The Market Value of Government of Canada Debt: A Comment on the Importance of Correct Valuation of Non-Marketable Debt

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The market value of Government of Canada debt: a comment on the importance of correct valuation of non-marketable debt

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Abstract. Previous attempts to provide market value estimates for both U.S. and Canadian government debt have included non-marketable debt items – specifically, savings bonds – as part of total 'marketable' debt. Inclusion of non-marketable debt items as part of marketable debt presents a significant valuation problem. This article demonstrates that previous studies on the market value of Canadian government debt used valuation methodologies that significantly underestimated the value of non-marketable debt. A more accurate valuation methodology is provided.

La valeur au marché de la dette du gouvernement du Canada: commentaire sur l'importance d'un évaluation correcte de la part de la dette pour laquelle il n'y a pas de prix du marché. Certains efforts pour établir des estimés de la valeur au marché de la dette gouvernementale au Canada et aux Etats-Unis ont inclus comme portion de la dette pour laquelle il y a 'prix du marché' des composantes comme les obligations d'épargne du Canada qui n'ont pas de 'prix au marché,' L'inclusion de ces items dans la valeur au marché de la dette pour laquelle il y a des prix du marché crée des difficultés importantes. Cet article montre que les études précédentes de la valeur au marché de la dette governementale canadienne ont utilisé des méthodologies qui ont sous-estimé considérablement la valeur de la portion de la dette pour laquelle il n'y a pas de 'prix au marché.' L'auteur suggère une méthodologie qui promet une évaluation plus précise.

Two recent articles in this *Journal* (Boothe and Reid (1986) and Cox and Haslag (1986)) extend the work of Seater (1981) and Cox and Hirschhorn (1983) on U.S. government debt to the Canadian case. While useful, this work contains a significant inconsistency: both studies define the outstanding 'market' value of government debt to include both non-marketable and marketable debt. Given the important role played by non-marketable debt financing, particularly in the 1970s, it is demonstrated here that the methods selected by Boothe and Reid (**BR**) and Cox and Haslag (**CH**) for 'market' valuation of non-marketable debt substantially affect their results for the total market value of Government of Canada debt. To show this, the first section

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reviews both the history of non-marketable debt financing by the Government of Canada and the valuation procedures used by **BR** and **CH** for non-marketable debt. In the second section specific inconsistencies between their valuation methods and valuation methods that take more accurate account of the specific features of Canadian non-marketable debt are identified. In conclusion, estimates of non-marketable debt valuation discrepancies are presented which indicate that **BR**'s and **CH**'s results fail to account for major debt management events in 1974 and 1981 as well as producing as much as a 10 per cent variation in the *total* market value of Government of Canada debt.

BACKGROUND

Non-marketable debt financing played an important role in the total financing requirements of the Canadian government over the 1948–86 period.[1] Specifically, use of Canada Savings Bonds (CSBs) as a financing source has grown from a small fraction of the par value of outstanding Government of Canada debt in the late 1940s and early 1950s, to around one-quarter to three-tenths of outstanding debt in the mid-to-late 1960s. The percentage of total financing attributable to CSBs peaked in the mid-1970s: in 1975 over 40 per cent of the par value of outstanding Government of Canada securities was in CSB form. However, because CSBs are non-marketable, it is not possible to observe directly the 'market' price at which the instrument would be traded. Hence, the importance of CSBs as a government-funding source implies that results for the total *market* value of the government's debt will be sensitive to significant errors in the method selected to value CSBs. If CSBs were undervalued by, say, 15 per cent in 1975 then the total market value estimate for the debt would be in error by around 7.0 per cent. In addition, if the errors associated with mis-pricing of CSBs fluctuated considerably, the usefulness of the market value measure would be further reduced.

While both make passing reference to the problems of valuing CSBs, **CH** and **BR** use different methods to arrive at an estimate for the market price of CSBs. Following Seater, **CH** adjust the par value of outstanding CSBs by a factor which attempts to account for changes in the market value of other government debt. Specifically, **CH** use the ratio of the market value of marketable government securities to the par value

1 Since March 1966 non-marketable securities issued by the Government of Canada have included Canada Savings Bonds (CSBs) and special non-marketable securities owned by the Unemployment Insurance Commission (UIC) and by the Canada Pension Plan (CPP). (The latter two items were overlooked by **CH** (p. 477).) From 1966 to the end of 1985, over 99 per cent of the outstandings of non-marketable debt was in the form of CSBs. On 31 December 1971 the UIC non-marketable securities were redeemed and the funds transferred to the Unemployment Insurance Account of the Government of Canada. From that date the UIC has not held non-marketable securities. Prior to 1966 the Government of Canada had issued other types of non-marketable securities – most notably, War Savings Certificates and Refundable Tax. By 1966 the outstandings for these instruments were zero. From the end of 1985 to April 1987 CPP-held non-marketable securities have increased from $375 million to $1.945 billion or from less than 1 per cent of non-marketable debt to 4.25 per cent. This increase in CPP-related issue has resulted largely from a reluctance by provincial governments to purchase provincial debt for the CPP accounts primarily because of high interest-rate levels.

of government securities as an adjustment factor to value CSBs (p. 480). Both **CH** and **BR** recognize that there are problems with this approach. For example, as noted by Seater and **CH** the measure is sensitive to differences in the maturity structure of CSBs and marketable debt. However, this effect is not likely to be large. More importantly, as **BR** point out, this measure is asymmetric – accounting for increases in wealth when interest rates fall but, because of both the encashability feature of CSBs and the periodic upward readjustments in coupon payments, incorrectly accounting for decreases in wealth when interest rates rise.[2]

With a view to avoiding the problems associated with adjustment factors, **BR** take the straightforward approach of valuing CSBs at par. Unfortunately, this approach has the opposite type of asymmetry from that arising in the **CH** case; that is, wealth increases due to interest rate decreases are ignored. In addition, the use of par value ignores the likelihood that the intrinsic value of CSBs will generally be greater than par if CSBs could be valued in the market. This undervaluation arises primarily from an ignoring of three important features of CSBs: encashability, the 'coupon-ratchet' effect, and the accumulation of coupons on CSBs with a compounding option. Encashability ensures that par value provides a lower bound for the value of a CSB (for a rational investor). If the value of the CSB falls below par, the rational investor will cash in the CSB and invest the resulting funds at a higher rate. In other words, encashability also provides an implicit put option for the CSB holder – this put option must be valued in order to arrive at an accurate estimate of the value of CSBs.

The CSB coupon-ratchet effect arises because of asymmetries in the coupon adjustment process. The ratchet effect was particularly important for the 'old-style' fixed-coupon CSB when interest rates were rising. In this case, the coupon on the new CSB issue was usually higher than coupons on previous issues. Long-term CSB investors could take advantage of the higher coupon by cashing in the old issue and purchasing the new issue. However, once interest rates started to fall, long-term CSB investors would prefer to hold onto previous issues because of the higher coupons. More formally, while the average[3] coupon on outstanding CSB issues would adjust upward when interest rates rose, the average coupon would be resistent to downward readjustments when interest rates fell.

In addition to a ratchet effect based on falling interest rates and a fixed-coupon CSB, in the face of increases in interest rates the coupons on outstanding CSB offerings have, on occasion, been adjusted upward between campaigns in order to avoid the debt management difficulties associated with high CSB redemption levels. A notable instance occurred in 1974 when the CSB coupon was increased twice, in May and again in September, in order to stem the flow of redemptions. This type of ratchet

2 More accurately, interest rates must rise above the weighted average coupon on past debt issues before the **CH** adjustment factor exceeds one. If, for example, coupons on past debt issues were in the 3–4 per cent range when prevailing market interest rates were in the 10 per cent range, then **CH**'s adjustment factor would be well below one. However, from encashability, CSBs cannot be valued below par. Hence, **CH**'s adjustment-factor-based estimate of the market value of CSBs can produce perverse results.

3 The average would be a weighted average, where the weights would be a function of the amounts outstanding.

effect occurs even with the 'new-style' CSB with annual coupon readjustment, where the coupon has at various times been increased between campaigns. As a result of ratcheting, there is a distinct tendency for the average coupon on outstanding CSBs to be at or above the market – indicating that, when valued as a straight bond, CSBs are generally premium bonds. This is particularly so for the 'old-style' CSB.

VALUING CBSS

Theoretically, a CSB is a compound security – a combination of a straight bond and a put option. Actual valuation is complicated by further factors, such as accumulated coupon payments on the stock of compound interest CSBs, occasional readjustment of CSB rates through cash bonuses and the limited alternative investment opportunities available to CSB investors in the period up to the late 1960s. In addition, the terms and conditions for the CSB have not remained constant over time.[4] Since the introduction of CSBs in 1946, there have been four major structural changes: the offering of a 'stepped-up' fixed-coupon structure in 1956, the introduction of a compound-interest option in 1966, the elimination of attached coupons in 1977, and, most importantly, the introduction of the (annual) floating-coupon CSB with a limited 'stepped-down' extendibility feature in 1981.[5] In practice, the latter change reduced the effective maturity of the CSB to more closely approximate that of a one-year debt instrument, thereby substantially reducing the value of the 'ratchet effect' to CSB purchasers.[6]

To illustrate a more accurate methodology for evaluating the degree of mis-pricing of the CSB stock by **CH** and **BR**, consider the par value of available CSB issues outstanding at year-end 1976 given in table 1. For this date, **BR** valued CSBs at par ($16.590 billion) and **CH** valued CSBs at $16.312 billion. In order to illustrate how a more accurate valuation procedure would affect these estimates, the valuation problem will be divided into three parts: the 'straight'-bond component, the put-option component, and the compound-interest component. Regarding the straight-bond component, the objective is to value the cash flow associated with a given CSB issue using the interest rates for similar maturity Government of Canada bonds that prevailed at the time of the CSB campaign. To do this, the year-end 1976 CSB stock must initially be decomposed into two parts – CSBs issued prior to 1974 (series up to S28) and CSBs issued in 1974 and after (S29-31). This is because all CSBs issued prior to 1974 had a coupon increase to 10.5 per cent until maturity effective 1 September 1974. On the other hand, at year-end 1976, series S29-31 that were issued in 1974 and later years had effective yields of from 9.13 per cent to 9.75 per cent, respectively.

4 Background on the development of CSBs up to 1977 can be found in 'Canada Savings Bonds,' *Bank of Canada Review* (October 1977). 23–31.

5 De facto, the use of the stepped-down coupon structure with annual readjustment of coupons began in 1979. However, the 1981 dating for the change is used, because this is when this type of coupon-setting practice was formally announced.

6 The introduction of extendibility at a lower coupon rate added a form of call option feature to CSBs. For a complete valuation of CSBs, the call option should be valued. However, the call option feature has been ignored in the valuation given here.

TABLE 1
Straight-bond valuation for CSBs issues outstanding – 1976

Series[b]	Estimated price	Par value outstanding	Estimated market value
	($100 par)	($ millions)	
S20 (1)	$102.6	$27.1	$27.79
SR and S24 (2)	104.9	2705.2	2838.1
CS (3)	107.1	147.3	157.8
S22 & S26 (4)	108.3	952.7	1031.5
S25 (5)	109.98	828.4	911.1
S23 (6)	109.1	287.2	313.4
S29 (7)	106.4	5211.7	5545.2
S27 (8)	111.3	736.3	819.3
S30 (8)	104.96	3073.5	3225.9
S28 (9)	112.2	431.2	484.0
S31 (9)	103.9	2147.3	2230.0
Total		16,547.9	17,584.1

a The following interest rates were used: one to three years: 7.75; three to five: 8.00; five to ten: 8.50; actual values were 7.80, 8.06, and 8.38, respectively.
The Par Value of Outstandings differs slightly from Actual Values because estimates for year-end 1976 were used.
b Number in brackets indicates term to maturity of the series.
SOURCE: Bank of Canada

Table 1 provides a breakdown of the calculations for the straight-bond component of each of the CSB series for 1976. The theoretical market price is calculated from the standard pricing formula for a marketable bond, using as the yield to maturity the Bank of Canada's average bond yield for bonds in that maturity class as of November 1976.[7] The market value of outstandings for each series is arrived at by converting the price to a factor and multiplying by the par value of outstandings. The resulting calculations reveal that the straight-bond component of CSBs was undervalued by 6 per cent by **BR** and by more than 6 per cent by **CH**. This translates into approximately a 3.0 per cent error in the *total* value of gross federal government debt, owing to the undervaluation of the straight-bond component alone. What is perhaps most disturbing is that while CSBs were premium bonds during 1976, **CH**'s method of estimating the market value of CSBs *reduced* the value of CSBs to be below par value.

The valuation method used in table 1 was applied to all CSB issues for the 1966–83 period (table 2). As indicated in table 2, when CSBs are valued as straight bonds, the

7 A number of simplifying assumptions were made in calculating the estimated market prices (e.g., yields to maturity were rounded to the nearest quarter of a decimal point) which may have produced second-order deviations from exact values. More importantly, the market value of accumulated interest and promised cash bonuses is also ignored. This factor is particularly significant for compound-interest bonds. Given this, because the resulting prices indicate that all the CSBs issues involved in the 1976 campaign were premium bonds, the estimated-market-value estimate given in table 2 is a more effective lower-bound estimate on 'market value' than are the measures suggested by **BR** and **CH**.

TABLE 2
CSB straight-bond valuation ($ millions)

Campaign	Series	Par value O/S	Estimated straight-bond value	Per cent difference
1966	S10–S21(CS)	$6074	5699.2	−6.2
1967	S11–S22	6307	6152.3	−2.5
1968	S11–S23	6324.7	6228.2	−1.5
1969	S12–S24	6672.1	6275.6	−5.9
1970	S13–S25	7366.8	7754.7	5.3
1971	S13–S26	9891.6	10954.5	10.8
1972	S13–S27	11094.9	11807	6.4
1973	S17–S28	10341.7	10970.3	6.1
1974	S17–S29	13156.3	14928.8	11.9
1975	S17–S30	15806.3	16775.3	7.1
1976	S20–S31	16547.9	17584.1	6.3
1977	S21–S32	18220	19548	7.3
1978	S21–S33	19943	19759.3	1.0
1979	S22–S34	18731.2	18386.1	−1.8
1980	S23–S35	17311.2	15935.2	−8.0
1981	S23–S36	25446.6	23358.9	−8.2
1982	S27–S37	33490.3	31657.5	−5.5
1983	S27–S38	39466.5	36738.5	−6.9

methods used by **CH** and **BR** result in considerable valuation discrepancies. An examination of the source of these discrepancies in more detail reveals that a number of important changes in the use of CSBs as a financing instrument took place over the period. In the early part of the period covered by table 2, CSB coupons were generally set below the rate available on Government of Canada bonds with similar maturity.[8] As a result, the market value estimates indicate that CSBs were selling at a discount on a straight-bond basis. Because interest rates were stable to slightly upward trending, terms for the new CSB issue could be set close to but slightly above that of the last issue. This resulted in an issuing cycle where a large part of outstanding CSB issues was concentrated in the most recently issued series; that is, despite the stated maturity, many long-term investors rolled their CSBs holdings into the new issue, taking advantage of the encashability feature of CSBs.

The terms on new issues continued to be more favourable until a coupon peak was reached with the S24 series.[9] For the period covered by table 2, the peak in the coupon with the S24 series is the first significant instance of the 'coupon ratchet' effect affecting CSB redemption activity; the S24 series was by far the largest of outstanding CSB issues from its inception in 1969 up to the major change in CSB terms in 1974 (e.g., in 1973, S24 outstandings were $3951.8 million while the new issue, S28, sold

8 This coupon-setting strategy may have been made possible by the limited alternative investment opportunities available to small savers in the 1960s.

9 While the S24 series had a shorter maturity than the S25 (eight years versus eleven years), the S24 series had a coupon structure that was slightly more than twenty-five basis points higher than the S25.

$1044.7 million). The impact of the ratchet effect induced by the S24 series is reflected in the shift in CSB straight-bond values from discount to premium. The across-the-board increase in CSB coupons on outstanding issues in 1974, triggered by redemption activity, provides another example of the significance of coupon ratcheting. The coupon increases in May and September of 1974 were followed by a downturn in rates. As a result, until the upswing in interest rates starting in 1978, straight-bond CSB values were decidedly above par.[10] This, combined with the larger percentage of *total* debt financed by CSBs results in a considerable undervaluation of total debt by **CH** and **BR** for much of the 1970s.

The considerable interest rate volatility in the late 1970s and early 1980s produced pressures for a restructuring of CSB terms. In 1979, much as in 1974, the coupons were raised on all outstanding CSB issues. However, unlike the situation in 1974, the coupon increase was stepped down – a higher coupon (12 per cent) was offered for 1979–80 while a lower coupon (10.5 per cent) was offered for the remaining term maturity. The continuation of the high interest rates that produced the increase in CSB coupons in 1979 continued into 1980, when the promised 10.5 per cent coupon had to be raised to 11.5 per cent for 1980–1. The formal announcement of the change to a 'stepped-down' coupon structure combined with annual rate setting came in 1981 when the annual coupon was again raised (to 19.5 per cent). The introduction of annual CSB coupon rate setting is significant because of the implied reduction in the effective maturity of CSBs. In addition, annual CSB coupon setting should generally reduce the value of the CSB put option. This follows because, with annual coupon setting, the value of the CSB should be closer to par.

Given an upward-sloping yield curve and relatively stable interest rates, reduction in effective maturity should result in an interest cost savings over time. These savings are reflected in the shift, around 1979, from premium to discount of the straight-bond CSB values. However, while this shift in the market value estimate for CSBs is consistent with the valuation of CSBs for previous years, the market values for the period of stepped-down coupons are incompatible with actual pricing (by rational investors). This is because the stepped-down coupons are not effective coupon rates – only minimum coupon rates. Hence, valuing the future cash flows on the basis of the stepped-down coupon ignores the potential upward revision in coupons which may take place in later years. A more accurate procedure is to value the cash flow for only one year, assuming that the CSB value in one year will be equal to par. On this basis, CSB straight-bond values were revised for 1979–83 (table 3). As might be expected, with the exception of 1981 (19.5 per cent coupon) the resulting calculations reveal that floating coupon CSBs have straight-bond values close to par.

As noted, the straight-bond portion does not account for the full market value of CSBs; that is, the straight-bond value provides only a (rational) lower bound on the market value of CSBs. For more accurate 'market' valuation, it is also necessary to account for both the value of the put option component and the value of coupon payments accumulated on compound interest CSBs. Valuation of the put option

10 On a technical level, 1978 is significant because it marks the beginning of a seven-year maximum maturity on CSB issues.

TABLE 3
Revised CSB straight bond values: 1979–83 ($ millions)

Campaign	Price	Outstandings	Estimated market value	Per cent difference
1979	99.55	18,731.2	18,648.0	−0.045
1980	101.24	17,311.2	17,527,2	1.24
1981	105.29	25,446.6	26,791.8	5.29
1982	101.35	33,490.5	33,945.1	1.35
1983	101.38	39,466.5	40,009.6	1.38

component of CSBs is considerably more difficult than valuation of the coupon income associated with CSBs. Yet, even crude estimates of the 'market value' of the put option are useful, because the value of the put option component of CSBs may have been considerable for certain years, for example, when straight-bond CSB values were at or below par. Theoretically, the size of the CSB put option premium is dependent on the relationship between the coupon rate and the prevailing level of rates, the shape of the term structure (a proxy for expectations about the direction of rates), the term expiration of the option, and the expected volatility of interest rates. If, for example, the prevailing CSB coupon is substantially above the market indicating that the straight-bond portion of the CSB is well above par, then the put option portion of the CSB will be relatively low in value. On the other hand, if the straight-bond portion of the CSB is close to or below par, then the put option may have considerable value depending on the other variables in the put option valuation function.

The general problem of pricing a savings bond has been examined by Brennan and Schwartz (1979) using techniques from the continuous-time-pricing theory for options. Using the continuous-time-pricing approach to specify pricing equations, Brennan and Schwartz provide empirical estimates for the value at issue of the CSB put option / straight-bond combination, as well as redemption values at various dates, for series S12–S31 over the 1964–76 period. Brennan and Schwartz estimated that between 1964 and 1976 the average put option premium on a *new* CSB issue was $5.29 per $100 par value with the total premium (straight plus option) averaging $6.74.[11] No information is given on the size of the put option premium for outstanding issues. Using Brennan and Schwartz's results as a benchmark, assumptions about the lower bound to the value of the put option straight-bond combination were made. The resulting calculations appear in table 5.

The final adjustment needed to arrive at a more precise lower bound to the 'market' value of CSBs is to incorporate the accumulated interest which has accrued on

11 The Brennan and Schwartz methodology for evaluating the straight-bond component differs somewhat from the procedure above. Specifically, their method of decomposing the total value of the option into straight-bond and option premium tends to produce somewhat different values for the straight-bond component relative to the procedure used for tables 1 and 2.

TABLE 4
Breakdown of outstanding CSBs into C and R bonds, November 1984 ($ millions)

Issue date	Series	Original sales	Current O/S	C	R
1977	S32	2834.9	405.3	262.1	143.2
1978	S33	7537.1	2925.2	1300.8	1623.4
1979	S34	5048.8	2049.5	677.4	1371.6
1980	S35	3360.4	1385.0	602.5	782.3
1981	S36	12806.4	7938.3	3164.0	4772.6
1982	S37	11383.1	7141.9	2388.2	4752.7
1983	S38	11663.9	8139.8	3065.5	5073.7
1984	S39	12753.3	12751.5	5170.5	7581.0

SOURCE: Bank of Canada

outstanding compound interest CSBs.[12] Valuation of unpaid compound interest is required for accurate market valuation because the interest that is due, but not paid, represents a future cash flow which would be valued by the market. This is important, because by allowing the government the ability to delay interest payments, compounding is another feature which distinguishes CSBs from other methods of financing. Unfortunately, while the compound interest option has been available from 1966, it is difficult to account accurately for the portion of outstanding CSBs that was uncashed and compounded until the explicit introduction of C (compound-interest) and R (regular-interest) bonds in 1977.[13] The importance of CSB-C issues is indicated in table 4. As indicated, anywhere between 30 to 45 per cent of the outstandings of a given CSB series was held in compound-interest form. Based both on the history of the holding of CSB-C bonds between 1977 and 1984 and on unpublished Bank of Canada analysis which indicated as much as one-half of coupon-bond holders (1967–74) had taken advantage of the compounding option, the impact of compounding was extrapolated for 1967–77 on the basis of 35 per cent of the outstandings for a given issue held in compounded form.

Table 5 provides a final tabulation for the various discrepancies between **CH**'s and **BR**'s 'market' value estimates for CSBs and the 'market' value estimates based on the adjustments examined here. By construction, put option values substantially less than those given by Brennan and Schwartz have been selected in order to avoid

12 The value of promised cash bonuses to be paid at maturity has already been accounted for in the calculation of the straight-bond values. In addition, the value of intrayear unpaid coupons on regular CSBs has been ignored. This is in keeping with the treatment of the unpaid semi-annual coupons on direct marketable bonds.

13 This is because the compound-interest option, which was introduced in 1966, applied only to coupons on regular-coupon and Special Fully Registered issues that were not cashed. If the coupons were cashed, the interest was not compounded. (The compounding option was not available on S29–S31). Hence, to account accurately for the unpaid interest due to compounding prior to 1977 it is necessary to have much more detailed information about CSB redemption activity than is available. In addition, a compound-interest bond will have a slightly different valuation than a coupon bond, because coupons are not received over the life of the compound interest bond. For computational purposes, the small pricing discrepancies between compound and coupon CSBs have been ignored in calculating the straight-bond values given in tables 1 and 2.

TABLE 5
Impact of valuation method on the total debt estimates of **CH** and **BR**

Year end	Estimate bond value	Estimate put option	Compound value	Impact on total debt	
				BR	**CH**
				(per cent)	(per cent)
1966	5699.2	199.5	0		1.2
1967	6152.3	184.6	38.4	0.3	2.9
1968	6228.2	186.8	41.2	0.5	3.1
1969	6275.6	219.6	60.4	−0.6	3.3
1970	7754.7	116.3	133.6	2.7	4.5
1971	10594.5	54.8	214.8	3.5	4.1
1972	11807.0	177.1	392.8	4.5	5.8
1973	10970.3	164.6	720.5	4.1	6.2
1974	14928.8	74.6	767.4	8.1	10.2
1975	16775.3	251.6	1103.8	6.4	9.4
1976	17584.1	263.8	1306.3	6.4	7.8
1977	19548.0	293.2	1399.3	6.3	7.0
1978	19759.3	494.0	1190.0	2.3	3.8
1979	18648.0	326.3	1858.6	3.3	5.6
1980	17527.2	306.7	1952.1	3.3	5.5
1981	26791.8	200.9	1866.2	3.9	6.6
1982	33945.1	594.0	1869.2	2.7	3.1
1983	40009.6	701.7	2442.6	2.6	2.6

NOTES: For the estimated Straight-bond values, results for tables 2 and 3 have been condensed, that is, for 1979 and after it is assumed that CSBs are valued as one-year instruments. For the put option values per $100 of straight-bond value for 1966–9, a value of between $3.00 and $3.50 was chosen, depending on the amount below par or the straight-bond value. For 1970–7, a value of $1.50 was used for SB values between +5 and +10 per cent above par, a value of $0.50 was used for values 10 per cent. For 1978, $2.50 was used and for 1979–83 a value of $1.75 was used for each year except 1981, when 0.75 is used.

For the compound values, to avoid calculation complications associated with the compounding rate adjustments, compounding was done at the prevailing one-to-three-year rate. This procedure will usually result in small, second-order deviations from the actual value of unpaid, compounded coupons.

upward biasing. Regarding compounding, the value of unpaid, compounding coupons is seen to be substantial – reflecting one important feature of CSBs for government financing. The result is that both **CH**'s and **BR**'s estimates of the total market value of Government of Canada debt are seen to differ markedly from estimates that account for more accurate market valuation. Because the price index adjustment factor used by **CH** generally was below one throughout the period examined, this resulted in a valuation for CSBs that was below the par value estimate used by **BR**. Hence, **CH**'s estimates deviate most from the more accurate 'market' values given in table 5. In 1974, the year with the greatest discrepancy, **CH**'s total market value estimate deviated by over 10 per cent from the table 5 estimate.

SUMMARY

The valuation methods selected for pricing non-marketable debt were shown to have a

significant impact on the results of both **CH** and **BR** regarding the total market value of Government of Canada debt. In particular, the price-index adjustment method used by **CH** produced particularly perverse results; that is, the par value of CSBs was usually adjusted downward to arrive at a 'market' value estimate when upward revisions were more consistent with accurate market valuation. On the whole, when non-marketable debt is accounted for using more accurate pricing methods, the market value of the debt for the 1970s was found to be higher relative to other periods. In addition, the analysis helps identify important debt-management events such as the revision of CSB terms in 1974, the introduction of compound-interest bonds in 1966 and the shift to floating-coupon setting.

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