered. A number of the countries in the top 10, could be found in the bottom 10 in the previous year. As Sec. 2.1 demonstrates, if the firm is unable to accurately forecast the future expected return, then the rational procedure is to use the minimum variance weights. Based on the results provided by Eun and Resnick, this intuition seems to work well empirically.

The CAPM and International Diversification

The CAPM is a central paradigm of modern Finance. This *ex ante* model can be stated as:

$$E[R_j] = R_F + \beta_j \left[E[R_M] - R_F\right]$$

where $E[R_j]$ is the expected return on asset j, $E[R_M]$ is the expected return on the closed economy market portfolio, $\beta_j = (cov[R_j, R_M]/var[R_M])$ and R_F is the riskfree rate of interest. Among other uses, the CAPM provides a methodology for estimating discount rates to be used in capital budgeting problems and the like. The CAPM also implies an investment strategy for rational investors. More precisely, if the CAPM is true, it follows that the tangency portfolio associated with the capital market line is the market portfolio. Operationally, the two fund separation theorem requires that the rational investor will hold portfolios composed of riskless assets and the market portfolio. Risk averse investors will be net long the riskless asset while high levels of risk tolerance would permit leveraging, i.e., borrowing by shorting the riskless asset in order to leverage the position in the market portfolio. This powerful and persuasive *ex ante* investment philosophy is confounded by the observation that diversification using international and domestic assets will shift the efficient frontier upward when compared with diversification using only domestic assets, as in Graph 6.1. Various problems arise immediately concerning the specification of the global market portfolio, the riskless asset, and how currency risk will be handled.

The difficulties that have been identified with operationalizing the Markowitz mean-variance model lend considerable support to efforts to generalize the CAPM, which is typically derived under closed economy assumptions. Various approaches can be taken to generalizing to the international context. The particular approach selected depends on a key assumption about the handling of exchange rate risk. Solnik (1974) and Sercu (1980) assume that exchange risk is unhedged. It is observed that the international CAPM is unlike the closed economy CAPM where systematic risk is related to the relationship between the individual asset returns and the market return, with appropriately adjustment for the riskless rate of return. In the international case, there is an additional risk dimension associated with the exchange rate. The additional risk is captured by modelling the return on the exchange rate using a "pure exchange risk asset". For a non-US investor, the pure exchange risk asset is the US Tbill.[15] In addition, the return on the domestic market portfolio is replaced by the global market portfolio.

Using * to denote the foreign (US) rate, it follows that the domestic risk free rate, R_F, can be expressed as: $R_F = R_F* + e$. As before the cross product term has been ignored for simplicity. Using a replicating portfolio argument, Uppal and Sercu (1995) are able to derive the international CAPM as:

$$E[R_j] = R_F + \beta_j \, (E[R_W] - R_F) + \gamma_j \, (R_F^* + e - R_F)$$

where $E[R_w]$ is the expected return on the world market portfolio and $\gamma_j = cov[R_j, e]/var[e]$. If $\gamma_j = 0$, then the international CAPM reduces to a variation of the closed economy CAPM, albeit with the global market portfolio replacing the domestic market portfolio. Yet, this presentation is constructed with a two-country format, e.g., the domestic asset/foreign asset portfolio encountered previously. Where there are $n + 1$ countries, then there would be n exchange rate γ's to estimate, all of which would have to equal zero in order have the international CAPM reduce to the closed economy CAPM. In practice, this model extends the bivariate regression of the closed economy CAPM to a multivariate regression involving exchange rates and the market portfolio.

The international CAPM represents a substantive theoretical extension of the closed economy model. Whether this theoretical contribution translates into improved empirical performance is, at present, an open question. The difficulties inherent in estimating *ex ante* asset pricing models are well known and it will be difficult to resolve the validity of the closed or open economy version.[16] Nevertheless, the questions involved are fundamental and require attention. In particular, are assets priced in domestic markets or in international markets? By estimating β's using the domestic market index, the convention is to assume a closed economy model. This approach avoids the problem of specifying the world market portfolio, a seeming quandary for the international CAPM. This and other difficulties associated with implementing the international model have meant that the international CAPM has received surprisingly little attention in empirical studies, e.g., Uppal and Sercu (p.607-8).

Why is the specification of the global market portfolio a quandary for the international CAPM? Consider the case of a Dutch investor. Extending the approach of specifying the closed economy market portfolio, presumably the world market portfolio would be a value weighted portfolio of the world's capital assets, which is proxied by a value weighted world equity portfolio. Though the precise value weights would depend on valuations at a specific point in time, at present such a portfolio would be more than 60% in US equities, with a further 15-20% in Japanese equities. The Dutch component in such a global index would be negligible. Hence, the international CAPM dictates that the expected return on a Dutch capital asset, e.g., an equity security, will be determined by the expected return on a world market portfolio that is not capable of capturing the Dutch country-specific risk. In effect, country specific risk, outside of pure exchange rate risk, will not be priced. This puts considerable pressure on international capital flows to equalize returns across countries. The upshot is that the international CAPM lacks the intuitive appeal needed for practical applications.

Another approach to specifying an international CAPM is suggested by extending the Arrow-Debreu theorem, a fundamental result from microeconomics about the Pareto optimality of a perfect market economy under uncertainty. This theorem requires complete markets: a traded security for each source of uncertainty. The resolution of uncertainty is provided by market participants hedging the uncertainty using the traded contingent claims. Given this, there is also considerable empirical evidence in favour of fully hedged versus unhedged portfolios in specifying the international CAPM. Early research on this issue by Madura and Reiff (1985) demonstrates that, *ex post*, efficient frontiers of fully hedged portfolios dominate unhedged portfolios.

Tables 6.9 and 6.10, from Eun and Resnick (1994), provide empirical results on the *ex ante* performance of hedged and unhedged portfolios. For US$ returns, hedged portfolios dominate unhedged portfolios for bonds. This confirms similar results found in Glen and Jorion (1993). Yet, for yen returns, the *ex ante* results are ambiguous. It is not possible to determine whether hedged or unhedged portfolios are superior. This incongruity in results seems to be due primarily to the differing performance of hedge profitability. For both reference currencies, *ex ante* unhedged returns are found to be generally higher than hedged returns, as reflected in the spreads between the mean returns for the hedged and unhedged portfolios for the two reference currencies. Yet, the Sharpe ratio gains seem to originate primarily from the reduction in return volatility.

The results for bonds do not carry forward to stock portfolios or stock/bond portfolios. If anything, for yen returns, unhedged portfolios of stocks generally outperformed hedged portfolios. Results for stock portfolios using US$ returns are ambiguous. There is little difference either in the mean returns or the Sharpe ratios. Eun and Resnick (1994, p.149) also make an insightful if unexplored observation about the *ex ante* results:

> The portfolio weight vectors under hedging for US and Japanese investors look remarkably similar for all asset classes. The reason for this is that regardless of which numeraire currency is used, a particular hedging strategy will identify a similar *ex ante* optimal investment weight vector to the extent that the forward premiums or discounts are small relative to the local currency mean returns.

Glen and Jorion (1993) use a variation on this result to design conditional hedging strategies, based on the forward premium, that can capture a significant increase in *ex ante* portfolio performance.

As initially conceived, the benefits of diversification were related to unhedged portfolios of securities, the rationale being that hedging would be redundant. If enough currencies were included, all sources of country specific currency risk would be diversified away leaving only the systematic currency component. However, a number of authors have argued that hedged portfolios may be a more appropriate method of capturing the full benefits of diversification, e.g., Perold and Schulman (1988), Fosburg and Madura (1991), Eaker, et.al. (1991), Filatov and Rappoport (1992), Adler and Jorion (1992). Others have argued against holding specific securities such as hedged foreign bonds, e.g., Burik and Ennis (1990).[17] On balance, though considerable insight has been achieved, at this point the issues surrounding the appropriate method of implementing mean-variance efficient international portfolio diversification are still largely unresolved.

6.4 Currency Swaps and Fully Hedged Borrowing

History of Currency Swap Trading

The history of currency swap trading is difficult to trace, if only because it is difficult to identify precisely when swap trading began. In a sense, swap trading is inherent in the activities of all financial intermediaries. For example, when a Singapore customer enters a local bank and purchases a Canadian dollar GIC (term) deposit, a current cash flow (S$) is being exchanged for a future cash flow, the return of principal and interest at maturity that depends on the future of the C$. This type of transaction has ancient roots. One example is the 16th century bill of exchange. This security involved the payment (receipt) of cash in one geographical location for return (payment) of principal plus interest, to be paid in another geographical location, in the currency of that location. In modern parlance, this transaction is a zero coupon currency swap or a short dated foreign exchange swap. Such contracts reach back to antiquity, e.g., there are laws regarding bills of exchange to be found in the Code of Hammurabi. Such contracts were actively traded on the 16th century Antwerp bourse, e.g., de Roover (1948).

Numerous modern writers date the beginning of swap trading to the mid-1970s for interest rate swaps and the early 1980s for currency swaps. This historical interpretation is based on a subtle change in the definition of a swap transaction, narrowing the focus to derivative contracts that are bundles of spot and forward exchange contracts. Swaps, such as those traded in the short dated foreign exchange market, would not be included under this 'modern' swap definition. Such contracts will be referred to as foreign exchange swaps, as opposed to currency swaps, to avoid semantic confusion. Under this historical interpretation, currency swap financing evolved from the shortcomings associated with parallel and back-to-back loans, e.g., Price and Henderson (1988) and AntI (1986). While a number of proprietary currency swap deals were done prior to 1981, the first widely publicised deal was a currency swap between IBM and the World Bank. Among other factors, currency swaps were an important legal advance over parallel and back-to-back loans that are governed by securities law, in which default provisions are unclear. Swaps fall within the realm of contract law in which default provisions are more straightforward.

While initially motivated by security and accounting difficulties arising from borrowing in different currencies, by the early 1980s it was widely recognised that currency swaps could be used for numerous purposes. With the growth of swap financing, intermediaries have been willing to take initially unmatched, i.e., dealer, swap positions. The result has been a liquid market where various types of currency swap quotes are available on a regular basis. In turn, this has permitted increasingly sophisticated swap trades to be executed, while at the same time requiring techniques for hedging the temporary dealer swap risk to be "engineered". To facilitate the standardisation of swap terms, the International Swap and Derivatives Association (ISDA, formerly the International Swap Dealers Association) has been formed by the major players.

To facilitate the increased liquidity in the currency swap market, the ISDA provided a uniform set of definitions and contracts to standardise swap trading (ISDA, 1987, 1991). From these early beginnings, the growth of swap trading, particularly for interest rate swaps, has been phenomenal, Examining Table 6.11 there was over US$22 trillion in notional principal of outstanding interest swaps and US$1.8 trillion in currency swaps. The use of notional principal is somewhat misleading, as the actual cash flows are dramatically less, especially for interest rate swaps where there is usually no exchange of principal at initiation and maturity. Table 6.12 indicates that the interest rate swap activity was not concentrated solely in US dollars. The relative importance of interest rate swaps compared to currency swaps, and the 'building block' character of many actual swap transactions, dictates that the mechanics of interest rate swaps get some attention, as well examining currency swaps.

Table 6.12 **Notional Principal of Interest Rate Swaps by Currency (in billions US$)**

	OUTSTANDING AMOUNTS		NEW SWAPS	
CURRENCY	***95/12***	***97/12***	***95/12***	***97/12***
Deusche mark	1,438.9	3,278.2	984.5	2,593.3
Italian lira	405.4	1,128.4	217.3	953.0
Japanese yen	2,895.9	4,313.1	2,259.3	2,899.8
Pound sterling	854.0	1,456.6	433.4	1,034.1
US dollar	4,371.7	6,078.1	2,856.5	4,387.0
Other	2,844.8	6,036.9	1,947.8	5,199.9
TOTAL	12,810.7	22,291.3	8,698.8	17,067.1

Source: BIS webpage.

The Mechanics of Swap Trading

While numerous variations on swap contracts are possible, e.g., Beidleman (1992), Das (1994), the basic principle of a swap is an exchange of cash flows that are deemed to be equal in value at the time the swap is initiated. Given this prerequisite, two basic types of “plain vanilla” swaps are available (Abken 1991): currency swaps and interest rate swaps.[18] In an interest rate swap, the net cash flows being exchanged are based on fixed and floating interest rate borrowings. One borrower issues fixed rate debt and exchanges the resulting periodic net debt payments with another borrower issuing floating rate debt. A currency swap involves exchanging net cash flows arising from debt issues denominated in different currencies. Principal values are exchanged at initiation and maturity, with both these cash flows, together with the periodic payments, being valued at the spot exchange rate prevailing at the time the swap was initiated. In ‘building block’ fashion, these basic types of swaps are often combined in practical applications, resulting in an exchange of borrowings in different currencies that can involve either fixed or floating interest rates.

Table 6.11 **Currency and Interest Rate Swaps: Annual activity and outstanding,** ($billions of notional principal)

	INTEREST RATE SWAPS		CURRENCY SWAPS		INTEREST RATE OPTIONS		TOTALS	
Yr ending	***Activity***	***Oustandings***	***Activity***	***Outstandings***	***Activity***	***Outstandings***	***Activity***	***Oustandings***
1987	$387.8	$682.80	$85.8	$182.80			$473.6	$865.6
1988	568.1	1,010,20	122.6	316.80		$327.30	690.7	1,654.3
1989	833.6	1,502.60	169.6	434.80	$335.5	537.30	1,338.7	2,477.7
1990	1,264.3	2,311.50	212.7	577.50	292.3	561.30	1,769.3	3,450.3
1991	1,621.8	3,065.10	328.4	807.20	382.7	577.20	2,332.9	4,449.5
1992	2,822.6	3,850.80	301.9	860.40	592.4	634.50	3,716.9	5,345.7
1993	4,104.6	6,177.30	295.2	899.60	1,117.0	1,397.60	5,516.8	8,474.5
1994	6,240.9	8,815.60	379.3	914.80	1,513.2	1.572.80	8,133.4	11,303.2
1995	8,698.8	12,810.70	454.1	1,197.40	2,015.4	3,704.50	11,169.3	17,712.6
1996	13,678.2	19,170.90	759.1	1,559.60	3,337.2	4,722.60	17,774.5	25,453.1
1997	17,067.1	22,291.30	1,135.4	1,823.60	3,978.4	4,920.10	22,180.9	29,035.0

Sources: ISDA and BIS web pages.

Exhibits 6.1-6.4, from Shapiro (1992) provide an illustration of the mechanics of a plain vanilla interest rate swap. The situation being described is stylized and is not meant to capture all the various rationales for doing an interest rate swap. Exhibit 6.1 describes the initial situation: there are two financial institutions with a duration gap mismatch. Counterparty *A* has an asset portfolio composed of fixed rates mortgages with an average maturity of 5 years, paying a fixed rate cash flow of 13.25%. Counterparty *B* has a asset portfolio composed of floating rate commercial loans, again with a 5 year maturity, paying LIBOR + 75 basis points. Against these assets, *A* has 5 year floating rate liabilities requiring payment of LIBOR + 50 basis points while *B* has a 5 year fixed rate liabilities of 11%. Par values of all the assets and liabilities are equal. The duration gap for both counterparties is apparent. *A* has mismatch between the longer duration cash flow of assets and the shorter duration of the liability cash flow. *B* is facing the opposite situation.

As indicated in Exhibits 6.2-6.3, Big Bank is able to step Into this situation and resolve the duration gap mismatch by providing an interest rate swap to both parties. This step captures two essential features of a plain vanilla interest swap: the transactions are intermediated by a swap broker or dealer; and, there is no initial exchange of cash flows, the plain vanilla interest rate swap only involves (net) payments of the periodic coupons. Subject to certain arbitrage restrictions, the fixed-to-floating/floating-to-fixed rates at which the swap is initiated, together with the intermediary's margin, will be determined by market conditions. In the example, the intermediary is quoting a fixed-to-floating swap of 11.35% in exchange for LIBOR with a floating-to-fixed swap quoted as LIBOR in exchange for 11.25%.

The benefits of the swap to *A* are apparent. *A* will receive 11.35% on the asset portfolio, and pay 11.35% to the intermediary and receive LIBOR. *A* then adds a 50 basis points to this LIBOR payment to satisfy the liability cash flow requirement. *A* has gone from a situation where end of period cash flows were uncertain, with a downside risk that LIBOR rates would move up, to a situation where there is a locked-in spread of 140 basis points: (13.25 - 11.35) - (LIBOR + 50bp - LIBOR) = 140 basis points. A similar result holds for *B*. The LIBOR part of the LIBOR plus 75 basis return on the asset portfolio is swapped for 11.25% fixed, of which 11% is dedicated to pay the fixed rate liability, permitting *B* to lock in a spread of 100 basis points. Prior to the swap, *B* was exposed to the risk that interests would fall, reducing the payments on the *B*'s asset portfolio. The final situation is depicted in Exhibit 6.4 where it can be observed that Big Bank is able to clear 10 basis points at

Despite being grouped in the category of swaps, currency swaps are substantively different than interest rate swaps. Exhibit 6.5 illustrates the transactions involved on one side of a currency swap. The two key features distinguishing currency swaps and interest rate swaps are evident. Unlike interest rate swaps, currency swaps do involve an ***exchange of principal at initiation and maturity***. The textbook objective of a currency swap is to acquire a borrowing in some target currency. As such, the receipt of that currency at the initiation of the swap is required. The other key feature is that the spot exchange rate on the trade initiation date governs all the cash flows. In effect, the currency swap involves counterparty A borrowing in currency X and counterparty B borrowing in currency Y. Given that the market values of the two borrowings are deemed to be equal on the trade initiation date, *A* and *B* then exchange borrowings. In practice, values are equalized through the market determined currency swap rates, the quoted borrowing rates on the underlying debt issues.

In Exhibit 6.5, the fixed-to-fixed currency swap rates involve a 5% yen borrowing being exchanged for a 10% US dollar borrowing. These are the quoted swap rates. As discussed below, unless foreign and domestic interest rate levels are equal ($S=F$), these rates will differ from rates observed for (unswapped) borrowings done directly in the domestic and foreign debt markets. At $t=0$, the transaction is initiated with an exchange of principal, ¥5 billion for US\$40 million. This translates to a spot exchange rate of ¥/US\$ = 125. At each coupon payment date there is an exchange of coupons. The trade being described, borrows yen and swaps for US\$. Over the life of the swap, this involves receiving yen payments of 5% to offset the yen borrowing. This yen payment inflow is exchanged for a dollar outflow of 10%. This fixed-to-fixed exchange of cash flows are equal at each payment date, a consequence of $S(0)$ being applied to each cash flow. In practice, coupon payments are usually netted, using the spot exchange rate applicable at the time the coupon payments are being made. At $t = T$, the swap concludes with the payment of the final coupon and the return of principal.

EXHIBIT 23.1 Situations Confronting Counterparties A and B Before the Swap

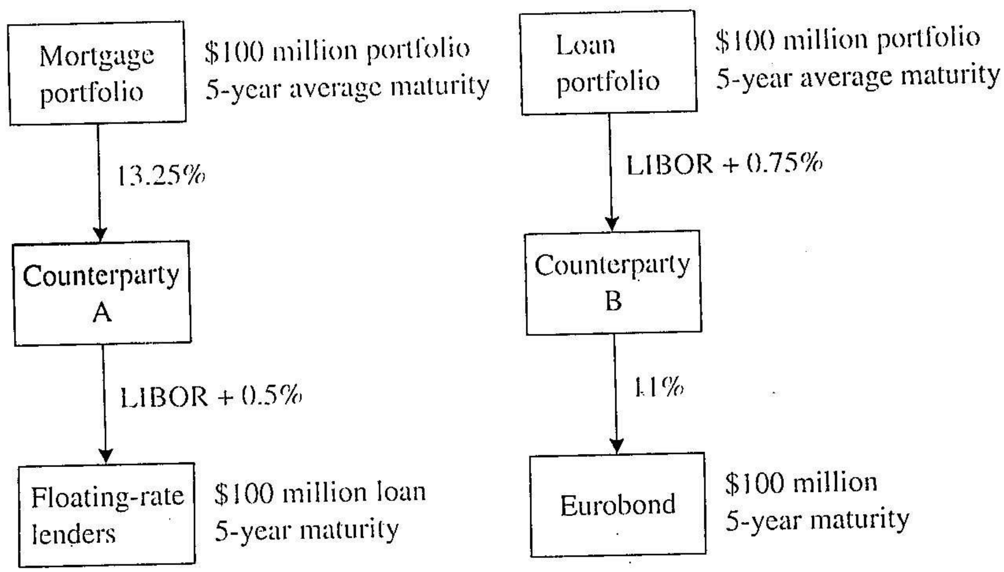


EXHIBIT 23.2 Counterparty A's Interest Rate Swap Agreement with BigBank

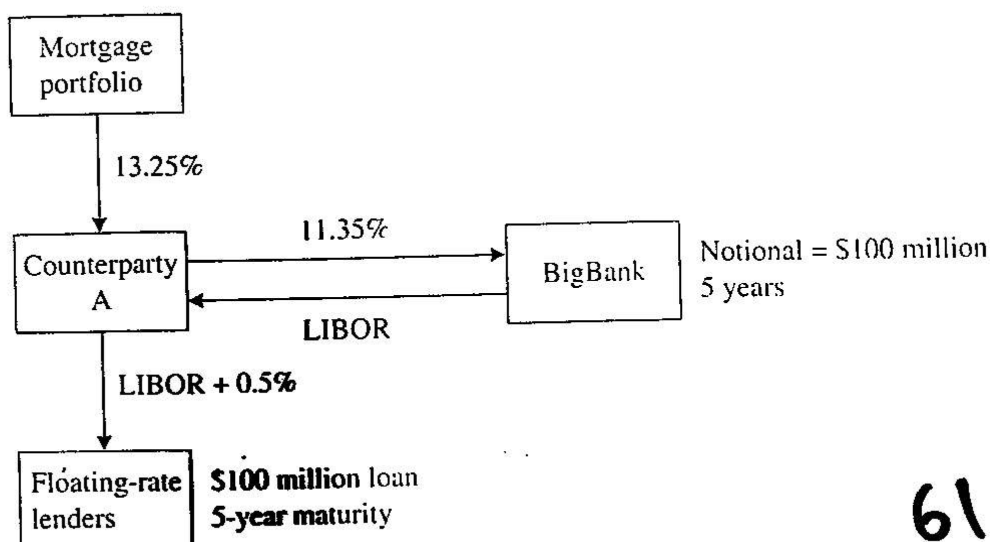


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EXHIBIT 23.3 Counterparty B's Interest Rate Swap Agreement ...

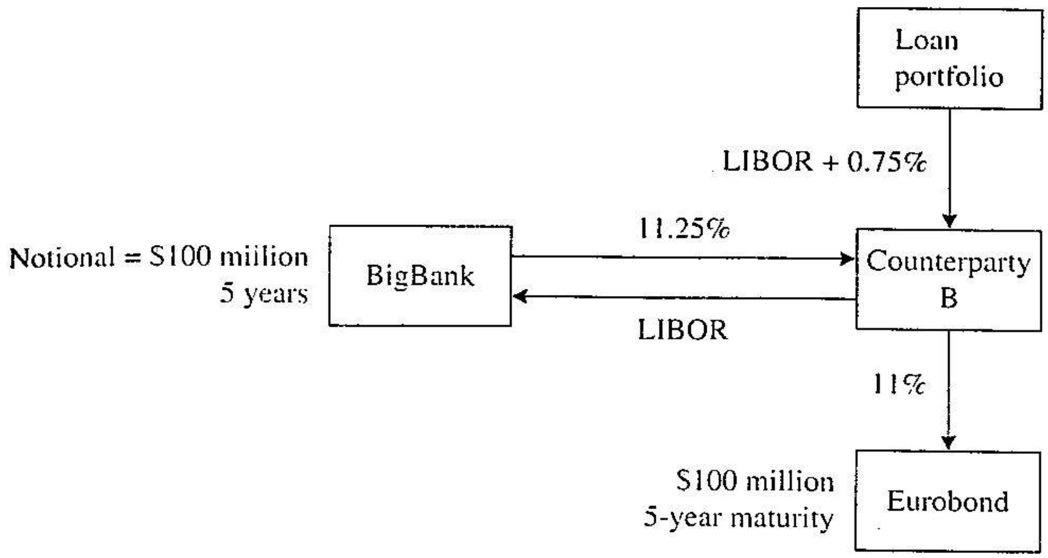


EXHIBIT 23.4 Classic Swap Structure

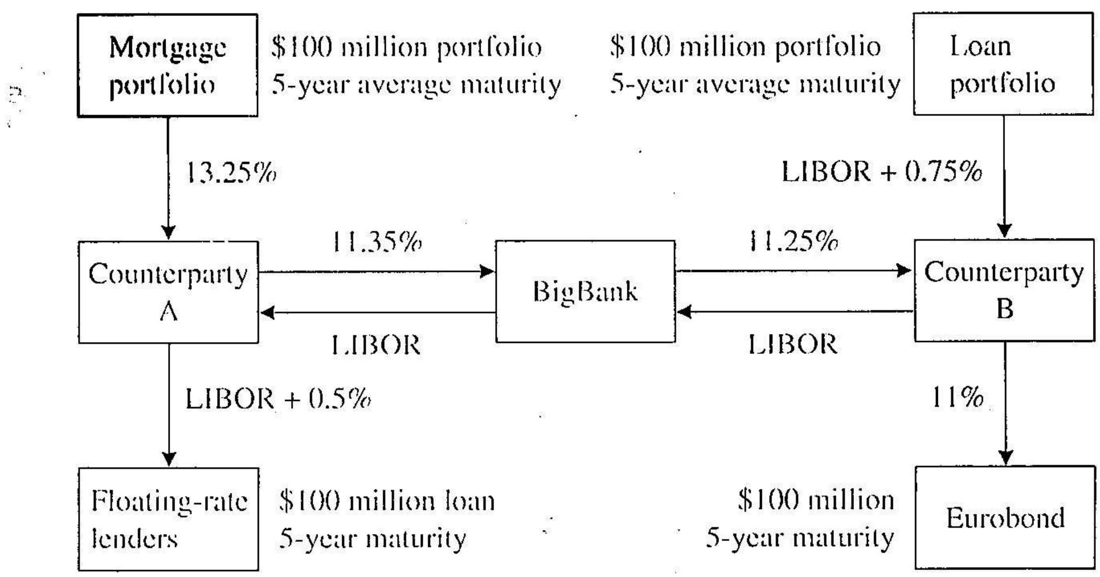
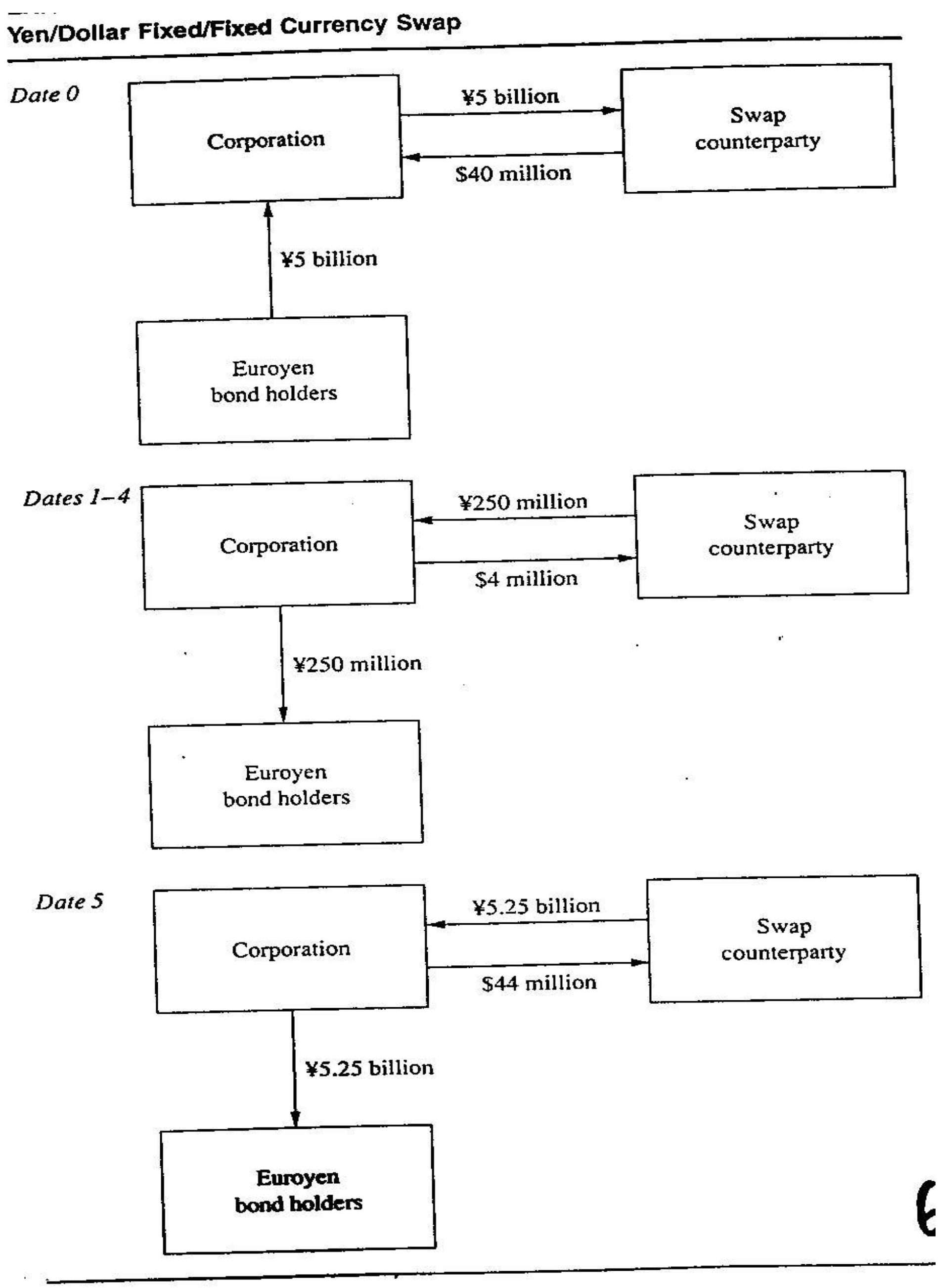


Figure 6.10 **Diagrammatic Illustration of a Plain Vanilla Fixed-to-Fixed Currency Swap**

Assumptions: Borrower issues a fixed rate ¥5 billion Eurobond issue at $r_{¥} = .05$. Through the currency swap intermediary, this is exchanged for a fixed rate US$40 million borrowing with $r_{\$} = .10$. The spot exchange rate used to value the transactions is (¥5 billion/US$40 million) = 125¥ / US$



Fully Hedged Borrowings and Currency Swaps

In a fixed-to-fixed currency swap, an agreed upon (usually spot) foreign exchange rate applicable to the closing date governs the exchange of principal at initiation and maturity, as well as the periodic net interest payments. In turn, the fixed periodic net payments are determined by negotiations, usually based on the interest differential in the different currencies prevailing at the time of closing, i.e., the issuer subject to the highest interest rate will usually receive the net payment. On the other hand, in a fully hedged foreign borrowing, the borrowing is made directly in the target currency and the principal is exchanged into the desired currency at the current spot rate. The resulting (fixed) coupon payments and return of principal are then fully hedged using the forward market. In addition to generating a different sequence of cash flows from a currency swap, this type of borrowing also depends on the availability of the appropriate forward exchange quotes.

Ignoring the issue of long-term forward market liquidity, there are often practical benefits to doing currency swaps instead of fully hedged borrowings. For example, in many countries investor preferences favour domestic credits, e.g., in Switzerland or Japan a well-known domestic corporation is likely to get a significantly lower borrowing rate than, say, a United States or Canadian corporation that does not have a substantial international reputation. A similar situation could prevail in reverse in the United States or Canada. In this case, borrowers requiring funding in foreign currency can exploit the borrowing advantage in their domestic market and swap into the desired currency, thereby significantly reducing foreign borrowing costs for both issuers.

A variation of differential credit assessment is differential credit spread compression. In this case. there are inter-country differences in the short- and long-term borrowing rate spread between strong and weak credits. Such differentials would generate cross-currency swap opportunities. In a fully hedged borrowing, which can be used either to acquire domestic or foreign funds, the "counterparty" is the forward foreign exchange market. Hence, the potential to exploit funding advantages arising from differential credit assessments is only indirectly available, ie, insofar as these benefits are reflected in long-term forward exchange rates. In the case of a fully hedged (foreign) borrowing to acquire domestic funds, credit assessment may have a negative effect.

In addition to funding advantages arising from differential credit assessments across countries, there are a number of other factors that could favour a currency swap over a fully hedged borrowing. For example, the ability to structure a currency swap as a series of foreign exchange transactions instead of as a foreign borrowing can lead to accounting and taxation advantages, both for the borrower and, particularly in the case of banks, for lenders (eLf?;, Hull 1987, 1989). Other possible factors favouring a currency swap could include: the borrowing corporation wanting to conserve forward exchange lines of credit for other purposes; favourable pricing, ie, given the borrower' s view of future exchange rates and interest rates (in the case of cross-currency interest rate swaps), the swap may be cheaper; and finally, regulatory restrictions imposed on overseas borrowings can be avoided.

To illustrate the fully hedged transaction, consider the following sequence of C$/US$ FX rates quoted by the Royal Bank of Canada, for Aug. 8, 1994:

$$S_0 = 1.3778 \quad F(0,1) = 1.3960 \quad F(0,2) = 1.4198$$
$$F(0,3) = 1.4428 \quad F(0,4) = 1.4633 \quad F(0,5) = 1.4833$$

where $F(0,T)$ is the forward exchange observed at $t=0$ for delivery at $t=T$. Assume, for example, that the Canadian borrower raised C$137,780 using a 5 year fixed coupon borrowing. When translated at the spot FX rate, the principal value of the borrowing would provide US$100,000. If the fixed C$ borrowing had a coupon of, say, 10% paid annually then the resulting fully hedged US$ cash flows would be:

$t=1$: C$13,778/1.3960 = US$9869.63 $t=2$: C$13,778/1.4198 = US$9704.18
$t=3$: C$13,778/1.4428 = US$9549.49 $t=4$: C$13,778/1.4633 = US$9415.70
$t=5$: (C$13,778 + C$137,780)/1.4833 = US$102,176

This sequence of uneven US$ cash flows can be compared with the US$ cash flows from the fixed-to-fixed currency swap that would be: five ***equal*** annual coupon payments of (US$100,000)*($rs_T$) each period plus the return of principal US$100,000 at maturity.

Compared to a fully hedged C$ borrowing, the currency swap would require lower US$ cash flows at the beginning, with the difference progressively narrowing to the point where the currency swap cash flows are higher than the fully hedged borrowing with the highest differential occurring at maturity. The opposite situation would apply for the fully hedged US$ borrower. Comparing a fully hedged borrowing in US$ to acquire C$ with a fixed-to-fixed currency swap out of US$ and into C$, the fully hedged borrower would issue fixed coupon US$ debt, exchange at the spot exchange rate to acquire C$, and fully hedge each of the C$ cash flows required to make payments on the US$ issue. If the US$100,000 issue were made at, say, 9.25%, then using the 8/8/94 LTFX rates the cash flows would be:[19]

$t=1$: (US$9250)(1.3960) = C$12,913 $t=2$: (US$9250)(1.4198) = C$13,133.20
$t=3$: (US$9250)(1.4428) = C$13,345.90 $t=4$: (US$9250)(1.4633) = C$13,535.50
$t=5$: US$(100,000 + 9250)(1.4833) = C$162,051

The associated (rs_T C$ fixed)-to-(rs_T* US$ fixed) currency swap would have higher C$ cash flows at the beginning, narrowing to the point where the currency swap cash flows become lower at maturity.

As before, the perfect markets assumption is retained.[20] It is further assumed that there is no funding advantage for either currency swaps or fully hedged borrowing arising from implicit differences between quoted interest rates on C$/US$ swaps, interest rates quoted for direct borrowings in the cash market or the implied zero coupon rates associated with LTFX. This ***assumption***, that there is ***no funding advantage*** for either currency swaps or fully hedged borrowings, permits differences in borrowing rates to be analysed as deviations from the equilibrium conditions. For the fully hedged borrowing to acquire US$, the cash flows are denominated in US$ even though the actual debt payments are made in C$. Hence, the fully hedged borrower is providing a sequence of US$ cash flows in exchange for PUX_0. All cash flows are in US$ and can be discounted using US spot interest rates, appropriately adjusted for credit risk. Because the perfect markets assumption involves ***no default risk in LTFX*** and, as before, the relevant default risk associated with the inherent risk of the borrower generating sufficient US$ cash flows (that, due to institutional rigidities, may be different than that of the borrower's ability to generate C$ cash flows) is also ignored.

In a fully hedged borrowing, the bond is issued directly in the Canadian debt market, such that:

$$PC_0 = \sum_{t=1}^{T} \frac{CV}{(1 + r_T)^t} + \frac{MV}{(1 + r_T)^T} = \sum_{t=1}^{T} \frac{CV}{(1 + z_t)^t} + \frac{MV}{(1 + z_T)^T}$$

where z_T is the spot interest rate associated with a direct borrowing in the debt market at yield to maturity r_T. For a par bond, $CV = (r_T)(PC_0)$ and $MV = PC_0$. The associated fully hedged US$ cash flows that are used to pay the C$ borrowing are discounted using the US$ spot interest rates of the fully hedged borrower:

$$\frac{PC_0}{S_0} = PUX_0 = \sum_{t=1}^{T} \frac{[CV/F(0,t)]}{(1 + z_t*)^t} + \frac{[MV/F(0,T)]}{(1 + z_T*)^T}$$

The discounting is done with z_t* because a sequence of US$ cash flows is generating a US$ denominated borrowing unconstrained by the requirement of a specific yield to maturity. By equating the C$ cash flows (*CV*, *MV*) for two distinct sets of US$ cash flows and fully covering any unmatched future cash flows, it is possible to derive an absence-of-arbitrage condition under which the fully hedged borrowing and the currency swap will be equivalent. This condition is complicated by forward cover associated with the potential mismatching of the US$ cash flows and the need to adjust principal values at maturity. More expediently, it is also possible to derive a less complicated ***equilibrium*** condition connecting fully hedged borrowings and currency swaps that does not fully cover the mismatched cash flows.

Derivation of the main equilibrium condition for LTFX proceeds by dividing the equation for PC_0 by S_0 and equating with (9). For each $t = 1$ to T it follows:

$$\frac{CV/S_0}{(1 + z_t)^t} = \frac{CV/F(0,t)}{(1 + z_t*)^t} \quad \Rightarrow \quad F(0,t) = \frac{(1 + z_t)^t}{(1 + z_t*)^t} S_0$$

This condition must hold in equilibrium because the cash flows in the two borrowings, PC_0 and PUX_0, have been constructed to be equal in C$ terms at each point in time. The equilibrium condition connecting LTFX and the foreign and domestic spot interest rates involves exploiting the covered interest arbitrage condition for LTFX (see Sec. 4.2):

$$F(0,t) = \frac{(1 + zz_t)^t}{(1 + zz_t^*)^t} S_0$$

where, as before, zz_t* and zz_t are the implied ***zero coupon*** interest rates associated with covered interest arbitrage trades between time *0* and time *t*. Using this condition, the equilibrium condition for LTFX derived from the fully hedged borrowing becomes, for each $t \in [1,T]$:

$$\frac{1 + z_t}{1 + z_t*} = \frac{1 + zz_t}{1 + zz_t*}$$

In words, for each *t*, ***the spot interest rate agio derived from the foreign and domestic debt markets must equal the implied zero coupon interest agio from the LTFX market***.

QUESTIONS

1. Outline appropriate questions to be addressed by a commercial or chartered bank undertaking a financial futures hedging decision. Explain in detail the appropriate hedging strategies for the following:

a) In April, a bank wants to "lock-in" today's interest rate on a $1 million issue of 6-month negotiable CD's due to take place in three months.
d) A money market trader wants to hedge against possible capital losses on a 1 year Government of Canada tbill that is about to be purchased. The bill may be sold at any time between purchase and maturity.
c) In June a bank wants to "lock-in" today's interest rates on a $10 million purchase of 3 month Tbills in September.
d) A bond dealer expects interest rates to rise and wants to protect itself against capital losses on its Tbond inventory.
e) In June, a metals refinery wants to "lock-in" today's price on a purchase of 50,000 lbs. of copper cathodes due to take place in September.

Why will basis variation affect the performance of the hedge? Which of your answers involves a cross hedge?

2. Derive a "closed-form" expression for the risk-minimizing hedge ratio. In what sense is this ratio an optimal hedge ratio? How is your answer affected if the commodity being hedged is undetermined at the time the hedge is "put on", e.g., a wheat farmer hedging the output for a crop that has just been planted.

b) Assuming mean-variance agents, derive an expression for the optimal speculative position size. What happens to this position as the sensitivity of the agent to risk diminishes? Based on this, what can you conclude about the equilibrium in a market dominated by risk-neutral speculators?

3. Derive the profit function for a hedger facing a stochastic output. What are the related expected profit and variance of profit? How would the profit function be affected by the addition of output (e.g., crop) insurance?

4. What is portfolio insurance and what role do stock index futures play in insuring portfolios? What role did stock index futures play in the October 1987 market break? Identify and explain some factors that restrict the execution of stock index futures arbitrages.

5. From Sec. 6.2, discuss the relationship between the mean-variance optimal hedge ratios, h^*_{MV}, which is given in terms of R_f, and (Q_H^*/Q_S), which is given in terms of F(1,T).

6. In calculating the *ex post* rate of return, what is the difference between the arithmetic average and the geometric average? Describe situations where these two methods of calculating rates of return will differ.

7. Suppose a UK zero coupon bond was purchased by a US investor and held for one year. The bond was purchased for £500 when the £/$ exchange rate was $2. (The bond cost $1000.) Due to increases in UK interest rates, the bond was sold for £475 when the exchange rate was $2.20.

a) What was the rate of return on the bond in £?
b) What was the rate of return on the bond in $?
c) What portion of the $ return was due to exchange rate changes?

8. Derive eq. (4) from p. 146 of Eun and Resnick (1994):

$$Var(R_{p,\$}) \approx \sum_{i=1}^{N} \sum_{j=1}^{N} X_i\, X_j\, Cov(R_i,\, R_j)$$
$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} X_i\, X_j\, Cov(e_i,\, e_j)$$
$$+\ 2 \sum_{i=1}^{N} \sum_{j=1}^{N} X_i\, X_j\, Cov(R_i,\, e_j)$$

NOTES

1. In this example, as well as those presented in Tables 6.2 and 6.3, interest expenses are calculated assuming a 90-day period where the actual days between dates are: 9/13/1999 to12/13/1999 = 91 days; 12/13/1999 to 3/13/2000 = 91 days; 3/13/2000 to 6/19/2000 = 98 days; 6/19/2000 /to 9/12/2000 = 85 days.

2. Because h_t enters with a minus sign, this defines a short futures position to be a positive quantity.

3 The transformation from terminal wealth, W_{t+1}, to terminal profit, π_{t+1}, follows because the expectation of $U[W_t]$ reduces to a constant that does not affect the optimization. In addition, it will always be assumed that the only state(conditioning) variables of interest are R_s and R_f. However, in general, this need not be true.

4. More precisely, the variables exhibit autoregressive condition heteroskedastic (ARCH) behaviour. Estimating and testing in ARCH models is currently an active area of research in econometrics (e.g., Engle (1982)). In addition to the ARCH problem, conventional estimates of hedge ratios typically confuse ex ante and ex post distributions.

5. A number of sources have shown that ARCH models are a specific form of random coefficient model.

6 Numerous studies have approached this problem using advanced estimation techniques, e.g., Scott (1989).

7 A number of possible approaches could be taken after the single asset case is considered. Some recent work that has addressed this issue include Filatov and Rappoport (1992), Adler and Jorion (1992) and Black (1989). For example, Black and others claim that there is a "universal currency hedge" that applies to all investors, regardless of nationality. The theoretical underpinnings for this result are derived from the international CAPM that prescribes that all investors should hold a combination of the riskless asset and the world market portfolio, with currency exposure fully hedged. In practice, the optimal hedge ratios are estimated using regression analysis. Unfortunately, a number of theoretical and empirical criticisms have been made of this approach have been offered, e.g., Solnik (1991, p.29-30).

8 The σ_{ij} term is interpreted as being a covariance when $i \neq j$ and as a variance when $i = j$. Because the basic optimization problem is quadratic, it follows that the optimal solutions will take the form of an ellipse or a parabola, e.g., Francis (). Consider the case where the $\{W_i\}$ are restricted to be non-negative, then the solution will be an ellipse. At any given target level of expected return, there will be two values of σ that solve the optimization problem. In evaluating the solutions, it is conventional to ignore the optimal solution that has the higher level of σ and consider only the portfolios that have the lowest σ.

9 Markowitz (2000) reviews the historical development of the model.

10 It is possible to derive the optimal solutions in a number of different ways. In particular, it is possible to

11. Various other writers, e.g., Maldanado and Saunders (1981), have demonstrated the instability of inter-country correlations for periods longer than two quarters.

12 Such is the reason for using moving sampling windows instead of using all the data available. For example, 100 years of monthly data produces estimates of the arithmetic average that would not be affected by an additional observation. Hence, the optimal weights would not change over time.

13 Eun and Resnick (1994) is not without shortcomings. For example, riskfree interest rates are set to zero (p.148) and there seems to have been some problems in identifying some of the *ex ante* optimal tangency portfolio for 7 of the 36 months in the US sample (p.149).

14 Coval and Moskovitz (1999) propose an alternative explanation for the home country bias.

15 The use of the Tbill to model the riskless asset is a common convention in presentations of the closed economy CAPM. This selection embeds an underlying assumption about the investment process driving the rational investor. In particular, the riskless asset is required to have no default risk or price risk. If the investor does rebalancing at regular three month intervals, then the purchase of a three month Tbill will represent a riskless return over that interval. If the rebalancing period is annual, then a 1 year Tbill is needed and so on.

16 Many of the difficulties associated with estimating the closed economy CAPM have been gathered under the heading of Roll's critique.

17 Others such as Levy and Lerman (1988) take the opposite view, arguing in favor of stock/bond portfolios composed largely of foreign bonds (West German for their 1960-80 data). The Levy and Lerman results also indicate mean-variance efficient portfolios that are heavily concentrated in a small number of risky assets, with many asset groups not held.

18 Various developments and combinations on these general types of swaps, not considered here, are possible, e8, cross-currency tloating-to-tloating interest rate. Anti (1986), Price and Henderson (1988), Miron and Swannell (1991) and IFR (1989) provide more in depth discussion of the various types. Swap trading techniques can be used in asset. as well as liability, management, e8, by combining a domestic fixed rate bond purchase with a swap. In addition, various combinations of swaps can be combined in .'completing" a given, intermediated transaction.

19 The rate of 9.25% is not necessarily consistent with absence-of-arbitrage for the 10% C$ offering used in the previous example. Given the sequence of forward exchange rates for different maturities, the precise fixed coupon US$ interest rate that is consistent with absence-of-arbitrage will depend on the sequence of spot interest rates in the US and Canadian debt markets so, at this point, the arbitrary rate of

9.25% can be chosen without significant loss of intuition.

20. The perfect markets assumption does suppress some important issues involved in comparing currency swaps and fully hedged borrowings. For example, assuming that **transactions costs are zero** favours the fully hedged borrowing. This because the currency swap involves only one transaction while the fully hedged borrowing involves a sequence of forward exchange transactions together with the initial and terminal debt market and foreign exchange transactions. This is an extension of a similar result observed for short-term CIP, e.g., Clinton (1988).