# 4. Arbitrage and the Basis

## 4.1 The Cash and Carry Arbitrage

### Cash-and-Carry Arbitrage: The Case of Gold

For storable commodities, the behavior of the basis and the futures basis is determined by the execution of ***cash-and-carry arbitrage*** trades. The use of the term arbitrage in this context has a technical meaning. What is colloquially called an "arbitrage" by market practitioners may, under a more technical definition, be little more than a potentially profitable trading strategy with limited risk of losses. An ***arbitrage*** opportunity is defined here as: ***a riskless trading strategy that generates a positive profit with no net investment of funds***.[1] By construction, for any commodity there will be two associated arbitrage trades: ***the long arbitrage***, where the arbitrage transactions involve holding a long position in the cash commodity; and, ***the short arbitrage***, where the arbitrage involves holding a short position in the cash commodity. In some presentations, the long arbitrage is referred to as the cash and carry arbitrage, with the short arbitrage being the reverse cash and carry.

A fundamental requirement of financial market equilibrium is that observed prices do not permit arbitrage opportunities. In other words, arbitrage profits cannot be positive. This requirement is considerably stronger than the conventional notion of "market efficiency" that requires that security price changes be serially uncorrelated. Arbitrages impose restrictions on spot and derivative prices that, under perfect market assumptions, have to be satisfied. Practical considerations dictate that the short and long arbitrages will provide upper and lower bounds on the derivative price. In turn, these boundary restrictions can be used to formulate speculative trading strategies, as in Chapters 3 and 5.

By design, arbitrages are defined only in terms of ***current*** prices and interest rates. If dependence on expected future values were admitted, the trading strategies would not be riskless. For much the same reason, arbitrages cannot involve a net investment of funds. Where a purchase or short of spot commodities is involved, the arbitrage is referred to as a cash-and-carry arbitrage. Where the purchase of a commodity is involved, a cash-and-carry arbitrage is executed by (risklessly) borrowing funds at the current interest rate and using those funds to purchase the spot commodity, simultaneously contracting to sell the commodity at some future date at the current futures price. Hence, the cash-and-carry arbitrage provides a relationship between the futures and spot prices that depends on the net price of carrying the commodity until delivery. Similarly, other derivative securities such as options on futures contracts also provide instances of arbitrage trades, that may or may not involve purchase of the physical commodity. In this fashion, arbitrages determine relationships among current prices of derivative securities.

In practice, the execution of cash-and-carry arbitrages and the associated behavior of the basis depends on the type of commodity under consideration. Various questions must be addressed to identify the details involved in a specific arbitrage. Is the commodity storable, storable with loss, or not storable? What costs are associated with storage? Is the commodity harvestable? Is there an off-setting carry return? To illustrate how arbitrage trading determining ***the basis*** for a specific commodity, it is convenient to develop the trades ***assuming that markets are perfect***. In other words, there are no transactions costs, either in the form of commission or bid/offer spreads. Other components of perfect markets include no taxes or other regulatory restrictions and equal lending and borrowing rates at riskless rates of interest. Relaxing these assumptions converts the cash and carry equality conditions to be derived into upper and lower bounds on forward prices associated with the long and short arbitrage conditions.

**What are Perfect Capital Markets?**

Various presentations of perfect capital markets are available, with different versions emphasizing elements that are of importance to the argument at hand. One particularly complete set is provided in Haley and Schall (1979).

A.1 **Costless capital markets**: No capital market transactions costs (including commissions and bid/offer spreads), no government restrictions which interfere with capital market transactions, and the costless ability to make financial assets infinitely divisible.

A.2 **Neutral Taxes**: There are no personal or corporate taxes.

A.3 **Competitive Markets**: There are many perfect substitutes for all securities of a firm at any point in time and there is no discrimination in the pricing of these securities such that any security can be acquired at the same market price by all investors. In addition, firms and investors are price takers in investing, borrowing and lending activities.

A.4 **Equal Access**: Investors and firms can borrow, lend and issue claims on the same terms. This assumption requires that borrowing and lending rates be equal.

A.5 **Homogeneous Expectations**: All capital market participants have the same expectations about relevant random variables.

A.6 **No Information Costs**: Firms and individuals have the same available information and this information is acquired at zero cost.

A.7 **No Costs of Financial Distress**: Firms and individuals incur no costs of financial distress or bankruptcy such as legal costs and disruption of operations. This assumption does not rule out the possibility of bankruptcy.

Perhaps the simplest cash-and-carry to consider is the case of gold, a commodity that is storable at low cost, earns no carry return and is not affected by harvests. Assuming for simplicity that there are no other storage costs other than financing charges, i.e., ignoring insurance, vault charges and administrative expenses, a 'long-the-spot' cash-and-carry arbitrage for deliverable gold is described in Figure 4.1.[2] Recalling that absence-of-arbitrage requires that the arbitrage profit be non-positive, $\pi \leq 0$, and observing that $Q > 0$ gives a ***long*** *cash-and-carry arbitrage* restriction on the gold futures price: $F(0,T) \leq S(0)\{1 + r(0,T)\}$. If this condition is violated, the long arbitrage can be profitably executed. When the equality is binding the futures price is said to be at *full carry*, i.e., to reflect the full carrying charges.

*Figure 4.1*: **Profit Function for a Long Gold Cash-and-Carry Arbitrage**

| ***DATE*** | ***Cash Position*** | ***Futures Position*** |
|---|---|---|
| $t=0$ | Borrow $\$[Q_G\, S(0)]$ at interest rate $r(0,T)$ and buy $Q_G$ ounces of gold at $S(0)$ for storage until $t=T$ | Short $Q_G$ units at $F(0,T)$ |

-- The cash gold position provide no pecuniary return between $t=0$ and $t=T$

| | | |
|---|---|---|
| $t=T$ | Deliver the $Q_G$ units against the maturing futures contract and use the proceeds to repay the maturity value of the loan, $\$[Q_G\, S(0)]\{1 + r(0,T)\}$ | |

In this case, the profit function can be specified:

$$\pi(0) \leq \{F(0,T) - S(0)(1 + r(0,T))\}\, Q_G$$

Consider the gold futures price data for Aug. 8, 1994 given in Figure 4.3. Examining the Dec 1994/Dec 1995 price relationship, for the associated closing prices of $382.30 and $403.20, the interest rate implied in the gold futures prices is 5.467%, while Jun95/Jun 96 contracts give an implied interest rate of 5.75% This is consistent with the rates offered on Euro-US deposits (LIBOR) for one year that, on Aug. 8, 1994 were trading at 5 13/16%. The recent relationship between these two rates is discussed further in Sec. 4.2. Examination of the futures price structures for other commodities reveals a range of different relationships. Silver, for example, exhibits a carry rate of 6.45% for the Dec 94/Dec 95. Soybeans exhibit 5.22% for the same Nov 94/Nov 95 differential. The price structure of copper is inverted, with prices for deferred delivery being lower than the nearby contracts. Most of the agricultural commodities exhibit some form of kinking or reversal in the direction of futures prices as delivery dates get more distant, e.g., the CBT wheat contract. The diversity of futures price spread behavior should be apparent. Full carry relationships are the exception, usually applying to the precious metals futures complex.

Because each futures contract involves both a long and a short position, the arbitrage relationship between cash and futures prices involves two trading strategies. In addition to the long-the-cash strategy described above which involves combining a fully leveraged purchase of the spot commodity with a short futures position, it is also possible to combine a short position in the cash commodity with a long futures position, a ***short cash-and-carry arbitrage*** trade. Hence, cash-and-carry arbitrage strategies for futures contracts are said to be *two-sided*, having both a long and a short arbitrage trade to be satisfied. While there are differences across commodities, in practice, due to restrictions on the ability to short the cash commodity, execution of the short *or reverse* cash-and-carry arbitrage can be substantially more difficult than the long arbitrage, that only involves purchase of the spot commodity.[3] In these cases, the cash-futures basis will be determined by what Siegel and Siegel (1990) call *quasi-arbitrages*, typically involving trades with natural hedges in the spot commodity. A classic example would be a jewelry manufacturer holding a gold inventory. If the futures price falls "too far" below full carry, then gold inventories would be sold (or purchases deferred), the funds invested, and the required gold inventory would be hedged using a long futures position. As will be illustrated in Sec. 6.1, banking activities provide a number of excellent examples of natural hedges in currency and other financial commodities.

In the case of gold, numerous mining companies engage in forward selling activities, particularly in North America (where approx. 30% of mine output is sold forward), Australia and, more recently, in South Africa (15% of mine output).[4] Forward sales by mines are combined with deferred delivery of spot transactions, that enable the forward selling mining company to defer or roll forward the delivery date, thereby benefitting from upward price movements. In addition to quasi-arbitrage, in recent years short selling requirements have been facilitated by the use of central bank gold stocks as a source of gold for loan. In addition to other charges, shorting costs involve make a leasing payment to the central bank or other market participant supplying the physical gold for the short. In addition to arbitrage activities, the gold leasing market is also affected by companies involved in gold loans seeking to cover a short term deficiency in physical supplies. Compared to the long arbitrage, the gold lease payment is an offset to the non-financing costs of storage that are associated with carrying a long commodity position through time.

*Figure 4.2*: **Profit Function for a Short Gold Cash-and-Carry Arbitrage**

| ***DATE*** | ***Cash Position*** | ***Futures Position*** |
|---|---|---|
| *t=0* | Borrow $Q_G$ ounces and sell at *S(0)*. Invest the funds received at interest rate *i(0,T)* | Long $Q_G$ ounces at *F(0,T)* |
| *t=T* | Take delivery of the $Q_G$ units against the maturing futures contract, pay with the proceeds of the investment, $\$[Q_G\, S(0)]\, \{1 + i(0,T)\}$, returning the $Q_G$ units to settle the short position | |

In this case, the profit function can be specified:

$$\pi(0) = \{S(0)(1 + i(0,T)) - F(0,T)\}\, Q_G$$

To execute the *short cash-and-carry arbitrage* for gold, the funds received from the sale of the borrowed gold will be invested at a different, probably lower, rate of interest than the long arbitrage. Taking *i(0,T)* to be the all-in lending rate and again ignoring incidental costs, the short arbitrage is described in Figure 4.2. Recalling that absence of arbitrage implies $\pi \leq 0$ and observing that $Q > 0$ gives the short cash-and-carry arbitrage restriction on the gold futures price: $F(0,T) \geq S(0)\,\{1 + i(0,T)\}$. If this inequality is violated the short arbitrage can be profitably executed.

*Figure 4.3* **Futures Prices for Selected Metals**

## METALS AND PETROLEUM

**COPPER-HIGH (CMX) – 25,000 lbs.; cents per lb.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Aug | 108.60 | 108.60 | 108.25 | 108.30 | + 1.65 | 116.00 | 75.30 | 159 |
| Sept | 107.25 | 109.10 | 107.25 | 108.60 | + 1.75 | 116.90 | 74.90 | 25,595 |
| Oct | | .... | .... | 108.75 | + 1.65 | 115.05 | 75.20 | 623 |
| Nov | 109.00 | 109.00 | 108.90 | 108.95 | + 1.60 | 112.80 | 77.75 | 513 |
| Dec | 107.70 | 109.40 | 107.70 | 108.95 | + 1.50 | 115.20 | 75.75 | 12,617 |
| Ja96 | 108.80 | 108.80 | 108.80 | 108.70 | + 1.45 | 111.30 | 76.90 | 341 |
| Feb | | .... | .... | 108.45 | + 1.40 | 111.30 | 87.85 | 245 |
| Mar | 107.90 | 108.50 | 107.80 | 108.25 | + 1.40 | 113.70 | 76.30 | 2,492 |
| Apr | | .... | .... | 107.90 | + 1.35 | 110.40 | 90.10 | 143 |
| May | 106.70 | 107.90 | 106.70 | 107.55 | + 1.35 | 111.40 | 76.85 | 1,062 |
| June | | .... | .... | 107.15 | + 1.30 | 107.30 | 106.70 | 100 |
| July | 107.05 | 107.05 | 107.05 | 106.75 | + 1.30 | 112.50 | 78.00 | 814 |
| Sept | | .... | .... | 105.95 | + 1.30 | 110.05 | 79.10 | 621 |
| Dec | | .... | .... | 105.15 | + 1.30 | 109.00 | 88.00 | 853 |
| Mr96 | | .... | .... | 104.15 | + 1.30 | 105.00 | 99.20 | 134 |

Est vol 12,000; vol Fri 10,861; open int 46,371, –1,025.

**GOLD (CMX) – 100 troy oz.; $ per troy oz.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Aug | 377.00 | 377.80 | 376.40 | 376.70 | – .60 | 415.00 | 341.50 | 1,107 |
| Oct | 379.80 | 380.00 | 379.10 | 379.30 | – .60 | 417.00 | 344.00 | 10,661 |
| Dec | 382.60 | 383.10 | 381.90 | 382.30 | – .60 | 426.50 | 343.00 | 91,590 |
| Fb95 | 386.00 | 386.30 | 385.70 | 385.70 | – .60 | 411.00 | 363.50 | 11,195 |
| Apr | | .... | .... | 389.00 | – .60 | 425.00 | 385.50 | 6,497 |
| June | 392.10 | 392.20 | 392.10 | 392.30 | – .60 | 430.00 | 351.00 | 10,014 |
| Aug | | .... | .... | 395.90 | – .60 | 412.50 | 380.50 | 4,191 |
| Oct | | .... | .... | 399.50 | – .60 | 413.30 | 401.00 | 1,077 |
| Dec | 403.30 | 403.30 | 403.30 | 403.20 | – .60 | 439.50 | 358.00 | 4,807 |
| Fb95 | | .... | .... | 407.00 | – .60 | 424.50 | 412.50 | 1,249 |
| Apr | | .... | .... | 410.90 | – .60 | 430.00 | 418.30 | 1,141 |
| June | | .... | .... | 414.90 | – .60 | 447.00 | 370.90 | 2,865 |
| Dec | | .... | .... | 427.40 | – .60 | 447.50 | 379.60 | 2,696 |
| Ju97 | | .... | .... | 440.20 | – .60 | 456.00 | 436.00 | 1,168 |
| Dec | | .... | .... | 453.20 | – .60 | 477.00 | 402.00 | 1,392 |
| Ju98 | | .... | .... | 466.80 | – .80 | 489.50 | 483.90 | 1,553 |
| Dec | | .... | .... | 480.30 | – 1.00 | 505.00 | 468.00 | 1,388 |

Est vol 14,000; vol Fri 25,886; open int 154,591, +394.

**SILVER (CMX) – 5,000 troy oz.; cents per troy oz.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Aug | | .... | .... | 510.3 | – 4.0 | 558.0 | 525.0 | 0 |
| Sept | 512.0 | 514.5 | 510.5 | 511.3 | – 4.0 | 590.5 | 376.5 | 73,304 |
| Dec | 519.0 | 521.5 | 517.5 | 518.3 | – 4.0 | 597.0 | 380.0 | 29,258 |
| Mr95 | 529.0 | 529.0 | 526.0 | 526.0 | – 4.0 | 604.0 | 416.5 | 6,751 |
| May | | .... | .... | 531.3 | – 4.0 | 606.5 | 418.0 | 3,834 |
| July | 539.0 | 539.0 | 539.0 | 536.9 | – 4.0 | 610.0 | 403.0 | 3,279 |
| Sept | 545.0 | 545.0 | 545.0 | 542.8 | – 4.0 | 615.0 | 493.0 | 647 |
| Dec | 553.0 | 553.0 | 553.0 | 552.0 | – 4.0 | 628.0 | 434.0 | 2,114 |
| Jl96 | | .... | .... | 574.4 | – 4.0 | 630.0 | 524.0 | 943 |
| Dec | 596.0 | 596.0 | 596.0 | 592.0 | – 4.0 | 670.0 | 454.0 | 1,238 |
| Jl97 | | .... | .... | 618.5 | – 4.0 | 655.0 | 588.0 | 483 |
| Dec | | .... | .... | 639.2 | – 4.0 | 695.0 | 502.0 | 307 |
| Dc98 | | .... | .... | 688.5 | – 4.0 | 731.0 | 694.0 | 107 |

Est vol 14,000; vol Fri 16,133; open int 122,386, –1,126.

Source: Wall Street Journal, Monday, Aug. 8, 1994.

The combination of the long and short arbitrage conditions imposes upper and lower boundaries on the gold futures price:

$$S(0)\ \{1 + r(0,T)\} \geq F(0,T) \geq S(0)\ \{1 + i(0,T)\}$$

In an idealized world where $r(0,T) = i(0,T) = r$, the equality condition is binding and $F(0,T) = S(0)\ \{1 + r\}$: the futures price will be fully determined at $t=0$ by the current spot price and the cost of financing. This idealized result requires that: lending and borrowing rates be equal; there are no short sale costs or restrictions; the commodity earns no carry return and is costless to store; and, transactions costs such as those associated with the bid/offer spread are ignored. When the futures prices for gold obey this condition, the commodity is at full carry.

Given the idealized full carry model for gold, it is useful to evaluate what happens to basis behaviour as maturity of the contract approaches. Defining $\Delta X \equiv X(1) - X(0)$ and substituting $X(1) = X(0) + \Delta X$ permits the profit function for a one-to-one long spot/short futures gold position to be expressed as:

$$\pi/Q = \{F(0,T) - S(0)\} - \{F(1,T) - S(1)\}$$
$$= \{r(0,T)\ S(0)\} - \{r(1,T)\ S(1)\} = -\{r(0,T)\ \Delta S + S(1)\ \Delta r\}$$

This result is useful for interpreting the profitability of inventory hedges. Recalling that $r(i,j)$ is not annualized but, rather, reflects the actual return over the i to j holding period, $r(0,T)$ will typically be larger than $r(1,T)$. This is due to the one period reduction in the number of days until maturity for both the loan and the futures contract. Hence, even if $\Delta S = 0$ and the level of annualized interest rates is unchanged, there is a time decay in the basis associated with $\Delta r < 0$ that is fundamental to understanding hedge profit determination. There is a 'time clock' at work in a hedge, acting to reduce the difference between the spot and futures price. The time clock continues to wind down to the point where $r = r(T,T) = 0$ and $F(T,T) = S(T)$. Significantly, this time decay is ***not*** present in the futures basis.

**Factors Impacting the Basis: Generalized Cash-and-Carry Arbitrage Conditions**

As it turns out, the idealized, perfect market conditions required for the long and short arbitrages to be combined and produce an equality condition do not occur in practice. Allowing for the conditions associated with actual markets converts the equality conditions to upper and lower boundaries on the futures (or forward) price provided by the long and short arbitrage conditions. Due to restrictions on the ability to execute the short arbitrage trade, the lower bound on the futures price will often provide a weaker boundary than the upper bound that is determined by the long arbitrage. In some cases, the short arbitrage cannot be executed and the cash-and-carry is said to be *one-sided*. An example of this situation is described in Sec. 4.2. In other cases, most notably for some non-storable commodities, neither the short or long cash-and-carry arbitrage can be executed. Commodities in this group have included potatoes, eggs and onions. The price performance of futures or forward contracts for these commodities is decidedly more erratic when compared to storable commodities.[5] Without the ability to do cash-and-carry arbitrages, price determination relies on expectations about future spot prices. At times, this can create market clearing problems, due to lack of liquidity on one side of the market. Empirically, while a number of different types of contracts have been offered over the years, there are currently only a few futures contracts for truly non-storable commodities.

In addition to the truly non-storables, certain other commodities do not satisfy strict requirements of store ability, such as Treasury bills and feeder cattle, insofar as a delivery in one contract month cannot be carried for delivery against the next contract as would be the case for, say, gold or silver . However, in these cases, non-storability does not significantly affect cash-futures price determination because there are other similar commodities that can be used to do the cash-and-carry arbitrage. For example, in the case of Tbills, even though a 3 month Treasury bill taken for delivery cannot be carried 3 months to the next delivery date (when the Tbill will be maturing), the current 6 month Treasury bill is available to do the arbitrage. Similarly, for feeder cattle, there are numerous opportunities to purchase feed stock which is at a point in the growth cycle that the stock will qualify for future delivery. On balance, it is safe to say there is considerable diversity across commodities in the execution of the short and long cash-and-carry arbitrages.

For analytical purposes, the diversity of arbitrage execution across commodities can be captured by generalizing the profit function for the cash-and-carry arbitrage to include both fixed costs and variable carry costs and carry returns. More precisely, for a transaction starting at $t=0$ and ending at $t=T$:

$$F(0,T) = S(0)\ \{1 + cc(0,T) + CC(0,T) - cr(0,T) - CR(0,T)\}$$
$$= S(0)\ \{1 + r(0,T) + oc(0,T) + CC(0,T) - d(0,T) - cy(0,T) - CR(0,T)\}$$

where $cc(0,T)$ is variable carry costs, expressed as a rate, $cr(0,T)$ is the variable pecuniary and non-pecuniary returns to holding the commodity, also expressed as a rate, $r(0,T)$ is the interest rate portion of carry costs, $CC(0,T)$ is the fixed cost component of carry costs, $cc(0,T) - r(0,T) = oc(0,T)$ is that portion of variable carry costs not attributable to interest carrying costs, $d(0,T)$ is the pecuniary interest or dividend return associated with carrying the commodity, $cy(0,T) = cr(0,T) - d(0,T)$ is that portion of $cr$ not associated with pecuniary interest or dividend return and $CR(0,T)$ is any fixed return earned while holding the commodity. This approach allows for all elements of cost to be introduced, including allowance for spoilage of the commodity during storage. Many commodities, even other metals such as copper and aluminum, have a significant storage component in carry cost.

Historically, the agricultural and industrial commodities were associated were associated with futures trading. For these commodities, it is possible to consider carry costs to be either relative (variable) or absolute (fixed). Relative carry costs are dependent on the value of the commodity and will change as the value of the commodity changes. The most significant relative cost is usually interest expenses. Absolute carry costs are associated with the quantity of the commodity to be stored and do not vary as the price of the commodity changes. These costs include storage, wastage and insurance. It is important to recognize that the basis will behave quite differently depending on whether carry costs are predominately relative, as in the case of gold and other financial commodities, or absolute, as with the grains and industrial commodities.

It is possible to define $ic(0,T) = cc(0,T) - cr(0,T)$ combining the carry cost and carry return elements. For notational convenience, this was done in Chap. 3. However, for present purposes, it is expedient to ignore fixed costs and to work with the additional variable, $cr(0,T)$, to explicitly allow for potential returns the commodity may provide during the arbitrage period. The return captured by $cr$ can be either pecuniary or non-pecuniary:

$$F(0,T) = S(0)\ \{1 + cc(0,T) - cr(0,T)\}$$

Financial futures such as Tbond and stock index futures contracts provide an example where this formulation is useful because of the pecuniary return in the cash and carry arbitrage associated with holding the commodity. Following the discussion of the long spot/short futures inventory hedge for gold, this formulation can be directly applied to the profit function for a inventory hedge for a general commodity. In this case, the profit function can be written:

$$\pi/Q\ = \{F(0,T) - S(0)\} - \{F(1,T) - S(1)\}$$
$$= [S(0)\{cc(0,T) - cr(0,T)\}] - [S(1)\{cc(1,T) - cr(1,T)\}]$$

Again, letting $\Delta X \equiv X(1) - X(0)$ and using $X(1) = X(0) + \Delta X$ permits this form of the profit function to be expressed as:

$$\pi/Q = S(0)\ \{\Delta cr - \Delta cc\} + \{cr(1,T) - cc(1,T)\}\ \Delta S$$
$$= S(0)\ \Delta ic + ic(1)\ \Delta S$$

This provides a general framework for examining the profitability of an inventory hedge as well as numerous other hedge trades. Application to specific cases requires correct interpretation of $cr$. For example, while there are no direct financial returns involved in carrying commodities such as wheat and other grains, there are other benefits. In these cases, $cr$ can be interpreted as the ***convenience yield*** from holding stocks. (Convenience yield is discussed in more detail in Sec. 3.1). This is based on the notion that stocks of a commodity provide some net benefit to the owner. When stocks are low, the convenience yield is high; when stocks are plentiful, the convenience yield is low. The *cr* may, at times be an important element of basis behavior, especially for commodities with high absolute carry cost such as the grains and industrial metals.

In practice, the role of *cr* depends intimately on the importance of absolute carry costs. Commodities with a cash-futures basis determined by relative carry costs, such as the financial futures and gold, usually adhere closely to the cash-and-carry arbitrage condition. When the cash-futures basis depends predominately on absolute carry costs, the arbitrage conditions provide only wide boundaries on the cash-futures basis, and convenience yield can act to off set the costs of carrying the physical. Numerous instances of this are provided in the futures price quotes. For example, this explains how harvestability affects the basis. Stocks of grain are most plentiful after the harvest. The value of stocks carried from one crop year to another will fall by the amount of the associated loss in convenience yield. This will be reflected in futures prices, typically resulting in a discontinuity in the futures price structure occurring with the harvest delivery contract, when stocks of grain will be high and the convenience yield low. This type of discontinuity can be seen in the futures prices for corn, soybeans and wheat. A similar situation occurs in the oil complex contracts and for copper.

**The Term Structure of Futures Prices**

At any time $t$, there are a range of futures prices for a given commodity. This is apparent from a casual inspection of the futures prices in chapter 3 and Figure 4.3. One unresolved question concerns the relationship between $F(t,T)$ and $F(t,N)$, $N \leq T$. Returning to the perfect markets assumption and ignoring fixed costs and returns, the cash and arbitrage conditions for any two futures prices can be specified:

$$F(0,N) = S(0)\,(1 + ic(0,N)) \qquad F(0,T) = S(0)\,(1 + ic(0,T))$$

Taking the ratio of $F(0,T)$ to $F(0,N)$ provides:

$$F(0,T)/F(0,N) = (1 + ic(0,T))/(1 + ic(0,N)) = \{(1 + ic(0,N))(1 + ic(0,T\text{-}N))\}/(1 + ic(0,N))$$

It follows that:

$$F(0,T) = F(0,N)\,(1 + ic(0,T\text{-}N))$$

This relationship holds between any two delivery dates.

The equation $F(0,T) = F(0,N)\,(1 + ic(0,T\text{-}N))$ can be conceptualized as a deferred cash and carry transaction. Consider a trade that is established at $t=0$ and is long $F(0,N)$ and short $F(0,T)$. At time $t=N$, the long position is settled by borrowing $Q\ F(0,N)$ at $r(N,T)$ and incurring other possible carry costs $oc(N,T)$ where $cc(N,T) = r(N,T) + oc(N,T)$. The borrowed funds are used to settle the long position by buying the spot commodity at the agreed price $F(0,N)$. This spot position is carried for $T\text{-}N$ periods, where applicable earning a carry return of $cr(N,T)$. At time $t=T$, the spot commodity is then used to settle the short position at $F(0,T)$. This sequence of transactions can be used to specify $ic(0,T\text{-}N)$ as the implied carry cost, reflected in $F(0,T)$ and $F(0,N)$ at time $t=0$, for a cash and carry arbitrage that will begin at $t=N$ and end at $t=T$. Observe that the actual implied carry earned on the cash and carry transaction between $N$ and $T$, $cc(N,T) - cr(N,T)$, will not be the same as that reflected in futures prices at time $t=0$.

### 4.2 Covered Interest Arbitrage

#### Early Forward Exchange Markets

Unlike some other commodities where trading in time bargains had been conducted at least since the 17th C., large scale forward trading in currencies did not develop until probably the late 1840's in Vienna (Einzig 1937).[6] The emergence of this trade can be attributed to wide fluctuations in the Austrian gulden during the 19th century. For a number of reasons, traditional methods of handling foreign exchange risk, such as trading in foreign bills of exchange, were inadequate for dealing with this exchange rate volatility. While forward trade in sterling developed much later, initial Vienna trading was in German mark notes with delivery dates up to six months in the future. Following the customs of the continental stock exchanges, forward contracts were settled at month end, with speculators covering their positions by settling the differences and renewing their forward contracts for the next end of month settlement. The impact of speculative activity on the foreign exchanges was evident in the movement of end of month exchange rates, which were impacted by speculative covering activities.

The presence of forward trading in Vienna gave impetus to the development of forward trading in Russian roubles, another unstable currency, and Austrian gulden in Berlin. Higher interest rates in Vienna led to active and fully covered interest arbitrage between the two centers. Forward trading also emerged in other centers, such as St. Petersburg. Active forward trading of British sterling, the international unit of account for the 19th and early 20th C., did not develop in London until just before WW I. The absence of a forward market in London can be attributed to the relative stability of sterling up to the period following WW I. In the US, there probably was forward trading in US dollars associated with the currency turbulence created by the Civil War. In the late 1880's, a forward market in French francs emerged in New York, New Orleans and Chicago. This trading was unusual in that there was no forward market for francs in France. In the period before WWI, there was also active forward currency trading in Latin America, Shanghai and Japan, where there had been trading in forward exchange since before the introduction of the gold standard.

An important element leading to the emergence of forward currency markets was instability in the foreign exchanges. Stabilization of the Austro-Hungarian currency (now the krone) in 1892 led to a reduction in commercial forward exchange transactions, even though there still was considerable forward trading associated with stock arbitrage. Forward dealing in currency gradually declined towards the end of the 19th C., being replaced by forward trading using bills of exchange. Vienna, Berlin and St. Petersburg all developed active markets in sterling bills. For many traders, foreign bills of exchange could provide the requisite currency hedging function. In addition, foreign bills were desirable instruments for conducting covered and uncovered interest arbitrage. "Interest arbitrage between Vienna and Berlin on the one hand and the Western European centers on the other, attained a high state of development in the 1890's, but it was usually the central European markets that took the initiative. Very little was known about the Forward Exchange business in London or Paris in those days. To secure a high yield on swap transactions in those days was still the privilege of the select few, who jealously safeguarded the secret of their knowledge....During the years that preceded (WWI), interest arbitrage combined with swap operations between Vienna and other markets assumed such dimensions that the Austro-Hungarian Bank at times considered it necessary to adopt special tactics to counteract its effect." (Einzig 1937, p.42)

**Keynes on Forward Exchange Rates**

John Maynard Keynes, in his *Tract on Monetary Reform*, was one the early popularizers in Britain of forward exchange trading. Writing in 1923, Keynes finds: "The nature of forward dealings in exchange is not generally understood. The rates are seldom quoted in the newspapers. There are few financial topics of equal importance which have received so little discussion or publicity. The present situation did not exist before the war (although even at that time forward rates for the dollar were regularly quoted), and did not begin until after the 'unpegging' of the leading exchanges in 1919, so that the business of the world has only begun to adapt itself. Moreover for the ordinary man, dealing in forward exchange has, it seems, a smack of speculation about it. Unlike the Manchester cotton spinners, who have learnt by long experience that it is not the hedging of open cotton commitments on the Liverpool futures market, but the failure to do so, which is speculative, merchants, who buy or sell goods of which the price is expressed in a foreign currency, do not yet regard it as part of the normal routine of prudent business to hedge these indirect exchange commitments by a transaction in forward exchange." (p.121)

A significant difference between a forward currency market and a forward market in bills was the deliverable instrument involved; one contract involved future delivery of foreign exchange and the other a short term debt instrument denominated in sterling, almost invariably involving an offshore issuer. The forward bill arose in a number of ways, typically with the objective of protecting against changes in the Bank rate. Because stability in the exchanges under the gold standard was usually achieved by significant fluctuations in the Bank rate, this source of risk was often greater than currency changes. For example, a bill secured by a crop to be harvested in the autumn could be sold forward in the spring. Delivery of the crop would give rise to a bill of exchange that would then be used for settlement of the forward position. There was a forward market in foreign sterling bills in London for decades before WWI, with delivery dates as far as six months in the future. The presence of the London forward discount market together with an active market in short term funds gave considerable impetus to the development of a forward foreign exchange market in sterling. Despite playing a significant role in the functioning of the forward markets, the speculative element in the markets was not officially encouraged.

### Covered Interest Arbitrage

While there are a number of possible arbitrages involving foreign exchange rates, covered interest arbitrage is sufficient to determine the forward foreign exchange rate. Section 4.1 gave a brief overview of the notion of a cash-and-carry arbitrage. While there are such arbitrages associated with virtually all futures and forward contracts, covered interest arbitrage is perhaps the most well known example of a cash-and-carry arbitrage. Covered interest arbitrage is based on the notion that, ***in markets where arbitrage is active and unrestricted, securities that differ only by currency of denomination should exhibit fully hedged returns that are approximately equal***. The resulting *covered interest parity* (CIP) condition establishes an arbitrage relationship between: domestic and foreign interest rates and the current forward and spot exchange rates. CIP arbitrages have been executed almost from the beginning of forward exchange trading, though as late as the 1920's Keynes observed that: "It must be remembered that the floating capital normally available, and ready to move from center to center for the purpose of taking advantage of moderate arbitrage between spot and forward exchange, is by no means unlimited in amount, and is not always adequate to meet the market's requirements." (Tract, p.129)

**A Stylized Example of Covered Interest Arbitrage**

Patrick Yamada, a trader in the foreign exchange department of Sanwa Bank, Singapore office, specializes in arbitraging U.S. dollars against Deutschemarks. He observes the following rates at 9:10 am Singapore time:

Spot rate: DM1.8200 = $1.0000
Three Month Forward Rate: DM1.8000= $1.0000

Yamada can borrow or invest U.S. dollars for three months at 9% per annum or Deutschemarks for three months at 5% per annum. He has a borrowing limit of $5,000,000 or the equivalent in DM.

a) Ignoring transactions costs, how can Yamada make a riskless arbitrage profit? Assume that Yamada desires to take any profits in dollars.

b) If the dollar three month interest rate on US dollars were 10%, instead of 9%, all other factors remaining the same, would Yamada still make a profit using the strategy outlined in a)? If not, is there another set of transactions which would provide an arbitrage profit?

c) If the transactions costs in a) or b) were above $7000 and were to be paid out of final proceeds, would this change the strategies described in a) or b)?

For expository purposes, development of the CIP condition requires a number of assumptions and definitions to be introduced. For example, the method of determining the exchange rate must be given as either units of domestic to foreign currency or the converse. To see the difference, consider that on Apr. 12, 1983, the $/^ exchange rate was 1.5285 while the ^/$ exchange rate was .6542.[7] Similarly, for the Aug. 27, 1992 cash exchange rates: the C$/US$ rate is 1.196 and the US$/C$ rate is .8361. Figure 4.4 provides the foreign exchange rate data from the *Globe and Mail* for Aug. 8, 1994. Following the convention on US currency futures markets, so-called ***US direct terms*** will be used that involves units of US$ to units of foreign currency (or domestic currency for non-US traders).[8] This will also apply to the spot exchange rate, meaning S(0) will be measured in US direct terms. If the opposite convention of quoting the exchange rate is used, so-called ***foreign direct terms***, this changes the identification of the foreign and domestic interest rate. However, the substance of the CIP condition is unchanged.

*Figure 4.4* **Selected Foreign Exchange Rates**

## CROSS RATES

| | Canadian dollar | U.S. dollar | British pound | German mark | Japanese yen | Swiss franc | French franc | Dutch guilder | Italian lira |
|---|---|---|---|---|---|---|---|---|---|
| Canada dollar | — | 1.3797 | 2.1289 | 0.8735 | 0.013610 | 1.0362 | 0.2550 | 0.7777 | 0.000875 |
| U.S. dollar | 0.7248 | — | 1.5430 | 0.6331 | 0.009864 | 0.7510 | 0.1848 | 0.5637 | 0.000634 |
| British pound | 0.4697 | 0.6481 | — | 0.4103 | 0.006393 | 0.4867 | 0.1198 | 0.3653 | 0.000411 |
| German mark | 1.1448 | 1.5795 | 2.4372 | — | 0.015581 | 1.1863 | 0.2919 | 0.8903 | 0.001002 |
| Japanese yen | 73.48 | 101.37 | 156.42 | 64.18 | — | 76.14 | 18.74 | 57.14 | 0.064291 |
| Swiss franc | 0.9651 | 1.3315 | 2.0545 | 0.8430 | 0.013135 | — | 0.2461 | 0.7505 | 0.000844 |
| French franc | 3.9216 | 5.4106 | 8.3486 | 3.4255 | 0.053373 | 4.0635 | — | 3.0498 | 0.003431 |
| Dutch guilder | 1.2858 | 1.7741 | 2.7374 | 1.1232 | 0.017500 | 1.3324 | 0.3279 | — | 0.001125 |
| Italian lira | 1142.86 | 1576.80 | 2433.03 | 998.29 | 15.554286 | 1184.23 | 291.43 | 888.80 | — |

**Mid-market rates in Toronto at noon, Aug. 8, 1994. Prepared by the Bank of Montreal Treasury Group.**

| | | $1 U.S. in Cdn.$ = | $1 Cdn. in U.S.$ = |
|---|---|---|---|
| U.S./Canada spot | | 1.3797 | 0.7248 |
| 1 month forward | | 1.3808 | 0.7242 |
| 2 months forward | | 1.3818 | 0.7237 |
| 3 months forward | | 1.3827 | 0.7232 |
| 6 months forward | | 1.3862 | 0.7214 |
| 12 months forward | | 1.3973 | 0.7157 |
| 3 years forward | | 1.4457 | 0.6917 |
| 5 years forward | | 1.4917 | 0.6704 |
| 7 years forward | | 1.5622 | 0.6401 |
| 10 years forward | | 1.6547 | 0.6043 |
| Canadian dollar | High | 1.3083 | 0.7644 |
| in 1994: | Low | 1.3990 | 0.7148 |
| | Average | 1.3712 | 0.7293 |

| Country | Currency | Cdn. $ per unit | U.S. $ per unit |
|---|---|---|---|
| Britain | Pound | 2.1289 | 1.5430 |
| 1 month forward | | 2.1294 | 1.5421 |
| 2 months forward | | 2.1297 | 1.5412 |
| 3 months forward | | 2.1297 | 1.5402 |
| 6 months forward | | 2.1320 | 1.5380 |
| 12 months forward | | 2.1385 | 1.5304 |
| Germany | Mark | 0.8735 | 0.6331 |
| 1 month forward | | 0.8739 | 0.6329 |
| 3 months forward | | 0.8751 | 0.6329 |
| 6 months forward | | 0.8787 | 0.6339 |
| 12 months forward | | 0.8891 | 0.6363 |
| Japan | Yen | 0.013610 | 0.009864 |
| 1 month forward | | 0.013650 | 0.009886 |
| 3 months forward | | 0.013728 | 0.009928 |
| 6 months forward | | 0.013873 | 0.010008 |
| 12 months forward | | 0.014211 | 0.010170 |
| Algeria | Dinar | 0.0436 | 0.0316 |
| Antigua, Grenada and St. Lucia | E.C.Dollar | 0.5119 | 0.3711 |
| Argentina | Peso | 1.38205 | 1.00170 |
| Australia | Dollar | 1.0213 | 0.7402 |
| Austria | Schilling | 0.12403 | 0.08990 |
| Bahamas | Dollar | 1.3797 | 1.0000 |
| Barbados | Dollar | 0.6933 | 0.5025 |
| Belgium | Franc | 0.04245 | 0.03077 |
| Bermuda | Dollar | 1.3797 | 1.0000 |
| Brazil | Real | 1.509519 | 1.094092 |
| Bulgaria | Lev | 0.0258 | 0.0187 |
| Chile | Peso | 0.003281 | 0.002378 |
| China | Renminbi | 0.1604 | 0.1162 |
| Cyprus | Pound | 2.8744 | 2.0833 |
| Czech Rep | Koruna | 0.0490 | 0.0355 |
| Denmark | Krone | 0.2220 | 0.1609 |
| Egypt | Pound | 0.4076 | 0.2954 |
| Fiji | Dollar | 0.9548 | 0.6920 |
| Finland | Markka | 0.2662 | 0.1929 |
| France | Franc | 0.2550 | 0.1848 |
| Greece | Drachma | 0.00578 | 0.00419 |
| Hong Kong | Dollar | 0.1786 | 0.1294 |
| Hungary | Forint | 0.01258 | 0.00912 |
| Iceland | Krona | 0.01972 | 0.01429 |
| India | Rupee | 0.04397 | 0.03187 |
| Indonesia | Rupiah | 0.000636 | 0.000461 |
| Ireland | Punt | 2.1068 | 1.5270 |
| Israel | N Shekel | 0.4531 | 0.3284 |
| Italy | Lira | 0.000875 | 0.000634 |
| Jamaica | Dollar | 0.04415 | 0.03200 |
| Jordan | Dinar | 1.9852 | 1.4388 |
| Lebanon | Pound | 0.000824 | 0.000597 |
| Luxembourg | Franc | 0.04245 | 0.03077 |
| Malaysia | Ringgit | 0.5346 | 0.3874 |
| Mexico | N Peso | 0.4074 | 0.2953 |
| Netherlands | Guilder | 0.7777 | 0.5637 |
| New Zealand | Dollar | 0.8340 | 0.6045 |
| Norway | Krone | 0.1999 | 0.1449 |
| Pakistan | Rupee | 0.04519 | 0.03275 |
| Philippines | Peso | 0.05276 | 0.03824 |
| Poland | Zloty | 0.0000603 | 0.0000437 |
| Portugal | Escudo | 0.00859 | 0.00623 |
| Romania | Leu | 0.0008 | 0.0006 |
| Russia | Ruble | 0.000661 | 0.000479 |
| Saudi Arabia | Riyal | 0.3679 | 0.2667 |
| Singapore | Dollar | 0.9164 | 0.6642 |
| Slovakia | Koruna | 0.0437 | 0.0317 |
| South Africa | Rand | 0.3821 | 0.2770 |
| South Korea | Won | 0.001719 | 0.001246 |
| Spain | Peseta | 0.01062 | 0.00770 |
| Sudan | Dinar | 0.0445 | 0.0322 |
| Sweden | Krona | 0.1787 | 0.1295 |
| Switzerland | Franc | 1.0362 | 0.7510 |
| Taiwan | Dollar | 0.0524 | 0.0380 |
| Thailand | Baht | 0.0553 | 0.0401 |
| Trinidad, Tobago | Dollar | 0.2475 | 0.1794 |
| Turkey | Lira | 0.0000441 | 0.0000320 |
| Venezuela | Bolivar | 0.00812 | 0.00589 |
| Zambia | Kwacha | 0.002090 | 0.001515 |
| European Currency Unit | | 1.6701 | 1.2105 |
| Special Drawing Right | | 1.9950 | 1.4460 |

The U.S. dollar closed at $1.3772 in terms of Canadian funds, down $0.0095 from Friday. The pound sterling closed at $2.1201, down $0.0182.

In New York, the Canadian dollar closed up $0.0050 at $0.7261 in terms of U.S. funds. The pound sterling was down $0.0026 to $1.5394.

Source: Globe and Mail, Monday, August 8, 1994.

**Solution to Yamada's Stylized Arbitrages**

a) Yamada can make an arbitrage profit by doing a ***long (DM)*** covered interest arbitrage. The arbitrage is short because it involves borrowing in US and investing in DM. This arbitrage involves the following sequence of transactions which will all be executed at 9:10 am Singapore time:

Borrow $5,000,000 for three months. In three months time, the amount owing on this borrowing will be: ($5 mil)(1 + (.09/4)) = $5,112,500

Exchange the $5 mil. at the spot exchange rate to get ($5 mil)(1.82) = 9.1 mil DM.

Invest the 9.1 mil. DM for three months. In three months time, the investment will mature to a value: (9.1 mil)(1 + (.05/4)) = 9,213,750 DM

Sell the maturing value of the DM investment for US dollars using a three month forward exchange contract. At the quoted forward exchange rate of 1.8, the DM investment will produce (9,213,750/1.8) = $5,118,750

In three months time, the DM investment will mature and the proceeds delivered on the forward exchange contract. The proceeds of the forward contract will be used to settle the maturing three month loan producing an arbitrage profit of $5,118,750 - $5,112,500 = $6250.

b) If the US interest rate is 10%, instead of 9%, then the cost of the US$ borrowing would be (5 mil)(1 + (.1/4)) = $5,125,000. Because this exceeds the covered return which could be received on the DM investment, the short arbitrage would not be profitable. However, in the absence of transactions costs, it would now be possible to do the **long** arbitrage, which would involve borrowing in DM and investing in the US. In this case the profit would be $5,125,000 - $5,118,750 = $6250.

c) The presence of a $7000 transaction cost would prevent either the long or the short arbitrage from being executed. This illustrates the point that covered interest arbitrage only provides upper and lower boundaries on the available combinations of interest rates and exchange rates that are consistent with absence of arbitrage at a specific point in time.

NOTE: In actual practice, the presence of transaction costs dictates that the spot and forward transactions will combined into one transaction, a foreign exchange swap.

The basics of the arbitrage trading strategy can be illustrated by considering a stylized cash-and-carry arbitrage trade between US dollars and a foreign currency for 1 year securities. If the covered foreign interest rate ***exceeds*** the rate on a comparable US security, the trade described in Figure 4.5 can be executed at $t=0$. Assuming perfect capital markets, this trade will generate an arbitrage profit by assumption because the amount received on the covered foreign investment will be more than the cost of the US dollar borrowing.

*Figure 4.5*: **Short Covered Interest Arbitrage Trade**

**At *t=0***

| US asset | Exchange Market | Foreign (Canadian) asset |
|---|---|---|
| Borrow *$Q* for 1 year at $r(0,1)$ | Buy *$Q/S(0)* Canadian dollars, spot | Invest *$Q/S(0)* for 1 year at $r^*(0,1)$ |
| | Sell *($Q/S(0))(1+r*(0,1))* Canadian dollars forward at $F(0,1)$ | |

**At *t=1*** Use the funds from the maturing foreign asset to settle the forward exchange position by paying the foreign currency and receiving US dollars. Use these dollars to settle the US dollar loan.

where: $F(0,1)$ = the 1 year forward exchange rate in US direct terms; $S(0)$ = the spot exchange rate in US direct terms; $r(0,1)$ = the domestic (US) interest rate on a 1 year zero coupon security (quoted on a 365 day basis); $r^*(0,1)$ = the foreign (Canadian) one year interest rate (quoted on a 365 day basis).

To see how the series of transactions in Figure 4.5 translates into an arbitrage profit function, consider that the fully covered value of the foreign asset at maturity is *F(0,1){$Q/S(0)}(1+r*)* while the amount to be repaid at maturity of the loan is *$Q (1+r)*. This produces the arbitrage profit function associated with the ***short arbitrage****:*

$$\pi_s(0) = F(0,1)\{\$Q/S(0)\}(1+r^*) - Q\,(1+r) \leq 0$$

The $\leq 0$ condition is required for absence of arbitrage.

To this point, much of the discussion of arbitrage transactions has assumed perfect markets. This assumption permits the profit functions for both the short and long arbitrages to be combined to produce an equality relationship involving forward and spot prices. When markets are not assumed to be perfect, as is the case in actual markets, then the short and long arbitrage conditions provide upper and lower boundaries on the futures or forward price. To see how this occurs, relax the assumption that lending and borrowing rates are equal by letting $y$ and $y^*$ denote the interest rates applicable to the ***long covered interest arbitrage trade*** done using covered Canadian borrowing to finance a US asset position. A sequence of transactions similar to those in Figure 4.5 produces the arbitrage profit function:

$$\pi_L(0) = Q\,(1+y) - F(0,1)\{\$Q/S(0)\}(1+y^*) \leq 0$$

In this case $y = y(0,1)$ is a lending rate and $y^* = y^*(0,1)$ is a borrowing rate, while for the short arbitrage $r$ is a borrowing rate and $r^*$ is a lending rate.

Manipulating the long arbitrage condition gives:

$$F(0,1) \geq \{(1+y)/(1+y^*)\}\, S(0)$$

while manipulation of the short arbitrage condition gives:

$$F(0,1) \leq \{(1+r)/(1+r^*)\}\, S(0)$$

The upper and lower arbitrage boundaries can now be expressed as:

$$\frac{1 + y}{1 + y^*} \leq \frac{F(0,1)}{S(0)} \leq \frac{1 + r}{1 + r^*}$$

These boundaries can be equivalently expressed as:

$$\frac{y - y^*}{1 + y^*} \leq \frac{F(0,1) - S(0)}{S(0)} \leq \frac{r - r^*}{1 + r^*}$$

The validity of these boundary conditions can be verified by observing that $y^* > r^*$ and $r > y$, because rationality requires that borrowing rates are always at least as high as lending rates. A number of studies further develop the upper and lower arbitrage boundaries by further relaxing the perfect market assumptions, particularly the assumption of no transactions costs, e.g., Deardorff (1969), Clinton (1988).

In order to arrive at the conventional equality conditions, assume that both the long and short arbitrages impose the binding restriction that $\pi=0$. This result is achieved by going back to the perfect markets assumption where $y=r$ and $y^*=r^*$ or, in words, lending rates and borrowing rates in each country are equal. Imposing this condition on the long and short arbitrage profit functions and manipulating gives an equality relationship. Recognizing that there are a wide range of possible maturity dates for forward contracts, the conventional CIP condition can be stated:[9]

$$F(0,T) = \frac{1 + r(0,T)}{1 + r^*(0,T)}\, S(0)$$

In terms of the cost of carry model in Section 4.1, $F(0,T) = \{1 + cc(0,T) - cy(0,T)\}\, S(0)$, the CIP condition can be expressed as:

$$F(0,T) = \{1 + \frac{r(0,T) - r^*(0,T)}{1 + r^*(0,T)}\}\, S(0) \qquad or \qquad cc(0,T) - cy(0,T) = \frac{r(0,T) - r^*(0,T)}{1 + r^*(0,T)}$$

where $r$ is the approximate carry cost and $r^*$ is the approximate carry return.

Extending this result to currency futures, the nearby contract can be taken as a replacement for the spot position:[10]

$$F(0,T) = \frac{1 + i(0,T-N)}{1 + i^*(0,T-N)}\, F(0,N) = \{1 + \frac{i(0,T-N) - i^*(0,T-N)}{1 + i^*(0,T-N)}\}\, F(0,N) \qquad (5.1)$$

where: $i(0,T\text{-}N)$ and $i^*(0,T\text{-}N)$ = the time 0 domestic (US) and foreign (Canadian) interest rates adjusted by $(T\text{-}N)/365$ to account for the trading horizon. (This notation takes no account of the term structure of rates implied by currency futures.) Figure 4.6 provides data for currency and Eurocurrency futures prices on Aug. 8, 1994. (See end of Chapter Questions for a useful exercise concerning this Table and the CIP condition.) The CIP relationships apply to money market instruments. A number of issues require attention before applying CIP to long term, $T > 1$ year, forward contracts.

In the discussion of cash-and-carry arbitrages in Sec. 4.1, it was demonstrated that, for a number of reasons, the short and long arbitrages would only provide upper and lower boundaries on the futures price. Only in idealized conditions would equality conditions be binding. In the case of CIP, transactions costs and other factors provide the basis for precisely specifying the boundaries. Clinton (1988) demonstrates the importance of considering how the arbitrage trade is executed in the foreign exchange market, correcting one of the numerous errors in the 1970's literature on CIP, e.g., Deardorff (1979).

Consider the short covered interest arbitrage trade described previously that involved both a spot and a forward foreign exchange transaction, in addition to the two security market transactions. Assuming no brokers fees were incurred on these transactions and that trading limits on forward exchange positions are not binding, the primary source of transactions costs for a CIP trade done in the forward market will be the bid/offer spread. (In practice, while there are futures contracts for a wide range of currencies, the bulk of covered interest arbitrage activity takes place in the inter-bank forward and cash markets.) In order to reduce the transactions costs associated with foreign exchange transactions, banks do matched spot/forward transactions by combining the trades in the form of a ***foreign exchange swap***.[11] This involves only one swap trade bid/offer transaction cost, instead of the two bid/offers arising from the spot and forward trades. Hence, to be consistent with the actual arbitrage execution, the stylized example above should have one swap transaction instead of a spot/forward combination.

**Bids and Offers in Foreign Exchange**

Quoting of bid/offer rates in the FX market is complicated because two units of account are being exchanged. This requires a convention to how the quotation method relates to which currency is being sold and which is being purchased. A further complication is the presence of conflicting market conventions for stating exchange rate quotes.

For example, say the bid/offer rates on the C$/US$ are 1.4955/1.4975 which means that FX dealers are willing to ***buy*** one US$ at the bid in exchange for 1.4955 C$. The dealer will give the customer 1.4955 C$ in exchange for a US$. Similarly, the dealer offers to ***sell*** one US$ to customers in exchange for 1.4975. This is consistent with the intuitive result that the bid rate will be less than the offer rate in order for the dealer to earn the spread, .0020 in this example.

The potential confusion arises because buying US$ involves selling C$. Buying one currency involves selling another. To see how to avoid possible confusion, consider the corresponding US$/C$ bid/offer quotes of .66778/.66867. Using these quotes, FX dealers are willing to ***buy*** one C$ in exchange for .66778 US$ and to ***sell*** one C$ in exchange for .66867 US$. Again the dealer will make the spread, which is .00089 when rates are expressed as US$/C$.

To avoid potential confusions that can arise, make reference to conventional bid/offer rates for other commodities. For example, the bid/offer on gold being $260/$261. These quotes are $/oz. and involve buying or selling gold in exchange for dollars. The ounce of gold that is being bought and sold appears in the denominator of the quote. Similarly, the bid/offer quotes for the currency that is being bought and sold is associated with the currency appearing in the denominator of the quote. If the exchange rate is quoted as US$/C$ then the bid/offer quotes will refer to C$ transactions.

A fundamental observation that can be made about the CIP condition concerns the number of variables involved in the CIP condition: given the spot exchange rate, two interest rates determine the forward rate. Given that there is only one forward-spot pairing for a given delivery date, there can only be one interest rate pair, domestic and foreign, that can determine the forward rate. At least since Einzig (1970), it has been recognized that the smallest empirical deviations from CIP occur when Euromarket interest rates are used (e.g., Marston 1976, Poitras 1989). Conditions in this market are close to providing the idealized conditions required for CIP to hold precisely. In addition, the primary participants in the Eurocurrency deposit markets are also the most important participants in the swap, forward and spot markets for foreign exchange.

Because interest rates for different money market securities tend to follow different paths, deviations from CIP for any given interest rate pairing can be considerable. This basic point was misunderstood by Frenkel and Levitch

(1975, 1977, 1981), Otani and Tiwari (1981), Bahmani-Oskooee and Das (1985), Sharpe (1984), Overturf (1986) and Prachowny (1970) who used other interest rates than Euro-deposit rates and incorrectly drew conclusions about excess deviations from covered interest arbitrage. In particular, when treasury bills are used to evaluate the CIP condition neither the short or long arbitrage is executable. (This follows because it is not possible for arbitragers to borrow in the treasury bill market). For this reason, it is not possible to draw any conclusions about deviations from arbitrage conditions when treasury bills are selected as the operative lending and borrowing rates.

In addition to pairing treasury bill rates together, there are a number of other pairings that can be considered. Interest rate pairings for securities that could be used by arbitragers for borrowing programs, such as US/Canadian commercial paper or bankers' acceptances tend to exhibit small (but still larger than Euro) deviations from CIP, consistent with institutional limitations on accessing these types of funds. To see this, consider the Canadian dollar exchange rates and money market interest rates on Aug. 8, 1994 given in Figures 4.6 and 4.9. The US direct terms spot is .7248 and the 3 month forward is .7232. This translates into a 3-month interest agio of 0.9977924. Using the appropriately adjusted US (4.77%, 360-day basis) and Canadian (5.61%, 365-day basis) 3 month BA rates, the interest agio is calculated by, first converting the US discount rate to the coupon yield equivalent, and then calculating the agio using annualized rates that have been divided by four (90/365 to be exact), to account for the 90 day maturity. The resulting value of 0.99826 is seen to differ from the arbitrage value by about 4 basis points (not annualized). Similar calculations for US (4.80%) and Canadian (5.73%) commercial paper gives 0.9980177, a smaller differential.

Other pairings to consider are where one of the rates can be used to borrow while the other cannot. These situations lead to *one-sided arbitrage* conditions. To see this, observe that the ready availability of arbitrage funds at the Euro-U.S. rate defines an arbitrage relationship between the Euro-U.S. rate and other non-Euro-currency assets.[12] For example, when the other rate is for treasury bills, whenever the covered domestic treasury bill rate rises above the Euro-U.S. rate arbitragers will borrow funds at the Euro-U.S. rate, convert the funds into domestic dollars and purchase domestic treasury bills. At the same time, the amount of funds to be received upon maturity of the treasury bill will be covered forward. If the maturity of the treasury bill and the Euro-deposit are the same, the trade will generate a theoretically riskless profit. This arbitrage establishes an *upper* bound on the domestic treasury bill rate-- the covered Euro-U.S. deposit rate.

**Triangular Arbitrage and Bid/Offer Rates**

Triangular arbitrage ensures consistency between exchange rate quotes for different currencies. More precisely, given the FX quote for currency A/currency B and the FX quote for currency B/currency C, triangular arbitrage ensures consistency of the FX quote for currency A/currency C. In practice, triangular arbitrage provides outer boundaries for the bid/offer rates that prevail for a given currency, relative to other traded currencies. The actual observed bid/offer rates will depend on competitive factors prevailing in the market for a given currency.

To see how triangular arbitrage constrains the bid/offer rate, consider the following quotes for S$/US$ and DM/US$: S$/US$ bid = 1.3410, S$/US$ offer = 1.3490 and DM/US$ bid = 1.4035, DM/US$ offer = 1.4100. To construct the offer rate for S$/DM consistent with triangular arbitrage involves starting with S$ and then exchanging these into US$ at the offer rate, 1.3490, and using these US$ to buy DM at the bid rate, 1.4035. The resulting calculation, 1.3490/1.4035 = .9612, which is the FX rate at which triangular arbitrage ***offers to sell*** S$ for DM. Similarly, for the bid rate to buy S$ in exchange for DM, involves selling DM for US$ at the offer rate of 1.4100 and using these US$ to buy S$ at the bid rate, 1.3410. The resulting calculation of 1.3410/1.4100 = .9511 is the FX rate that triangular arbitrage ***bids to buy*** S$ for DM. Hence, the triangular arbitrage bound on the bid/offer rate for S$/DM is .9511/.9612.

*Figure 4.6* **Currency Futures and Euro Deposit Futures**

## CURRENCY

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| **JAPAN YEN (CME) – 12.5 million yen; $ per yen (.00)** | | | | | | | | |
| Sept | 1.0012 | 1.0016 | .9850 | .9878 | – .0129 | 1.0408 | .8942 | 65,543 |
| Dec | .9985 | .9985 | .9920 | .9948 | – .0131 | 1.0490 | .9525 | 6,056 |
| Mr95 | 1.0130 | 1.0130 | 1.0000 | 1.0025 | – .0133 | 1.0560 | .9680 | 1,052 |
| Jun | .... | .... | .... | 1.0114 | – .0135 | 1.0670 | .9915 | 268 |
| Est vol 24,931; vol Fri 24,956; open int 72,941, +687. | | | | | | | | |
| **DEUTSCHEMARK (CME) – 125,000 marks; $ per mark** | | | | | | | | |
| Sept | .6336 | .6338 | .6301 | .6311 | – .0018 | .6595 | .5364 | 89,149 |
| Dec | .6310 | .6332 | .6308 | .6315 | – .0018 | .6606 | .5351 | 4,567 |
| Mr95 | .... | .... | .... | .6327 | – .0018 | .6595 | .5798 | 1,532 |
| Est vol 23,876; vol Fri 43,379; open int 95,294, +2,455. | | | | | | | | |
| **CANADIAN DOLLAR (CME) – 100,000 dlrs.; $ per Can $** | | | | | | | | |
| Sept | .7214 | .7258 | .7214 | .7255 | + .0052 | .7740 | .7068 | 32,802 |
| Dec | .7195 | .7242 | .7195 | .7239 | + .0055 | .7670 | .7038 | 2,534 |
| Mr95 | .7170 | .7221 | .7170 | .7221 | + .0058 | .7618 | .7020 | 700 |
| June | .7150 | .7180 | .7150 | .7188 | + .0061 | .7600 | .6990 | 377 |
| Est vol 15,365; vol Fri 3,365; open int 36,494, +1,096. | | | | | | | | |
| **BRITISH POUND (CME) – 62,500 pds.; $ per pound** | | | | | | | | |
| Sept | 1.5420 | 1.5456 | 1.5366 | 1.5382 | – .0028 | 1.5764 | 1.4440 | 31,389 |
| Dec | 1.5362 | 1.5400 | 1.5350 | 1.5360 | – .0028 | 1.5760 | 1.4400 | 766 |
| Mr95 | | .... | .... | 1.5340 | – .0020 | 1.5750 | 1.4530 | 152 |
| Est vol 6,930; vol Fri 11,704; open int 32,307, -343. | | | | | | | | |
| **SWISS FRANC (CME) – 125,000 francs; $ per franc** | | | | | | | | |
| Sept | .7518 | .7522 | .7478 | .7490 | – .0015 | .7817 | .6590 | 39,514 |
| Dec | .7495 | .7532 | .7495 | .7506 | – .0015 | .7840 | .6885 | 1,561 |
| Est vol 12,166; vol Fri 17,207; open int 41,106, +531. | | | | | | | | |
| **AUSTRALIAN DOLLAR (CME) – 100,000 dlrs.; $ per A.$** | | | | | | | | |
| Sept | .7415 | .7430 | .7385 | .7388 | – .0039 | .7467 | .6645 | 8,418 |
| Est vol 806; vol Fri 3,067; open int 8,472, +842. | | | | | | | | |
| **U.S. DOLLAR INDEX (FINEX) – 1,000 times USDX** | | | | | | | | |
| Sept | 89.65 | 90.23 | 89.61 | 90.11 | + .29 | 98.55 | 86.85 | 6,557 |
| Dec | 89.95 | 90.50 | 90.28 | 90.37 | + .29 | 99.00 | 87.08 | 3,108 |
| Est vol 2,300; vol Fri 2,711; open int 9,675, +784. | | | | | | | | |
| The index: High 90.02; Low 89.50; Close 89.91 +.28 | | | | | | | | |

Source: *Wall Street Journal*, Monday, August 8, 1994.

*Figure 4.6* **Currency Futures and Euro Deposit Futures** (cont.)

**STERLING (LIFFE) – £500,000; pts of 100%**

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 94.10 | 94.15 | 94.09 | 94.13 | + .02 | 95.25 | 90.10 | 97,877 |
| Dec | 93.33 | 93.42 | 93.33 | 93.40 | + .06 | 95.23 | 90.10 | 171,388 |
| Mr95 | 92.78 | 92.84 | 92.74 | 92.82 | + .06 | 95.10 | 90.70 | 64,708 |
| June | 92.31 | 92.37 | 92.28 | 92.37 | + .08 | 94.23 | 91.73 | 51,323 |
| Sept | 91.92 | 92.01 | 91.90 | 92.00 | + .09 | 94.81 | 91.46 | 42,165 |
| Dec | 91.63 | 91.72 | 91.61 | 91.70 | + .09 | 94.66 | 91.14 | 35,157 |
| Mr96 | 91.39 | 91.49 | 91.37 | 91.49 | + .11 | 94.61 | 90.82 | 23,179 |
| June | 91.20 | 91.30 | 91.20 | 91.29 | + .08 | 94.25 | 90.60 | 21,963 |
| Sept | 91.05 | 91.10 | 91.05 | 91.10 | + .06 | 93.64 | 90.40 | 11,912 |
| Dec | 90.88 | 90.92 | 90.88 | 90.92 | + .07 | 93.50 | 90.20 | 11,484 |
| Mr97 | 90.77 | 90.79 | 90.77 | 90.78 | + .04 | 93.33 | 90.05 | 9,549 |
| June | 90.65 | 90.69 | 90.65 | 90.66 | + .04 | 90.90 | 89.92 | 2,686 |

Est vol 33,164; vol Fri 55,930; open int 543,391, +5,775.

**EUROMARK (LIFFE) – DM 1,000,000; pts of 100%**

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 95.06 | 95.07 | 95.05 | 95.06 | – .01 | 95.54 | 91.81 | 169,353 |
| Dec | 94.96 | 94.97 | 94.94 | 94.94 | – .03 | 95.73 | 91.83 | 168,509 |
| Mr95 | 94.68 | 94.69 | 94.66 | 94.68 | – .02 | 95.83 | 92.45 | 155,996 |
| June | 94.35 | 94.37 | 94.33 | 94.35 | – .01 | 95.91 | 93.15 | 97,541 |
| Sept | 94.09 | 94.09 | 94.06 | 94.07 | – .03 | 95.83 | 93.62 | 63,420 |
| Dec | 93.81 | 93.82 | 93.78 | 93.81 | – .01 | 95.74 | 93.43 | 54,859 |
| Mr96 | 93.61 | 93.63 | 93.60 | 93.61 | – .02 | 95.62 | 93.25 | 38,230 |
| June | 93.46 | 93.47 | 93.43 | 93.46 | – .01 | 95.45 | 93.05 | 19,193 |
| Sept | 93.31 | 93.32 | 93.31 | 93.30 | – .03 | 94.26 | 92.82 | 8,911 |
| Dec | 93.12 | 93.12 | 93.12 | 93.10 | .... | 94.10 | 92.22 | 11,788 |
| Mr97 | .... | .... | .... | 92.98 | .... | 93.99 | 92.45 | 6,556 |
| June | 92.86 | 92.86 | 92.81 | 92.82 | – .01 | 93.10 | 92.29 | 4,327 |

Est vol 46,083; vol Fri 154,991; open int 798,683, –1,972.

**EUROSWISS (LIFFE) – SFr 1,000,000; pts of 100%**

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 95.62 | 95.64 | 95.61 | 95.61 | – .02 | 96.75 | 95.37 | 25,024 |
| Dec | 95.51 | 95.52 | 95.48 | 95.48 | – .04 | 96.80 | 95.14 | 12,280 |
| Mr95 | 95.31 | 95.31 | 95.27 | 95.27 | – .04 | 96.15 | 94.80 | 10,933 |
| June | 95.02 | 95.02 | 94.97 | 94.97 | – .05 | 95.22 | 94.42 | 3,237 |

Est vol 3,650; vol Fri 6,892; open int 51,474, +1,548.

**3-MONTH EURO LIRA (LIFFE) – ItI 1,000,000; pts of 100%**

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 91.30 | 91.30 | 91.26 | 91.27 | – .03 | 91.35 | 91.02 | 27,631 |
| Dec | 90.93 | 90.93 | 90.89 | 90.90 | – .05 | 91.00 | 90.74 | 48,506 |
| Mr95 | 90.52 | 90.54 | 90.51 | 90.53 | – .05 | 90.58 | 90.40 | 13,359 |
| June | 90.07 | 90.08 | 90.07 | 90.06 | – .06 | 90.15 | 89.97 | 12,035 |
| Sept | 89.85 | 89.85 | 89.85 | 89.80 | .... | 89.85 | 89.68 | 8,117 |
| Dec | 89.55 | 89.55 | 89.55 | 89.48 | – .03 | 89.55 | 89.41 | 3,336 |

Est vol 1,915; vol Fri 5,479; open int 112,984, –656.

Source: Wall Street Journal, Monday, August 8, 1994.

The arbitrage trade when the covered domestic treasury bill rate exceeds the Euro-U.S. rate is apparent. Yet, there is no practical arbitrage trade when the Euro-U.S. rate exceeds the covered domestic treasury bill rate. In this case, when the treasury bill rate falls below the covered Euro-U.S. rate, the implied covered interest arbitrage trade would involve borrowing at the domestic treasury bill rate, converting to U.S. dollars and investing in an appropriately dated Euro-US deposit-- simultaneously selling forward the funds to be received upon maturity of the Euro-U.S. deposit. This trade cannot be executed because only the domestic government has the ability to issue liabilities in the domestic treasury bill market. As a consequence, the connection between these two rates is limited, the Euro-US rate can move to a substantial premium over the covered Canadian. The size of this potential premium depends on what can be descriptively defined as *quasi-arbitrage* support where cash market investor activity assumes importance.

The need for quasi-arbitrage support extends to the case where US and domestic tbill rates are considered. For example, investors may not want the additional risk associated with a Euro-US investment, opting instead to buy US Tbills that are perceived by the market to be less risky than Canadian tbills (or Euros). Hence, substitution between US and Canadian Tbills by asset holders would be based on covered parity plus an adjustment for the differential risk characteristics of the two instruments. This type of quasi-arbitrage support will provide a weak lower bound on the Canadian tbill rates. For this lower bound, Tbill investors and other important players are required to react to the size of the covered differential with US Tbills or Euro-US deposits primarily by making portfolio adjustments. The adjustment process is conditioned by the level of overnight foreign and Canadian interest rates, tax rates, foreign exchange market conditions, Canadian monetary policy and so on.

### Forward-Forward Arbitrage and Swap Arbitrage

Forward-forward covered interest arbitrage determines the relationship between the price of forward contracts for different delivery dates. The arbitrage is interesting both in the method used to construct the underlying trades and in the associated covered interest arbitrage conditions. The arbitrage extends naturally to arbitrage between swap contracts. As illustrated in the description of the short forward-forward arbitrage in Figure 4.7, the trades involve careful selection of the principal amounts of the borrowings and the forward contracts. Extending the arbitrage to swap contracts imposes additional restrictions on the principal amounts. These additional restrictions can be satisfied by "tailing" the initial swap trades. This is accomplished using additional out-right forward positions to supplement the forward contracts embedded in the initial swap transactions.

*Figure 4.7*: **Short Forward-Forward Arbitrage**

***At t=0***

| ***US (Domestic) Market*** | ***Exchange Market*** | ***Foreign Market*** |
|---|---|---|
| Borrow $\$Q(1+r^*(0,N))$ at $r(0,T)$ | Sell US$ forward $Q\$(1+r^*(0,N))(1+r(0,N))$ at $F(0,N)$ | Borrow $(\$Q/F(0,N))(1+r(0,N))$ at $r^*(0,N)$ |
| Invest $Q\$(1+r^*(0,N))$ at $r(0,N)$ | | Invest $(\$Q/F(0,N))(1+r(0,N))$ at $r^*(0,T)$ |
| | Buy US$ forward $(\$Q/F(0,N))(1+r(0,N))(1+r^*(0,T))$ at $F(0,T)$ | |

***At t=N***

The US investment will mature to give $\$Q(1+r^*(0,N))(1+r(0,N))$ that is used to deliver on the forward position that matures at $t=N$. The amount of foreign currency received will be $(\$Q/F(0,N)(1+r^*(0,N))(1+r(0,N))$ which is the amount owing on the foreign borrowing maturing at $t=N$. ***The cash flows at t=N all cancel.***

***At t=T***

The $T$ period foreign investment will mature to $(\$Q/F(0,N))(1+r(0,N))(1+r^*(0,T))$. This amount is delivered against the forward contract to obtain US$ which can be used to settle the loan. The resulting US$ cash flow will have to be less than or equal to the maturing value of the US$ $T$ period loan in order to ensure absence of arbitrage opportunities.

At $t=0$, the forward-forward arbitrage involves borrowing and investing offsetting market values in both the foreign and domestic markets. In the US or domestic market, the amount borrowed and invested is equal to $\$Q(1 + r^*(0,N))$ while in the foreign market the amount borrowed and invested is $(\$Q/F(0,N))(1 + r(0,N))$. Because the maturity dates of the investments and borrowings are different, these cash market transactions result in a forward starting loan. For example, when the US investment matures at t=N, the investment will still have $T$-$N$ days to go. In the absence of the foreign transactions, the trader would raise the funds required by borrowing at $t=N$ at $r(N,T-N)$, an interest rate that is ***uncertain*** at $t=0$. The forward-forward arbitrage avoids this uncertainty by paying the balance due on the domestic loan using the fully covered proceeds of the foreign loan. This offset is achieved by careful selection of the market values of the initial foreign and domestic transactions.

The result of these transactions is a forward-starting covered interest arbitrage trade that commences at $t=N$. At $t=T$, the short arbitrage profit function provides the restriction:

$$\pi_s = \frac{\$Q}{F(0,N)}(1 + r(0,N))(1 + r^*(0,T))\ F(0,T) - \$Q(1 + r*(0,N))(1 + r(0,T)) \leq 0$$

Much as with the spot-forward covered interest arbitrage, this condition can be manipulated and combined with the long arbitrage condition to produce the perfect markets result:

$$F(0,T) = \frac{\dfrac{1 + r(0,T)}{1 + r(0,N)}}{\dfrac{1 + r^*(0,T)}{1 + r^*(0,N)}} F(0,N) = \frac{1 + i(0,T-N)}{1 + i^*(0,T-N)} F(0,N)$$

Closer examination of this condition reveals a direct connection between i and *i**, the interest rates determining the relationship between forward exchange rates for different delivery dates, and implied forward rates determined from the cash market yields *[r(0,T), r(0,N)] and [r*(0,T), r*(0,N)]*.

More precisely, the ***implied forward rate*** for money market securities can be defined:

$$1 + i(0,T-N) = \frac{1 + r(0,T)}{1 + r(0,N)} \quad \rightarrow \quad (1 + i(0,T-N))(1 + r(0,N)) = 1 + r(0,T)$$

In effect, the *t=0* implied forward rate *(i(0,T-N))* is the ***breakeven interest rate*** that will equate the buy and hold return on a *T* period security with the return earned on a rollover investment. The rollover investment involves buying an *N* period security, holding to maturity, and investing the proceeds at *t=N* in a *T-N* period security that matures at time *T*. Remembering that the arbitrage is done in the Euro markets is significant because the major currencies also have actively traded Euro futures/forward contracts. Because these contracts provide the market with a traded implied forward rate, this means that the relevant Euro futures interest rates for the appropriate maturity can be used as the operative interest rates in evaluating the forward-forward arbitrage condition.[13]

Where implied forward rates are calculated using money market securities, there is a subtle difference when compared to implied forward rates from bonds. For example, if *T* is six months and the annualized *T* period interest rate is 10% then *r(0,T)* = 5%. If N is three months and the associated *N* period rate is also 10% annualized, then *r(0,N)* = 2.5%. In this case, *i(0,T-N)*, which is the 3 month interest rate starting in three months, is determined as *(1.05) = (1.025)(1 + i(0,T-N)* or *i(0,T-N)* = 2.439%. Using a straight forward application of intuition associated with implied forward rate calculations from bond markets, it may seem that *i(0,T-N)* would have to be 2.5%, this does not happen because of the linear relationship among money market interest rates.

The forward-forward arbitrage extends naturally to the case where the forward contracts are embedded in swap transactions. However, in the swap arbitrage it is not possible to have both the spot/forward FX transactions as well as the cash market transactions in both the foreign and domestic markets at *t=0* to offset. Much as with the spot-forward covered interest arbitrage that is executed using a swap, this complication is handled by adjusting one of the forward positions using an outright forward contract.

For example, consider the case where the trader is short the swap for forward delivery at *t=N* and long the swap for delivery at *t=T*. Using Figure 4.7 as a reference point, having the *t=0* value of the spot FX transactions equal can be accomplished by having the spot FX values being exchanged at *t=0* set equal to *$Q(1+r(0,N))(1+r*(0,N))*. In this case, the trader has no position in spot currency. However, unlike the forward-forward arbitrage, the trader now has ***uncertainty*** associated with the principal value applicable to the forward transactions for *t=T*. The cash market transactions require that *($Q/F(0,N))(1+r(0,N))(1+r*(0,T))* units of FX be delivered for exchange into US$ at *F(0,T)*. But the corresponding spot FX transaction only provided for *($Q/S(0))(1+r(0,N))(1+r*(0,N))*. This leaves a residual forward position to be covered that is equal to *($Q/S(0))(1+r*(0,N))(r*(0,T)-r(0,N))*.

**Designing Speculative Trading Strategies**

*Figure 4.8*: **Bankers' Foreign Exchange Swap Speculation**

***At t=0***

***Nearby (N) Swap Contract***

Short the Swap that sells
$Q of domestic currency
in exchange for a forward purchase of
[*$Q/S(0)*] of FX at *F(0,N)*.

***Deferred (T) Swap Contract***

Long the Swap that buys
Q$ of domestic currency
in exchange for selling
forward [*$Q/S(0)]* of FX at *F(0,T)*

***NOTE: The $Q spot FX transactions at t=0 cancel.***

**At *t=N***

Receive delivery of domestic currency from the maturing nearby swap = *[$Q/S(0)]* at *F(0,N)* that is immediately swapped by entering into a *T-N* period swap that delivers *[$Q/S(0)] F(0,N)* of domestic currency in exchange for receiving *[(Q/S(0)) F(0,N)]/S(N)* at *F(N,T)* at time *T*

**At *t=T***

The swap from *t*=0 is required to deliver domestic currency of *[Q/S(0)]* at *F(0,T)* while the swap from *t=N* receives domestic currency of *[(Q/S(0))(F(0,N)/S(N)]* at *F(N,T)*

$$\pi(T) = [Q/S(0)]\ \{[F(N,T)/S(N)] - [F(0,T)/F(0,N)]\}$$

$$= [Q/S(0)]\ \{ (1 + ic(N,T)) - (1 + ic(0,T\text{-}N))\} = [Q/S(0)]\ \{ ic(N,T) - ic(0,T\text{-}N) \}$$

Observing that $ic = (r - r^*)/(1 + r^*)$ and substituting where appropriate reveals the connection between the profit function of bankers' swap speculation and the profit function for a 1-1 spread.

The CIP condition can be used to specify various types of speculative currency spread trading strategies, including speculative trades used by banks in the swap market. For illustrative purposes, the general results can be done using the CIP condition expressed in terms of forward contracts. Assuming that the CIP condition holds with equality, manipulation of the CIP equation gives:

$$i(0,T-N) = i = i^*(0,T-N) + (1 + i^*(0,T-N))\ \frac{F(0,T)-F(0,N)}{F(0,N)} = i^* + (1 + i^*)\ \frac{F(T)-F(N)}{F(N)}$$

Now, consider a first difference equation for this CIP condition:[14]

$$\Delta i - \{\Delta i^* + (1 + i^*)\ \Delta[\frac{F(T)-F(N)}{F(0,N)}] + \frac{F(T)-F(N)}{F(N)}\ \Delta[(1 + i*)]\} = 0$$

Assuming that the last term can be safely ignored for present purposes as it is of second order, expanding and manipulating gives an approximation to the condition desired, where *FPS(t) = {F(t,T) - F(t,N)}*:

$$\Delta i = \Delta i^* + (1 + i^*)\left[\frac{\Delta(FPS)}{F(N)} - \frac{FPS}{F(N)}\frac{\Delta F(N)}{F(N)}\right]$$

$$= \Delta i^* + (1 + i^*)\frac{\Delta(FPS)}{F(N)} - (i - i^*)\frac{\Delta F(N)}{F(N)}$$

This result can be rearranged to get the profit function for a one-to-one currency spread:

$$\Delta FPS = \Delta\{F(T) - F(N)\} = \frac{F(1,N)}{(1 + i^*)}\ \{\Delta i - \Delta i^*\} + \frac{(i - i^*)}{(1 + i^*)}\ \Delta F(N)$$

Examining this result, it can be seen that the profit function for a one-to-one currency spread depends on: the difference in changes of US and Canadian interest rates; and, the change in the level of the exchange rate. Insofar as *(i - i*)* is small, there is no need to tail the spread or, in other words, the $\Delta F$ term is effectively zero. In this case, the profit function will be dominated by the first term, the difference in the changes of domestic and foreign interest rates. Poitras (1997) provides a substantial development of these concepts.

Application of this profit function to swap speculation follows from inspection of Figure 4.8 where the profit function can be written as:

$$\pi(T) = \frac{Q}{S(0)}\left[\frac{F(N,T)}{S(N)} - \frac{F(0,T)}{F(0,N)}\right]$$

$$= \frac{Q}{S(0)}\{ic(N,T) - ic(0,T-N)\} = \frac{Q}{S(0)}\left[\frac{r(N,T) - r^*(N,T)}{1 + r^*(N,T)} - \frac{i(0,T-N) - i^*(0,T-N)}{1 + i^*(0,T-N)}\right]$$

$$\cong \frac{Q}{S(0)}\{(r(N,T) - r^*(N,T)) - (i(0,T-N) - i^*(0,T-N))\}$$

The logic associated with the profit function for the one-to-one currency spread can be applied directly to the swap speculation. Hence, the analysis of speculative profits on interbank swap speculation can be approached using the techniques developed for the general analysis of speculative trading strategies.

**Long Term Forward Exchange Rates**

Extending the Figure 4.5 conventional covered interest arbitrage formula for pricing short term forward exchange contracts to long term, N year forward contracts gives:

$$FF_N = \frac{(1 + z_N)^N}{(1 + z_N^*)^N}\ S$$

where: $FF_N$ is the *N* year, theoretical arbitrage consistent forward exchange rate in domestic direct terms, *S* is the spot exchange rate in domestic direct terms, $z_N$ is the N year domestic zero coupon interest rate, $z^*_N$ is the *N* year zero coupon foreign interest rate.[15] Consistent with an arbitrage relationship, all variables in the long term CIP equation are available at the same date, say *t=0*. Manipulation of the CIP condition leads immediately to an expression for the covered interest rate deviations:

$$z_N - z_N^* - \left[\left\{\frac{F_N}{S}\right\}^{1/N} - 1)(1 + z_N^*)\right] = deviation_N$$

where $F_N$ is the forward exchange rate quoted in the foreign exchange market. Figure 4.9 provides long term *FX* rates for Aug. 8, 1994 (see also Figure 4.4).

**Example: Solving the Canadian Spot Interest Rates**

Assume annual coupon payments

From Figures 4.6 and 4.10 pick the following bills/bonds

1 year tbill $z_1 = .0720$

6.5% 1 Aug 1996 $P_2 = 97.505$ ($y_2 = .07887$)
7.5% 1 Jul 1997 $P_3 = 98.225$ ($y_3 = .08197$)
6.5% 1 Sep 1998 $P_4 = 93.350$ ($y_4 = .08474$)
7.75 1 Sep 1999 $P_5 = 96.800$ ($y_5 = .08542$)

Two Possible Bootstrap Solution Techniques: the direct approach and the par bond approach.

The direct approach involves using the observed price and coupon to solve for the spot interest rate.

Solving for $z_2$: $97.505 = 6.5/(1 + z_1) + 106.5/(1 + z_2)^2$

Using this method, $z_2 = .0792$

For the Par Bond approach, use the result that when the stated yield to maturity equals the C/M then the bond sells at par:

$100 = 7.887/(1 + z_1) + 107.887/(1 + z_2)^2$

Using this method $z_2 = .0791428$

Why the difference, when the method would appear to be exactly the same?

Yields used are semi-annual, not annual, compounded while the prices/coupons are exact (except that annual coupon payments have been assumed in making the calculations).

The differences involved are generally small:

Par bond: $z_3 = .082423$ $z_4 = .0854697$ $z_5 = .0861374$

Price/Coupon: $z_3 = .08232$ $z_4 = .08595$ $z_5 = .08630$

*Figure 4.9* **Money Market Interest Rates**

# MONEY RATES

**ADMINISTERED RATES**

| | |
|---|---|
| Bank of Canada | 5.70% |
| Canadian prime | 7.25% |

**MONEY MARKET RATES**
(for transactions
of $1-million or more)

| | |
|---|---|
| 3-mo. T-bill(when-issued) | 5.58% |
| 1-month treasury bills | 5.21% |
| 2-month treasury bills | 5.40% |
| 3-month treasury bills | 5.50% |
| 6-month treasury bills | 6.10% |
| 1-year treasury bills | 7.20% |
| 10-year Canada bonds | 9.03% |
| 30-year Canada bonds | 9.18% |
| 1-month banker's accept. | 5.46% |
| 2-month banker's accept. | 5.56% |
| 3-month banker's accept. | 5.61% |
| Commercial Paper (R-1 Low) | |
| 1-month | 5.60% |
| 2-month | 5.68% |
| 3-month | 5.73% |
| Call money | 5.25% |

**Supplied by Dow Jones Telerate Canada**

**UNITED STATES**

NEW YORK (AP) — Money rates for Monday as reported by Telerate Systems Inc:

Telerate interest rate index: 4.820

Prime Rate: 7.25

Discount Rate: 3.50

Broker call loan rate: 6.00

Federal funds market rate: High 4.375, low 4.3125, last 4.3125

Dealers commercial paper: 30-180 days: 4.48-5.15

Commercial paper by finance company: 30-270 days: 4.43-4.71

Bankers acceptances dealer indications: 30 days, 4.45; 60 days, 4.64; 90 days, 4.77; 120 days, 4.86; 150 days, 5.05; 180 days, 5.12

Certificates of Deposit Primary: 30 days, 3.40; 90 days, 3.85; 180 days, 4.23

Certificates of Deposit by dealer: 30 days, 4.47; 60 days, 4.67; 90 days, 4.80; 120 days, 4.91; 150 days, 5.12; 180 days, 5.21

Eurodollar rates: Overnight, 4.25-4.375; 1 month, 4.50-4.5625; 3 months, 4.8125-4.875; 6 months, 5.25-5.3125; 1 year, 5.75-5.8125

London Interbank Offered Rate: 3 months, 4.75; 6 months, 5.1875; 1 year, 5.5625

Treasury Bill auction results: average discount rate: 3-month as of Aug. 8: 4.43; 6-month as of Aug. 8: 4.93

Treasury Bill, annualized rate on weekly average basis, yield adjusted for constant maturity, 1-year, as of Aug. 1: 5.51

Treasury Bill market rate, 1-year: 5.29-5.27

Treasury Bond market rate, 30-year: 7.53

Source: Globe and Mail, Monday, August 8, 1994.

*Figure 4.9* **Money Market Interest Rates (cont.)**

# MONEY RATES

Monday, August 8, 1994

The key U.S. and foreign annual interest rates below are a guide to general levels but don't always represent actual transactions.

**PRIME RATE:** 7 1/4%. The base rate on corporate loans posted by at least 75% of the nation's 30 largest banks.

**FEDERAL FUNDS:** 4 3/8% high, 4 3/16% low, 4 1/4% near closing bid, 4 5/16% offered. Reserves traded among commercial banks for overnight use in amounts of $1 million or more. Source: Prebon Yamane (U.S.A.) Inc.

**DISCOUNT RATE:** 3 1/2%. The charge on loans to depository institutions by the Federal Reserve Banks.

**CALL MONEY:** 6%. The charge on loans to brokers on stock exchange collateral. Source: Dow Jones Telerate Inc.

**COMMERCIAL PAPER** placed directly by General Electric Capital Corp.: 4.42% 30 to 44 days; 4.45% 45 to 59 days; 4.58% 60 to 74 days; 4.65% 75 to 89 days; 4.70% 90 to 119 days; 4.75% 120 to 149 days; 4.95% 150 to 179 days; 5.05% 180 to 265 days; 5.18% 266 to 270 days.

**COMMERCIAL PAPER:** High-grade unsecured notes sold through dealers by major corporations: 4.52% 30 days; 4.69% 60 days; 4.80% 90 days.

**CERTIFICATES OF DEPOSIT:** 3.73% one month; 3.90% two months; 4.09% three months; 4.50% six months; 4.91% one year. Average of top rates paid by major New York banks on primary new issues of negotiable C.D.s, usually on amounts of $1 million and more. The minimum unit is $100,000. Typical rates in the secondary market: 4.44% one month; 4.76% three months; 5.18% six months.

**BANKERS ACCEPTANCES:** 4.40% 30 days; 4.59% 60 days; 4.72% 90 days; 4.83% 120 days; 5.04% 150 days; 5.14% 180 days. Offered rates of negotiable, bank-backed business credit instruments typically financing an import order.

**LONDON LATE EURODOLLARS:** 4 9/16% - 4 7/16% one month; 4 3/4% - 4 5/8% two months; 4 7/8% - 4 3/4% three months; 5% - 4 7/8% four months; 5 3/16% - 5 1/16% five months; 5 5/16% - 5 3/16% six months.

**LONDON INTERBANK OFFERED RATES (LIBOR):** 4 9/16% one month; 4 7/8% three months; 5 5/16% six months; 5 13/16% one year. The average of interbank offered rates for dollar deposits in the London market based on quotations at five major banks. Effective rate for contracts entered into two days from date appearing at top of this column.

**FOREIGN PRIME RATES:** Canada 7.25%; Germany 5%; Japan 3%; Switzerland 7.50%; Britain 5.25%. These rate indications aren't directly comparable; lending practices vary widely by location.

**TREASURY BILLS:** Results of the Monday, August 8, 1994, auction of short-term U.S. government bills, sold at a discount from face value in units of $10,000 to $1 million: 4.43%, 13 weeks; 4.93%, 26 weeks.

**FEDERAL HOME LOAN MORTGAGE CORP.** (Freddie Mac): Posted yields on 30-year mortgage commitments. Delivery within 30 days 8.63%, 60 days 8.70%, standard conventional fixed-rate mortgages; 5.625%, 2% rate capped one-year adjustable rate mortgages. Source: Dow Jones Telerate Inc.

**FEDERAL NATIONAL MORTGAGE ASSOCIATION** (Fannie Mae): Posted yields on 30 year mortgage commitments (priced at par) for delivery within 30 days 8.57%, 60 days 8.66%, standard conventional fixed rate-mortgages; 6.85%, 6/2 rate capped one-year adjustable rate mortgages. Source: Dow Jones Telerate Inc.

**MERRILL LYNCH READY ASSETS TRUST:** 3.77%. Annualized average rate of return after expenses for the past 30 days; not a forecast of future returns.

Source: Wall Street Journal, Monday, August 8, 1994.

*Figure 4.10* **Long Term Forward Exchange Rates**

# Currency Cross Rates

Supplied by Royal Bank of Canada - indicative wholesale late afternoon rates

| | C$ | US$ | DM | Yen | £ | Fr. fr. | Sw. fr. | A$ |
|---|---|---|---|---|---|---|---|---|
| Canadian $ | ... | 1.37775 | 0.87026 | 0.01357 | 2.12070 | 0.25399 | 1.03164 | 1.01905 |
| U.S.$ | 0.72582 | ... | 0.63165 | 0.00985 | 1.53925 | 0.18435 | 0.74878 | 0.73965 |
| Deutschmark | 1.14908 | 1.58315 | ... | 0.01559 | 2.43686 | 0.29185 | 1.18544 | 1.17098 |
| Japanese yen | 73.69 | 101.53 | 64.13 | ... | 156.27 | 18.72 | 76.02 | 75.09 |
| British pnd. | 0.47154 | 0.64967 | 0.41036 | 0.00640 | ... | 0.11977 | 0.48646 | 0.48053 |
| French franc | 3.93722 | 5.42450 | 3.42640 | 0.05343 | 8.34966 | ... | 4.06177 | 4.01223 |
| Swiss franc | 0.96933 | 1.33550 | 0.84357 | 0.01315 | 2.05567 | 0.24620 | ... | 0.98780 |
| Australian $ | 0.98130 | 1.35199 | 0.85399 | 0.01332 | 2.08105 | 0.24924 | 1.01235 | ... |

# Forward Exchange Rates

Supplied by Royal Bank of Canada - indicative wholesale late afternoon rates

| Per US$ | Spot | 1-mo | 3-mo | 6-mo | 1-yr | 2-yr | 3-yr | 4-yr | 5-yr |
|---|---|---|---|---|---|---|---|---|---|
| Canadian $ | 1.3778 | 1.3789 | 1.3808 | 1.3844 | 1.3960 | 1.4198 | 1.4428 | 1.4633 | 1.4883 |
| Japanese yen | 101.53 | 101.33 | 100.89 | 100.10 | 98.58 | 94.53 | 91.83 | 89.62 | 87.33 |
| Deutschmark | 1.5832 | 1.5839 | 1.5839 | 1.5816 | 1.5754 | 1.7032 | 1.7531 | 1.8032 | 1.8532 |
| British pound* | 1.5393 | 1.5386 | 1.5367 | 1.5347 | 1.5275 | 1.5768 | 1.6118 | 1.6517 | 1.6868 |
| **Per C$** | | | | | | | | | |
| U.S.$ | 0.7258 | 0.7252 | 0.7242 | 0.7224 | 0.7164 | 0.7043 | 0.6931 | 0.6834 | 0.6719 |
| Japanese yen | 73.69 | 73.49 | 73.06 | 72.30 | 70.61 | 66.58 | 63.65 | 61.25 | 58.68 |
| Deutschmark | 1.1491 | 1.1487 | 1.1471 | 1.1424 | 1.1285 | 1.1996 | 1.2151 | 1.2323 | 1.2452 |
| British pound* | 2.1207 | 2.1214 | 2.1219 | 2.1246 | 2.1324 | 2.2386 | 2.3254 | 2.4169 | 2.5103 |

* inverted

**Mid-market rates in Toronto at noon, Aug. 8, 1994.**

| | | $1 U.S. in Cdn.$ = | $1 Cdn. in U.S.$ = |
|---|---|---|---|
| U.S./Canada spot | | 1.3797 | 0.7248 |
| 1 month forward | | 1.3808 | 0.7242 |
| 2 months forward | | 1.3818 | 0.7237 |
| 3 months forward | | 1.3827 | 0.7232 |
| 6 months forward | | 1.3862 | 0.7214 |
| 12 months forward | | 1.3973 | 0.7157 |
| 3 years forward | | 1.4457 | 0.6917 |
| 5 years forward | | 1.4917 | 0.6704 |
| 7 years forward | | 1.5622 | 0.6401 |
| 10 years forward | | 1.6547 | 0.6043 |
| Canadian dollar | High | 1.3083 | 0.7644 |
| in 1994: | Low | 1.3990 | 0.7148 |
| | Average | 1.3712 | 0.7293 |

*Figure 4.11* **Canadian Bond Interest Rates**

# CANADIAN BONDS

**Selected quotations, with changes since the previous day, on actively traded bond issues, provided by RBC Dominion Securities. Yields are calculated to full maturity, unless marked C to indicate callable date. Price is the midpoint between final bid and ask quotations Aug. 8, 1994.**

| Issuer | Coupon | Maturity | Price | Yield | $ Chg |
|---|---|---|---|---|---|
| **GOVERNMENT OF CANADA** | | | | | |
| CANADA | 4.75 | 15 MAR 96 | 95.625 | 7.708 | +0.300 |
| CANADA | 6.50 | 1 AUG 96 | 97.505 | 7.857 | +0.360 |
| CANADA | 7.75 | 15 SEP 96 | 99.795 | 7.652 | +0.420 |
| CANADA | 7.50 | 1 JUL 97 | 98.225 | 8.197 | +0.425 |
| CANADA | 6.25 | 1 FEB 98 | 93.600 | 8.418 | +0.450 |
| CANADA | 6.50 | 1 SEP 98 | 93.350 | 8.474 | +0.500 |
| CANADA | 5.75 | 1 MAR 99 | 89.850 | 8.486 | +0.600 |
| CANADA | 7.75 | 1 SEP 99 | 96.800 | 8.542 | +0.600 |
| CANADA | 9.25 | 1 DEC 99 | 102.550 | 8.634 | +0.600 |
| CANADA | 9.75 | 1 JUN 01 | 104.650 | 8.821 | +0.700 |
| CANADA | 9.50 | 1 OCT 01 | 103.425 | 8.835 | +0.700 |
| CANADA | 9.75 | 1 DEC 01 | 104.750 | 8.848 | +0.700 |
| CANADA | 8.50 | 1 APR 02 | 97.950 | 8.872 | +0.700 |
| CANADA | 7.25 | 1 JUN 03 | 89.650 | 8.973 | +0.700 |
| CANADA | 7.50 | 1 DEC 03 | 90.800 | 8.977 | +0.750 |
| CANADA | 10.25 | 1 FEB 04 | 107.750 | 9.014 | +0.850 |
| CANADA | 6.50 | 1 JUN 04 | 84.000 | 8.989 | +0.750 |
| CANADA | 9.00 | 1 DEC 04 | 99.863 | 9.017 | +0.955 |
| CANADA | 10.00 | 1 JUN 08 | 106.650 | 9.139 | +0.900 |
| CANADA | 9.50 | 1 JUN 10 | 102.850 | 9.152 | +0.950 |
| CANADA | 9.00 | 1 MAR 11 | 98.650 | 9.159 | +0.950 |
| CANADA | 10.25 | 15 MAR 14 | 109.400 | 9.204 | +1.000 |
| CANADA | 9.75 | 1 JUN 21 | 105.650 | 9.177 | +1.150 |
| CANADA | 8.00 | 1 JUN 23 | 88.150 | 9.174 | +1.000 |
| CMHC | 8.25 | 3 AUG 99 | 98.200 | 8.703 | +0.600 |
| REAL RETURNS | 4.25 | 1 DEC 21 | 94.500 | 4.606 | +0.125 |
| **PROVINCIAL** | | | | | |
| ALBERTA | 7.00 | 20 AUG 97 | 95.650 | 8.278 | +0.500 |
| ALBERTA | 6.50 | 1 SEP 99 | 99.190 | 8.700 | +0.620 |
| ALBERTA | 6.38 | 1 JUN 04 | 82.238 | 9.158 | +0.700 |
| B C | 7.00 | 9 JUN 99 | 93.700 | 8.621 | +0.550 |
| B C | 9.00 | 9 JAN 02 | 99.425 | 9.105 | +0.650 |
| B C | 9.00 | 21 JUN 04 | 98.400 | 9.248 | +0.750 |
| B C | 8.50 | 23 AUG 13 | 91.550 | 9.466 | +1.000 |
| B C | 8.00 | 8 SEP 23 | 85.225 | 9.505 | +1.000 |
| HYDRO QUEBEC | 9.25 | 2 DEC 96 | 102.025 | 8.254 | +0.350 |
| HYDRO QUEBEC | 10.88 | 25 JUL 01 | 106.775 | 9.517 | +0.800 |
| HYDRO QUEBEC | 11.00 | 15 AUG 20 | 108.725 | 10.049 | +1.350 |
| HYDRO QUEBEC | 9.63 | 15 JUL 22 | 95.825 | 10.073 | +1.250 |
| MANITOBA | 6.75 | 24 AUG 95 | 99.425 | 7.340 | +0.150 |
| MANITOBA | 7.00 | 19 APR 99 | 93.500 | 8.717 | +0.650 |
| MANITOBA | 7.88 | 7 APR 03 | 92.125 | 9.213 | +0.650 |
| MANITOBA | 10.50 | 5 MAR 31 | 109.400 | 9.569 | +1.350 |
| NEW BRUNSWIC | 7.00 | 17 MAR 98 | 95.225 | 8.570 | +0.400 |
| NEW BRUNSWIC | 8.38 | 26 AUG 02 | 95.250 | 9.224 | +0.550 |
| NEW BRUNSWIC | 8.50 | 28 JUN 13 | 90.375 | 9.613 | +0.850 |
| NEWFOUNDLAND | 10.13 | 22 NOV 14 | 101.025 | 10.002 | +0.950 |
| NOVA SCOTIA | 9.60 | 30 JAN 22 | 97.325 | 9.884 | +0.950 |
| ONTARIO HYD | 10.88 | 8 JAN 96 | 103.930 | 7.637 | +0.240 |
| ONTARIO HYD | 7.25 | 31 MAR 98 | 95.725 | 8.644 | +0.450 |
| ONTARIO HYD | 9.63 | 3 AUG 99 | 103.400 | 8.765 | +0.700 |
| ONTARIO HYD | 8.63 | 6 FEB 02 | 96.575 | 9.269 | +0.700 |
| ONTARIO HYD | 9.00 | 24 JUN 02 | 98.325 | 9.301 | +0.700 |
| ONTARIO | 8.75 | 16 APR 97 | 100.775 | 8.411 | +0.450 |
| ONTARIO | 9.00 | 15 SEP 04 | 97.200 | 9.434 | +0.800 |
| ONTARIO | 7.50 | 7 FEB 24 | 78.600 | 9.713 | +0.900 |
| P E I | 9.75 | 30 APR 02 | 101.125 | 9.535 | +0.550 |
| P E I | 11.00 | 19 SEP 11 | 108.300 | 9.976 | +0.900 |
| QUEBEC | 8.00 | 30 MAR 98 | 97.625 | 8.774 | +0.450 |
| QUEBEC | 10.25 | 7 APR 98 | 104.375 | 8.813 | +0.450 |
| QUEBEC | 10.25 | 15 OCT 01 | 103.550 | 9.549 | +0.750 |
| QUEBEC | 9.38 | 16 JAN 23 | 93.425 | 10.079 | +1.200 |
| SASKATCHEWAN | 9.88 | 6 JUL 99 | 103.975 | 8.852 | +0.700 |
| SASKATCHEWAN | 9.50 | 16 AUG 04 | 99.625 | 9.559 | +0.750 |
| SASKATCHEWAN | 9.60 | 4 FEB 22 | 98.300 | 9.779 | +1.100 |
| TORONTO -MET | 10.38 | 4 SEP 01 | 106.250 | 9.151 | +0.650 |
| **CORPORATE** | | | | | |
| ALTA ENERGY | 8.15 | 31 JUL 03 | 91.250 | 9.628 | +0.625 |
| BELL CANADA | 9.20 | 1 JUN 99 | 101.375 | 8.836 | +0.500 |
| BELL CANADA | 9.50 | 15 JUN 02 | 100.750 | 9.358 | +0.625 |
| BELL CANADA | 9.70 | 15 DEC 32 | 99.375 | 9.760 | +1.000 |
| BC TELEPHONE | 9.65 | 8 APR 22 | 99.875 | 9.661 | +1.000 |
| CDN IMP BANK | 7.10 | 10 MAR 04 | 85.375 | 9.454 | +0.625 |
| CAN TRUST M | 10.05 | 4 AUG 14 | 101.375 | 9.890 | +0.875 |
| CDN UTIL | 8.73 | 1 JUN 04 | 95.875 | 9.379 | +0.625 |
| CDN UTIL | 9.40 | 1 MAY 23 | 97.375 | 9.669 | +0.875 |
| FINNING LTD | 8.35 | 22 MAR 04 | 90.375 | 9.925 | +0.625 |
| IMASCO LTD | 8.38 | 23 JUN 03 | 92.625 | 9.628 | +0.625 |
| INTERPRV PIP | 8.20 | 15 FEB 24 | 85.000 | 9.757 | +0.875 |
| MOLSON BREW | 8.20 | 11 MAR 03 | 92.125 | 9.565 | +0.500 |
| MOLSON BREW | 8.40 | 7 DEC 18 | 87.000 | 9.811 | +0.750 |
| MAR TEL - TEL | 10.13 | 31 JUL 97 | 104.000 | 8.563 | +0.500 |
| NVA SCOT PWR | 6.50 | 15 DEC 98 | 91.750 | 8.828 | +0.500 |
| NVA SCOT PWR | 7.70 | 15 OCT 03 | 88.500 | 9.612 | +0.625 |
| NVA SCOT PWR | 9.75 | 2 AUG 19 | 99.375 | 9.817 | +1.000 |
| NRTH TELECOM | 7.45 | 10 MAR 98 | 96.000 | 8.776 | +0.375 |
| NOVA CORP | 8.30 | 15 JUL 03 | 92.250 | 9.511 | +0.525 |
| ROYAL BANK | 10.50 | 1 MAR 02 | 106.000 | 9.371 | +0.500 |
| SUNCOR INC | 7.40 | 23 FEB 04 | 86.375 | 9.616 | +0.625 |
| THOMSON CORP | 9.15 | 6 JUL 04 | 97.250 | 9.564 | +0.625 |
| TRANSCDA PIP | 9.45 | 20 MAR 18 | 96.625 | 9.818 | +0.875 |
| UNION GAS | 8.75 | 3 AUG 18 | 89.875 | 9.858 | +0.875 |
| WSTCOAST TRN | 8.50 | 4 SEP 18 | 87.750 | 9.837 | +0.750 |

## BOND INDEX

### ScotiaMcLeod Bond Indexes

| Index | Close | % chg | Yield | Chg | 52 wk High | 52 wk Low |
|---|---|---|---|---|---|---|
| Short | 222.00 | 0.38 | 8.249 | -0.14 | 231.91 | 215.83 |
| Mid | 232.90 | 0.59 | 9.071 | -0.10 | 258.78 | 223.77 |
| Long | 243.42 | 0.79 | 9.439 | -0.09 | 282.93 | 232.20 |
| Universe | 234.84 | 0.56 | 8.857 | -0.11 | 258.29 | 226.21 |

## BENCHMARK INTERNATIONAL BONDS

| Issuer | Coupon | Maturity | Price | Yield | $ chg |
|---|---|---|---|---|---|
| U.S. Treasury | 6¼ | Aug/23 | 84 29/32 | 7.54 | +3/32 |
| British gilt | 9 | Oct/08 | 104 24/32 | 8.41 | +12/32 |
| German | 6¾ | Sep/04 | 98.76 | 6.93 | -0.36 |
| Japan # 164 | 4.5 | 2003 | 96.54 | 4.630 | +0.03 |

Source: Globe and Mail, Monday, August 8, 1994.

As it turns out, empirical analysis of deviations from long term covered interest arbitrage is complicated by the market preference for using coupon bonds to raise long term funds. Hence, while the CIP condition is applicable to short term money market instruments, such as Eurodeposits, BA's, treasury bills and commercial paper, that feature zero coupons, it is not immediately applicable to markets that feature coupon bearing securities. Even though there are zero coupon instruments that are traded in the long term market, e.g., US Treasury strips (Gregory and Livingston 1992), the bulk of market liquidity is focused on trading coupon bearing securities.[16] Hence, coupon bonds are most representative of actual, long term market interest rates. Given this, appropriate calculation of the covered interest deviations requires that observed coupon bond rates be converted to zero coupon rates before being used to determine the deviations. This involves the use of a bootstrap technique to 'back-out' the zero coupon rates implied in the coupon bond term structure. A worked example of the bootstrapping methodology is given in the Example and, for semi-annual bonds, in the Figure 4.11.[17]

The Example provides the spot interest rates needed to find a solution for the following problem: On Aug. 8, 1994, the spot and *5 year* forward rates for the Canadian dollar/US dollar exchange rate were $1.3797 and $1.4917. Using interest rate information provided in Figures 4.9 and 4.10. what "arbitrage free" interest rate on 5 year zero coupon US dollar instruments would be consistent with CIP? To answer this question it is necessary to pick out the appropriate Canadian interest rates to use for determining the spot interest rates. For current purposes it is sufficient to assume that all bonds pay annual coupons. (The gain in precision using semi-annual calculations, as in the Appendix, is lost in the slippage required to identify bonds with appropriate maturities.) For the first spot interest rate using the one year treasury bill rate. For the appropriate bonds, select the Government of Canada bond with a maturity closest to August 4. The relevant bonds would be 6.5% of 1/8/96, the 7.5% of 1/7/97, the 6.5 of 1/9/98, and the 7.75% of 1/9/99.

In addition to limitations on issuing appropriate long term zero coupon instruments, there are a number of other institutional factors that impose significant restrictions on arbitrage trading of long term forward exchange contracts, e.g., Popper (1993). For example, because the bulk of trading activity involving currency hedged bond issues is done in the currency swap market, the relevant trading strategy to consider is between a fully hedged borrowing and the related currency swap.[18] While in the special case of a zero coupon currency swap these two trades are equivalent, this is not the case where coupon bonds are involved. The resulting trading strategies involve comparing cash flows that are unequal at future payment dates, creating a significant problem for specifying the arbitrage portfolio. Other factors such as taxes and transactions costs, e.g., wider bid/offer spreads and longer execution times, also complicate the analysis.

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*Figure 4.11* **Bootstrapping Spot Interest Rates for Semi-annual Bonds**

The bootstrapping technique, e.g., Smith (1991) and Iben (1992), is an iterative process for calculating implied zero coupon interest rates, so-called spot interest rates, from observed coupon bond rates. The process requires the observed yields for coupon bonds of each relevant term to maturity along the yield curve. In practice, spot rates would typically be extracted from the yield curve for federal government bonds, Treasury bonds in the US or Government of Canada bonds in Canada. Because these types of bonds pay semi-annual coupons, precision requires that the bootstrap be executed at semi-annual intervals.

Hence, for purposes of illustration, assume that the relevant bonds are sold at par and pay coupons semi-annually. Further assume that the observed six month yield is 8.87%. Taking this yield to be quoted ex-dividend means that the quoted yield is for a six month zero coupon bond. If the observed yield on a one year semi-annual coupon bond is assumed to be 9.04, for a $100 par value bond this implies a semi-annual coupon payment of 4.52. Given this, the iteration for solving a sequence of implied zero coupon rates begins by discounting the first semi-annual coupon payment at the six month, zero coupon rate and solving for the implied one year zero coupon rate. For a bond sold at par this requires that:

$$100 = \frac{4.52}{1 + \frac{.0887}{2}} + \frac{104.52}{(1 + \frac{z_1}{2})^2}$$

where $z_1$ is the implied one year zero coupon rate, that can be calculated as 0.090438.

Having solved for $z_1$, the next step in the iteration involves using $z_1$ to solve the implied zero coupon rate, $z_{1.5}$, using a 1.5 year par coupon bond. If the observed rate on 1.5 year coupon bonds is 9.155, then this implies a semi-annual coupon payment on a \$100 par bond of 4.5775. This leads to:

$$100 = \frac{4.5775}{1 + \frac{0.0887}{2}} + \frac{4.5775}{(1 + \frac{z_1}{2})^2} + \frac{104.5775}{(1 + \frac{z_{1.5}}{2})^3}$$

Substituting the value for $z_1$ determined previously and solving gives $z_{1.5} = 0.091629$. The next step in the iteration involves solving for $z_2$. Taking the observed two year yield to be 9.2% produces:

$$100 = \frac{4.6}{1 + \frac{z_{.5}}{2}} + \frac{4.6}{(1 + \frac{z_1}{2})^2} + \frac{4.6}{(1 + \frac{z_{1.5}}{2})^3} + \frac{104.6}{(1 + \frac{z_2}{2})^4}$$

This formula can be used to solve for $z_2$, using the previously computed values for $z_1$ and $z_{1.5}$. This iterative process continues until the zero coupon rate for the desired term to maturity is calculated. The relevant zero coupon rate can then used to do calculations involving long term covered interest arbitrage.

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### 4.3 When-Issued Arbitrage

#### The When-Issued Market for Government of Canada Treasury Bills

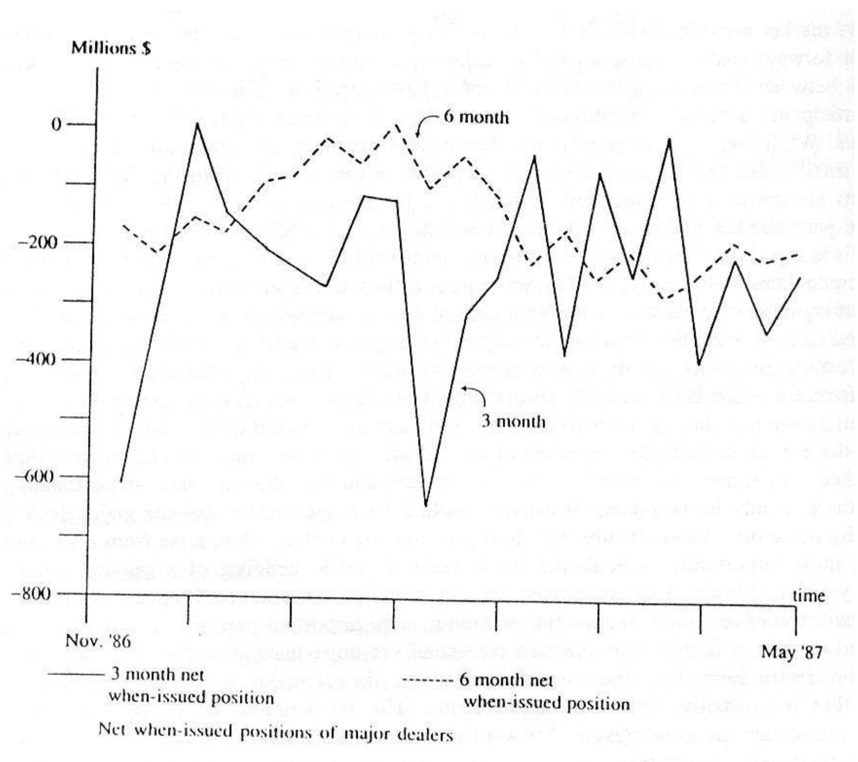
Each week, the Government of Canada issues 3 and 6 month treasury bills (tbills) at auction.[19] Barring holidays, this price discrimination auction now takes place on Tuesday with delivery (and maturity) of the tbills occurring on Wednesday. Announcement of size of the following week's offerings is made at the same time (2:00 p.m. Tuesday) as the results for that week's auction are announced. Unlike the US where limited public participation and noncompetitive tenders at the weekly Treasury bill auction are permitted, only banks and investment dealers eligible to act as primary distributors of new Government of Canada issues are eligible to submit only 'sealed' competitive tenders at the Canadian tbill auction. No commissions are paid on allotted tenders. Investors with "jobber" status are expected to actively participate at the auction.

Tbills to be auctioned at the next tender are traded on a ***when-issued*** (wi) basis from the time of the announcement of the size of the tender until the auction takes place. In other words, the wi tbill market is a forward market for tbills with maximum contract maturities of one week.[20] More precisely, a when-issued tbill (wi) is a contract either to purchase or sell tbills that are to be auctioned at the next tender. Settlement of wi transactions is made in tbills on the next business day following the auction. Typically, active wi trading begins on the Wednesday preceding the tender and continues up to the actual auction.[21] Active participants in the wi market are the investment dealers, the banks and major institutional investors. While there was some wi trade prior to 1978, the formal beginning of the *wi* market in Canada can be traced to the introduction of screen-oriented brokered wi trading in the Fall of 1978. By the early 1990s, more than half of trading in wi's is done through screen-oriented broker services.[22] The widespread use of screen-oriented trading means that wi activity is highly visible to other participants in the tbill market.

The wi market provides large tbill cash market participants with a relatively inexpensive method for forward trading Canadian tbills. Like other forward markets, there is an essential connection between forward trading and cash market participation. For example, only eligible auction participants, a significant subset of all wi traders, have the ability to effectively short sell wi positions. While there is some purely speculative participation by forward market participants, e.g., from small banks seeking to make a rate play without the costs of acquiring tbills, the bulk of contracts are initiated to reflect tbill purchases at the ensuing auction. This lack of small speculative participation makes the wi market unlike a tbill futures market. In the Canadian context, this is significant because the only futures contract for Canadian tbills has been delisted by the Toronto Futures Exchange.[23] The wi market differs in a number of other ways from a typical futures market: while the deliverable commodity is homogeneous, wi contracts are not standardized in size; a smaller performance deposit (margin) is required;[24] there is no "marking to market"; and, trading is not on a centralized exchange. Most importantly, the one week trading horizon for wi's differs markedly from the maturity dates typically associated with futures.

Regarding the liquidity of the market, there is at best only limited quantitative information. Typically, there is a considerable narrowing of the bid/offer for a wi contract as the auction date is approached. In terms of the net position of traders and the absolute size of outstanding contracts, there is only limited data. However, Figure 4.12 demonstrates that the major dealers are generally net short. Theoretically, net short positions by dealers could arise from a number of factors, most importantly: speculating on increasing yields, hedging of inventory against increasing yields and hedging of anticipated auction winnings. In practice, it appears as though the use of wi's to hedge anticipated auction winnings is an important part of the dealer's use of wi's. Based on this, it is useful to examine the actual arbitrage mechanics for wi's and verify what types of restrictions affect these trades in practice. In particular, transactions costs are a necessary, but not usually sufficient, adjustment. The introduction of transactions costs effectively transforms the arbitrage equality restriction into upper and lower arbitrage boundary restrictions providing an arbitrage band within which some variation from the frictionless arbitrage condition is permitted.

*Figure 4.12* **When Issued Positions of Major Dealers**



**The When-Issued Cash-and-Carry Arbitrage**

To derive the long cash and carry arbitrage condition for 3 month wi's, the profit function at $t=0$ involves the *purchase* of a (3 month + $N$ day) tbill financed at the term call loan rate. Term financing is required for the borrowing rate to be riskless. The call loan rate is used because this market typically provides the best combination of availability of funds and competitive rates. This purchase is simultaneously covered with the sale of a three month wi maturing in N days. In other words, the tbill being purchased is the 'old' six month tbill that is to be reopened at the next three month auction, $N < 7$. This cash tbill purchase is simultaneously covered with a *wi* short. Schematically, the long arbitrage trade is described in Figure 4.13. While this profit function is consistent with the idealized full carry model, it is somewhat misleading in the case of tbills in that $P(0)$ will increase over time due to the effect of maturity reduction. This is reflected in the $r(0,N)$ term that incorporates the return on the underlying tbill. In terms of yields, it can be shown (Poitras 1997) that the profit function can be approximated as:

$$\pi(0,N) = i^* - [i + (N/91)(i - R)] \leq 0$$

where $i^*$ is the promised yield at $t$=0 on the 91-day wi tbill, $i$ is the secondary market yield on the (to-be-reopened) 91+$N$ day tbill, and $R$ is the term call loan rate *with all interest rates expressed in annualized form*.

*Figure 4.13* **Derivation of the Yield Version of the Cash and Carry**

Assume that the tbill purchase can be financed at the term call (or term repo) rate, that the par values of the tbill and WI have been normalized to 1 and that the deliverable tbill has 91 days, i.e., only the 3 month wi case is being considered. Given this, then $P(0) = 1/(1 + (i(91 + N)/365))$ and $WI(0) = 1/(1 + (i^*(91/365))$ where $i$ is the tbill rate and $i^*$ is the $WI$ rate at time 0. The cost of carrying the tbill from time 0 to $N$ at the term repo rate is $P(0)((R)(N/365)) = P(0)\ R(0,N)$ where $R$ is the annualized term call rate.

With these assumptions and ignoring transactions costs, the no arbitrage profit condition is:

$$\pi(0) = WI(0) - [P(0)\ (1 + r(0,N))] = 0$$

where: $r(0,N) = R(0,N) - i(0,N) = (R(0) - i(0))(N/365)$

Evaluating individual terms:

$$(WI(0) - P(0)) = \frac{((i^* - i)(91/365)) + (i(N/365))}{(1 + (i(91 + N)/365))(1 + (i^*(91/365))}$$

Similarly, the interest expense can be derived:

$$P(0)\ R(0,N) = ((R(N/365)\ (1/(1 + (i(91 + N)/365))$$

After manipulating the two expressions, evaluation of the resulting *numerator* gives:

$$i^* - i \leq i(N/91) - ((R(N/91)(1 + (i^*(91/365)))$$

The result in the text follows from ignoring the second order term $R\ i^*(N/365)$.

In other words, the wi rate is bounded from above by the tbill rate and the cost of carrying the tbill between time *0* and *N*.

The profit function for the short cash-and-carry arbitrage requires the (3 month + *N* day) tbill has to be 'shorted'. The implied acquisition of the appropriate tbill can be done by doing a term *reverse* repo at annualized rate RR using the correctly dated tbill as underlying collateral. This tbill is then sold and the funds acquired used to cover the cost of doing the reverse. Simultaneously, the short tbill position is covered by taking an equal par value amount of long wi contracts. At delivery, the tbills acquired from the long wi position are used to cover the short position created by the reverse and the profit function is:

$$\pi(0,N) = i^* - [i + (N/91)(i - RR)] \geq 0$$

In words, the short cash arbitrage provides a lower bound on the wi rate. Hence, in addition to transactions costs, the upper and lower boundaries use different borrowing/lending rates.

While these profit functions provide theoretical upper and lower bounds on wi rates, the adherence of wi rates to the boundary restrictions depends on the ability of market participants to execute the arbitrages. From a trading perspective, there are considerably more impediments to executing the wi arbitrages than there are on similar arbitrages for, say, US tbill futures. In particular, unlike the term repo market in the US, the term call (or repo) market in Canada is quite thin. This is especially so for reverses. In practice, the long wi arbitrage would likely have to be financed in the overnight call market. This would require the cash position to be refinanced on a daily basis.[25] Given the short trading horizon, this will not severely restrict execution, only the level of riskiness. On the other hand, the reverse would likely require the participation of an institutional tbill account that would require some form of short sale fee. By implication, as is in a number of other financial markets, the short arbitrage provides a significantly weaker restriction than the long arbitrage. This means that large deviations from zero arbitrage would be tend to be positive.

In order to assess the performance of the wi cash-and-carry trade, Poitras (1991) uses cash and wi transactions in which the bid/offer spread[26] as well as broker commissions on the wi are taken into account. Assuming wi bid/offer spreads of 2 basis points, cash tbill spreads of 1 basis point, 1 basis point in wi broker commissions, and 1 basis point for the bid/offer on the financing rate, transactions costs provide a 5 basis point buffer on either side of the theoretical arbitrage value. Given this, Poitras (1991) provides evidence that, when account is taken of transactions costs, wi rates have on a number of occasions deviated substantially outside of the arbitrage boundaries of 5 basis points in the direction of the long arbitrage, 10 basis points in the short arbitrage direction, positive arbitrage profits were observed. Given a number of qualifications, detailed examination of the empirical evidence reveals that the instances of unexploited arbitrage opportunities arose mostly during unsettled market conditions. Especially when markets are unsettled, speculative traders use the wi market as an inexpensive vehicle to make an interest rate play. Because of the limited amount of arbitraging activity, speculative trade is able to move the wi rate significantly outside the arbitrage bounds. Based the prevalence, timing and size of the differentials, it appears that the wi market is under-arbitraged.

### 4.4 Stock Index Arbitrage

Due to the role that stock index futures played in the market crash of Oct. 1987, stock index arbitrage has, arguably, received the most public attention of all the futures arbitrages. Careful analysis of the specific mechanics of the arbitrage transactions was required to determine the connections between trading in the cash and futures markets for equities. The general public ignorance of stock index futures reflected in the public discussion surrounding the crash of 1987 is understandable. Trading in stock index futures is a product of the modern Renaissance in derivative security trading. Not only is exchange trading of stock index future contracts relatively new, the underlying commodity is not the garden variety agricultural or industrial commodity, featuring multiple delivery specifications of a particular commodity. Stock index futures are written on an index, an appropriately weighted bundle of underlying securities. Arbitrage transactions in this commodity generate anywhere from dozens to hundreds of trades in the underlying equities.

*Figure 4.15* **Profit Function for a Long S&P 500 Cash-and-Carry Arbitrage**

| ***DATE*** | ***Cash Position*** | ***Futures Position*** |
|---|---|---|
| $t=0$ | Invest $\$Q_S$ in the Stock Index at $S(0)$ financed with a loan at the broker call loan interest rate, $i(0,T)$ | Short $\$Q_S$ of the Stock Index at $F(0,T)$ |

--Position will earn the dividend yield $d(0,T)$ on the S&P between $t = 0$ and $t = T$

$t=T$ Sell the $\$Q_S$ of the stock index at $S(T)$ to receive the value of the stock index futures $F(T,T) = S(T)$. Cash settle the futures position, repaying the broker call loan with the net proceeds of the investment and dividend income, $\$[Q_S\, S(0)]\,\{1 + i(0,T) - d(0,T)\}$

Because the value of the cash and futures positions at $T$ are equal, in this case the cash-and- carry arbitrage profit function can be specified:

$$\pi(0) = \{F(0,T) - S(0)(1 + i(0,T) - d(0,T)\}\, Q_S \leq 0$$

The first stock index futures contract, based on the Value Line Index, was introduced in Feb. 1982 on the KCBT. The most important stock index futures contract, the S&P 500 traded on the CME/IMM, was introduced shortly thereafter in April 1982. A raft of stock index futures contracts has appeared since that time, starting with the introduction of the NYSE Composite on the NYFE in May 1982 and the Major Market Index on the CBT in 1984. More recently, there has been the introduction of foreign indexes traded on US exchanges, such as the Nikkei 225 on the CME. This has been accompanied by the trading of domestic equity indexes on futures markets around the world, including markets in Japan, Hong Kong, Holland, Australia, England, France, Germany, Switzerland, and Canada. Another recent development has been the start of trading in the DJIA index futures in October 1997. The slow pace associated with the introduction of the DJIA was not due to a lack of interest in such a contract. On the contrary, perceiving considerable demand, the CBT had attempted to introduce a DJIA contract as early as July 1984. However, these plans were thwarted by Dow Jones and Company that initiated legal action to prevent trading of the contract. What ensued was a process lasting over a dozen years, ending with the CBT eventually introducing DJIA futures and options contracts.

Stock index futures were a success almost from the beginning. In conjunction with the Eurodollar contract and the oil complex contracts, the introduction of trading in stock index futures redefined the landscape of derivative security trading. By 1986, in terms of total number of contracts traded, the S&P 500 futures contract was the second most successful futures contract in the world. Stock index futures contracts are cash settled, eliminating complications associated with multiple delivery specifications. This simplification is offset by the differing methods by which stock indexes are calculated. A range of possible indexing schemes are available. The DJIA is price-weighted and the S&P 500 is value-weighted. Though indexes do not typically adjust for dividends, there are exceptions such as the DAX, which is a *performance index*. By reinvesting coupons back into the index, a performance index is cumulative dividend adjusted. The price behavior of such an index is similar to that of a non-dividend paying stock. The specifics of index construction are essential to actual execution of the arbitrage.

Given the practical importance of stock index arbitrage, there have been numerous studies on the arbitrage and the implications for derivative pricing, e.g., Eytan and Harpaz (1986), Neal (1996). There have also been a range of studies on program trading, e.g., Grossman (1988), Harris et al. (1994), Hill and Jones (1988), Kamara et al. (1992). As discussed in Sec. 9.3, program trading is a collection of strategies in which computerized trading techniques are an integral part. Program trading includes dynamic portfolio insurance, stock index futures arbitrage and dynamic hedging of equity option books. Analysis of stock index arbitrage activities surrounding the crash of 1987 generally supported the notion that such activities were stabilizing, e.g., Tosini (1988), though such conclusions were based primarily on the narrow implications of the arbitrage transactions.

To better appreciate how the cash-and-carry arbitrage condition for stock indexes, consider the arbitrage trades involved for a stylized stock index future described in Figure 4.15. For stock index arbitrage, $cc(0,T)$ is given by the broker call loan rate and $cr(0,T)$ is given by the dividend yield. Consistent with all the financial commodities, the cash-and-carry arbitrage condition only takes account of borrowing charges paid and pecuniary returns earned while holding the commodity during the period of the arbitrage. Complications arise with stock index arbitrage because the dividend yield, $d(0,T)$ is not known at $t=0$ and, for distant delivery dates, there may be some difficulty in obtaining broker call loan financing without a substantial ***haircut***. The uncertainty of the dividend yield introduces some risk into the arbitrage while the haircut requires the trader to finance a fraction of the position, again undercutting the pure arbitrage notion because a net investment of funds is required. In order to reduce transactions and other execution costs, additional uncertainty may be intentionally introduced by trading only a sub-component of the index that is deemed to track the index sufficiently well.

*Figure 4.16* **Stock Index Futures Prices**

# INDEX

**S&P 500 INDEX (CME) $500 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 457.35 | 458.45 | 457.00 | 457.95 | + .55 | 485.20 | 436.75 | 195,724 |
| Dec | 460.00 | 460.75 | 459.60 | 460.40 | + .60 | 487.10 | 438.85 | 16,365 |
| Mr95 | | .... | .... | 463.70 | + .60 | 479.00 | 441.45 | 2,859 |
| June | | .... | .... | 467.20 | + .70 | 472.70 | 449.50 | 1,357 |

Est vol 33,151; vol Fri 55,440; open int 216,305, +1,921.
Indx prelim High 458.30; Low 457.10; Close 457.89 +.80

**S&P MIDCAP 400 (CME) $500 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 170.20 | 171.80 | 170.10 | 171.70 | + 1.50 | 186.70 | 161.50 | 12,140 |

Est vol 365; vol Fri 682; open int 12,239, +96.
The index: High 171.12; Low 169.84; Close 171.12 +1.15

**NIKKEI 225 STOCK AVERAGE (CME)—$5 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 20750. | 208.50 | 207.50 | 208.20 | + 2.45 | 21775. | 16240. | 22,596 |
| Dec | 208.60 | 209.10 | 208.60 | 209.10 | + 2.45 | 21800. | 17030. | 477 |

Est vol 2,039; vol Fri 848; open int 23,074, +155.
The index: High 20638.93; Low 20502.20; Close 20635.83 +114.13

**GSCI (CME)—$250 times GSCI nearby prem.**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Oct | 179.40 | 179.40 | 178.30 | 178.50 | .... | 184.40 | 171.90 | 4,731 |
| Dec | | .... | .... | 182.30 | + .20 | 187.20 | 178.70 | 736 |

Est vol 476; vol Fri 1,618; open int 5,467, −738.
The index: High 178.82; Low 176.93; Close 177.30 −.38

**MAJOR MARKET INDEX (CME)—$500 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Aug | 385.50 | 386.45 | 385.30 | 386.20 | + 1.05 | 388.55 | 369.90 | 3,206 |
| Sept | 386.20 | 386.60 | 386.20 | 386.50 | + 1.05 | 399.35 | 360.55 | 709 |
| Dec | | .... | .... | 388.10 | + 1.05 | 400.45 | 361.75 | 187 |

Est vol 107; vol Fri 1,361; open int 4,102, +800.
The index: High 386.98; Low 385.05; Close 386.69 +1.64

**NYSE COMPOSITE INDEX (NYFE)—500 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 252.65 | 253.30 | 252.50 | 253.10 | + .55 | 267.90 | 241.00 | 3,587 |
| Dec | 253.70 | 254.20 | 253.70 | 254.10 | + .60 | 264.50 | 244.15 | 261 |

Est vol 1,374; vol Fri 2,609; open int 3,962, +214.
The index: High 253.05; Low 252.50; Close 252.99 +.49

**KR-CRB INDEX (NYFE)—500 times index**

| | Open | High | Low | Settle | Chg | High | Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| Sept | 232.70 | 233.15 | 231.80 | 232.05 | − .95 | 241.50 | 221.10 | 2,101 |
| Dec | 234.15 | 235.00 | 233.80 | 233.90 | − .95 | 241.00 | 225.00 | 1,757 |
| Mr95 | | .... | .... | 235.75 | − .95 | 242.75 | 228.70 | 968 |
| May | | .... | .... | 237.25 | − .95 | 240.75 | 238.75 | 655 |

Est vol 184; vol Fri 240; open int 5,481, +9.
The index: High 232.31; Low 231.03; Close 231.05 −.93

Source: Wall Street Journal, Monday, August 8, 1994.

As noted, stock index futures are a commodity without the complications of multiple delivery specifications and cheapest deliverable grades. Stock index futures contracts are cash settled. As indicated in Figure 4.15 the bulk of market interest centers on the nearby contracts, reducing the difficulties of obtaining longer term financing for arbitragers but complicating the use of strategies that aim to replicate the long term payoff on the underlying equity index by forming portfolios of cash Tbills and stock index futures. Such portfolios have to be rolled over at regular intervals, creating a slippage between the underlying equity index return and the replicating portfolio. The arbitrages for money market futures, such as Treasury bills and Eurodollars examined in Sec. 4.3, also do not have the problem of multiple delivery specifications. The most important contract, in terms of volumes, which features significant multiple delivery is the US Tbond contract.

## Questions

1. a) Explain the arbitrage underlying the covered interest parity theorem discussed in Sec. 4.2. What assumptions are being made about both the execution of the arbitrage and the underlying securities?

b) Will CIP hold for all types of money market instruments? Which money market security will produce the smallest deviations from the covered interest parity conditions? Why? What institutional characteristics of Bankers' Acceptance, commercial paper and treasury bills would make it difficult for CIP transactions to be instantaneously executed in those markets? Be as complete as possible in explaining your answer.

c) On March 1, 1990 the spot and 3 month forward rates for the Canadian dollar (per US dollar) were \$1.1922 and \$1.2072 respectively. What "risk-free" discount rate on U.S. dollar instruments would be consistent with the interest-rate-parity theorem if the 3 month (annualized) risk-free rate on Canadian dollar instruments was 13.10%?

d) On Aug. 8, 1994, the spot and 5 year forward rates for the Canadian dollar/US dollar exchange rate were \$1.3797 and \$1.4917. Using interest rate information provided in Tables 2.2.3 and 2.2.5 what "arbitrage free" interest rate on 5 year zero coupon US dollar instruments would be consistent with CIP?

2. On Aug. 8, 1994, the price of the September Treasury bill futures is 95.24. The spot market for 91-day Treasury bills is quoted at a discount rate of 4.45, while the 182-day rate is 5.08. Estimate the forward rate on 91-day Treasury bills and compare it with the rate implied by the futures market. Repeat your calculations using similar quotes for Eurodollar cash and futures prices.

3. Define the following:

i) basis; ii) location basis; iii) maturity basis; iv) contango v) backwardation; vi) arbitrage; vii) cash and carry arbitrage

Explain how the cost of carry affects the relationship between spot and futures prices. How do changes in the cost of carry affect the basis over the life of a futures contract? What factors are significant for determining the quality basis for Treasury bond futures?

4.a) Are forward prices unbiased predictors of future spot prices?

b) It is often stated that futures price levels follow random walks. What is the relationship of this hypothesis with the hypothesis that forward prices are unbiased predictors of future spot prices?

c) If futures prices are at full carry, is this inconsistent with the hypothesis the futures prices are unbiased predictors and the there are zero expected profits to speculation? If so, what type of trading strategy could be used to profit from this discrepancy?

5. From Sec. 4.1, rework the condition $S(0)\ \Delta ic + ic(1)\ \Delta S$ to include fixed as well as variable cost components. Explain the impact of the additional terms on the pricing of forward and futures prices?

6. Using data from Table 4.2. calculate the size of the deviation from CIP. To do this requires the CIP condition to be manipulated to get:

$$i(0,T) - i^*(0,T) - \{\frac{F(0,T)}{S(0)} - 1\}(1 + i^*(0,T)) = Deviation$$

To do the calculations it is sufficient to use the nearby interest rate futures for the interest rate.

7. Extend the results of Sec. 4.2 to the calculation of banker's profits from doing foreign exchange swap transactions over three delivery periods. Is there a substantive difference in the solution? Hint: Are implied forward interest rates unbiased predictors of future interest rates? Consider the case of a locking in a three year return vs. locking in a two year and then rolling into an uncertain one year security vs. locking in for one year and then taking on either another one year or locking in for two years.

## NOTES

1. The terminology "self-financing trading strategy" is also used, typically in mathematical finance. An arbitrage trade is similar to the notion of hedge portfolios which are central to the Black-Scholes models of Chap. 8 and 9. By construction, hedge portfolios involve net investment of funds, which are then hedged using options trading strategies. If the initial purchase of stock was fully financed, the hedge portfolio would be an arbitrage portfolio. Hence, the hedge portfolio can be constructed as an arbitrage.

2. The interest rate $r(i,j)$ is assumed to have been adjusted for the period over which interest is paid. This means that the interest rate has *not* been annualized.

3. As demonstrated in Secs. 2.3 and 3.3., in some cases neither the long or the short cash-and-carry is executable.

4. These numbers are approximate and date from the early 1990's. During the 1990's, the general downward trend in the gold price level combined with a full contango futures price structure contributed to a significant increase in hedging activities by gold mining companies. In 1990, the South African Reserve Bank introduced a forward selling facility called the Stabilised Contango Scheme that involves a mining company selling forward a specific amount of gold over either a 12 or 24 month period, at a fixed price, thereby fixing the producer price for gold output. The best single source for following developments in the gold market is annual <u>Gold</u> publication by Gold Fields Mineral Services.

5. A number of the relevant studies on this point can be found in A. Peck, <u>Selected Writings on Futures Markets</u>, Chicago: CBT, 1977.

6. This statement refers only to forward foreign exchange contracts that are the same form as contracts traded in the modern era, a pure agreement to exchange currency at a specified later date at an exchange rate that is determined on the date that the contract is created. It would be possible to effectively achieve a forward exchange transaction using bills of exchange. Such trades could have been done at a much earlier time, possibly even in ancient times. However, bills of exchange did have certain features that would pose real complications, such as a geographical separation in the contract initiation and settlement locations.

7. The notational convention selected here, e.g., to use \$/£ to mean number of US\$ per £, is not general. There are various sources, including certain textbooks and journals, that use the ***opposite*** convention where \$/£ would mean the number of £ per \$. For example, using the reverse quotation US\$/MR 1.7220 would mean 1.7220 Malaysian ringit per US\$. Another less confusing convention involves using five significant digits when the FX rate is less than one and four significant digits when the FX rate is greater than one.

8. More generally, "domestic direct terms" is defined as units of domestic currency to units of foreign currency, e.g., for Canadian investors this would be \$C/\$US (Stigum 1981). However, the Canadian dollar futures contract is traded in \$US/\$C. Hence, to avoid complications, the US is considered the "domestic" country for present purposes.

9. In the stylized form $F = S\ (1+ic)$, the CIP formula implies $(1+ic)=(1 + \{(r-r^*)/(1+r)\})$. The term $(1+r)/(1+r^*)$ is sometimes referred to as the *interest agio*. Depending on the relative sizes of r and r*, the interest agio can be either greater or less than one. When the interest agio is less than $1\ (r < r^*)$, then $F(0,T) < S(0)$ and the spot currency is said to be at a *premium*. When the interest agio is greater than one $(r > r^*)$, the $F(0,T) > S(0)$ and the spot currency is said to be at a *discount*. The difference between the spot and forward rate, $S(0)$ - $F(0,T)$ is known as the *swap rate*, and is the "price" of doing a swap transaction. Finally, $\{F(0,T) - S(0)\}/S(0) = \{r - r^*\}/(1+r^*)$ is known as the *forward exchange agio*.

10. By the nature of trade in futures on the IMM, these relationships will also hold for forwards. There are specific institutional arrangements within the IMM to ensure that a direct link is maintained between the currency futures and outright forward market. An arbitrage role is performed by traders whose function is to purchase and sell futures contracts at the IMM, simultaneously covering the positions in the forward market with a commercial bank. Because the IMM does not release information figures on this type of arbitrage trading, market participants such as central banks rely on the commercial banks to provide information on the activities of these IMM traders.

11. Numerous excellent references on this subject are available including Duffey and Giddy and Kubarych (1983). In practice, the volume of swap transactions is far greater than for forwards. A minor complication arising from the use of swaps involves the need to adjust the increment the forward position to account for the difference in principal values due to the interest paid. Stigum (1983, p.167) recognizes this point: "On swap transactions, interest payments generate a residual foreign exchange exposure. For example, if a bank takes in a 3-month DM deposit and swaps it into dollars, the bank assumes a foreign exchange risk because it is committed to pay interest in DM on the DM deposit at maturity, while it will earn interest at maturity in dollars on the dollars it has loaned. If the bank chooses to avoid this risk, it can lock in a fixed spread on the overall swap by buying DM (selling dollars) *forward* in an amount equal to the interest to be paid in DM." In terms of Figure 2.2.1, the swap would involve a spot exchange of US\$Q for $\$Q/S(0)$ Canadian dollars combined with a return of the $\$Q/S(0)$ in 1 year at $F(0,1)$. To make the transaction riskless, the trader would have to enter an outright forward contract for an additional $[\$Q/S(0)]\ r^*(0,1)$ *at* $F(0,1)$.

12. Following Kreicher (1982), institutional factors such as reserve requirements and deposit insurance costs may affect the rate determination processes in the Euro and domestic markets. As a result, appropriate adjustments have to be made to domestic rates before examining the arbitrage relationship. These institutional factors are not as important in the Canadian case because of the limited institutional restriction on Canadian banks operating in the Euro-US markets (though foreign banks arbitraging the Euro-US/Canadian t-bill differential will be subject to the requirements imposed by their country of origin).

13. To see that the Euro futures rate is a substitute for the implied forward rate observe that the rollover strategy can be constructed as investing in a one period Eurodeposit and simultaneously entering into a long Eurodeposit forward contract with a delivery date that matches the maturity date on the one period Eurodeposit. For example, at $t = 0$ the investor buys a 6 month Eurodeposit at $Q\$$ and simultaneously goes long a 6 month forward contract for a Eurodeposit with an initial 6 month maturity. The principal value $(M)$ for the forward contract is set to equate the maturity value of the 6 month Eurodeposit ($\$Q\ (1 + r(0{,}6 \text{ months})))$. In other words, $M = (\$Q\ (1 + r(0, 6\ months))(1 + e(0, 6\ months))$, where $e(0,T)$ is the $t=0$ interest rate quoted on the Eurodeposit forward contract that requires delivery of a 6 month Eurodeposit in 6 months. For the return on this rollover strategy to equal the buy-and-hold return, $e(0,T)$ has to be the implied forward rate.

14. For ease of exposition, it is assumed that the relevant variables satisfy the conventional calculus regularity conditions. In practice, this assumption requires that the futures positions be continuously adjusted.

15. Domestic direct terms is defined as units of domestic currency to units of foreign currency.

16. In addition, due to pricing anomalies, there are difficulties associated with using the observed zero coupon yields quoted for US Treasury strips, Daves and Ehrhardt (1993).

17. There are two methods for determining the spot interest rate. The Example describes the par bond approach using semi-annual coupon bonds. This method uses the result that when a bond sells at par then the coupon percentage is equal to the stated yield to maturity. In this case, par value can be used for the price of the bond being used to determine the spot interest rates and the yield to maturity used as the coupon. The alternative approach is to work directly with the observed prices and coupons. Instead of using par value, the price used in the observed price and the stated coupon is used for the coupon. There are some computational advantages to using the par bond approach.

18. The relevant trading strategies and arbitrage conditions are discussed in Popper (1993) and Poitras (1993, 1999).

19. The Government of Canada also typically issues 12 month tbills at the weekly auction that are also traded on a wi basis. However, for part of the sample 12 month tbills were only issued bi-weekly creating sampling problems. In addition, the 12 month cash and wi markets are the thinnest of the three available maturities. For these reasons, the 12 month maturity was not included in the analysis.

20. The structure of the Canadian tbill auction process differs somewhat from that in the U.S. For example, as well as having different auction days and somewhat different auction procedures, there is a three day settlement lag in the US and only a one day lag in Canada. As a result, in the US wi Treasury bill (Tbill) trading often refers to trade in new Tbills in the period between auction and settlement (Stigum 1983). In Canada, wi tbill trade refers only to the trade in to-be-issued tbills occurring between the announcement of tranche sizes and the auction. Wi trading also occurs in instruments other than tbills-- such as Government of Canada marketable bonds. Finally, there is no formal restriction on wi trade limiting the maximum maturity to one week. Rather, it is the requirements of tbill traders that determine the available contract maturities. As the Canadian tbill market grows in size and sophistication, longer maturity (e.g., 2 weeks to delivery) wi trade may emerge.

21. While there is some wi trade on the auction date, in the period between auction and settlement the market is primarily concerned with sorting out of cash positions arising from the auction. Hence, the market does not usually start focusing on wi trading for the next auction until the day following the auction.

22. In screen-oriented trading, bids and offers are listed on computer terminal screens in the offices of subscribers to the service. Bids, offers and amounts for wi trades are entered directly onto the screen by the trader. Trades are executed by accepting one of the positions listed on the screen, again by making the appropriate terminal data input. Once a trade is executed, the broker is responsible for recording the relevant transaction information. Off-screen brokered trading of wi's is conducted like conventional OTC trading. Usage of wi brokers is significantly affected by the ability of investment dealers to do wi trade directly with their accounts. Typical broker commissions for wi's are one-half a basis point each way.

23. In 1989, the Montreal Exchange launched a new Government of Canada bond contract. More recently, a Bankers Acceptance contract has been offered. For more on other available hedging instruments see Poitras (1988). By draining off liquidity, the growth of the wi market has undoubtedly contributed to the lack of success of the tbill futures contract.

24. The margin requirements for wi's arise out of the Investment Dealers Association's Regulations. For IDA purposes, wi's are treated as contingent liabilities, much as with futures contracts. Current margin requirements are capital sufficient to cover 1% of the outstanding contingent liability.

25. The introduction of auctions of Receiver General Balances in 1986 has increased the supply of term call money, especially from the near banks. Hence, there may be scope for the development of the term financing in the future.

26. In market parlance, "the edge". While the trader does not have to give up the edge on each trade, the required passive trading tactics (e.g., putting out bids instead of hitting offers) would increase the execution time (risk). Active execution of the arbitrage would necessitate that the transactions costs be paid.