The when-issued market for Government of Canada treasury bills

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Abstract. By working directly with the profit functions for both arbitrage and speculative trades, this paper develops two distinct notions of market efficiency. These notions provide the basis for evaluating the performance of the when-issued (WI) market for Government of Canada treasury bills. Despite the maximum one-week maturity of WI forward contacts, a number of instances of inefficient pricing behaviour are observed.

Le marché avant l'émission des bons du trésor du gouvernement du Canada. En travaillant directement avec des fonctions de profit pour le commerce d'arbitrage et le commerce de spéculation, ce mémoire développe deux notions distinctes d'efficience des marchés. Ces notions fournissent des éléments pour évaluer la performance du marché avant l'émission des bons du trésor du gouvernement du Canada. Malgré l'échéance maximale d'une semaine des contrats à terme sur ce marché, on observe des cas de tarification inefficiente.

Each week, the Government of Canada issues three- and six-month treasury bills (tbills) at auction.[1] Barring holidays, the auction takes place on Thursday with delivery (and maturity) of the tbills occurring on Friday. Announcement of size of the following week's offerings is made at the same time (2:00 p.m. Thursday) as the results for that week's auction are announced. Tbills to be auctioned at the next tender are traded on a *when-issued* (WI) basis from the time of the announcement of the size of the tender until the auction takes place; that is, the WI tbill market

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1 The Government of Canada also typically issues twelve-month tbills at the weekly auction which are also traded on a WI basis. However, for part of the sample, twelve-month tbills were issued only bi-weekly, creating sampling problems. In addition, the twelve-month cash and WI markets are the thinnest of the three available maturities. For these reasons, the twelve-month maturity was not included in the analysis.

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is a forward market for tbills with maximum contract maturities of one week.[2] The central objective of the current study is to identify whether the interest rate generating process in the WI market is either 'arbitrage efficient' or 'speculative efficient.' These efficiency concepts are developed directly from the speculative and arbitrage profit functions associated with WI trade. Significantly, the resulting efficiency concepts impose different testable restrictions on the behaviour of WI rates. In the following, section I provides institutional background on the WI market. Section II develops the arbitrage relationships applicable to the WI market. Section III provides empirical evidence on the speculative efficiency of the WI interest rate determination process. Finally, section IV provides a summary of the important results contained in the study.

I. BACKGROUND

A when-issued tbill is a contract either to purchase or to sell tbills that are to be auctioned at the next tender. Settlement of WI transactions is made in tbills on the next business day following the auction. Typically, active WI trading begins on the Friday preceding the tender and continues up to the actual auction.[3] Active participants in the WI market are the investment dealers, the banks, and major institutional investors. While there was some WI trade prior to 1978, the formal beginning of the WI market in Canada can be traced to the introduction of screen-oriented brokered WI trading in the fall of 1978. Currently, more than half of trading in WIS is done through screen-oriented broker services.[4] The widespread use of screen-oriented trading means that WI activity is highly visible to other participants in the tbill market. For monetary and debt management purposes, in conjunction with the yield on the most recently issued tbills, the yields quoted in the WI market are often taken to be indicators of what tbill yields at the next auction will be.

2 The structure of the Canadian tbill auction process differs somewhat from that of the United States. For example, as well as having different auction days and somewhat different auction procedures, there is a three-day settlement lag (Monday to Thursday) in the United States and only a one-day lag in Canada. As a result, in the United States WI Treasury bill (Tbill) trading often refers to trade in new Tbills in the period between auction and settlement (Stigum 1983). In Canada, WI tbill trade refers only to the trade in to-be-issued tbills occurring between the announcement of tranche sizes and the auction. WI trading also occurs in instruments other than tbills – such as Government of Canada marketable bonds. Finally, there is no formal restriction on WI trade limiting the maximum maturity to one week. Rather, it is the requirements of tbill traders which determine the available contract maturities. As the Canadian tbill market grows in size and sophistication, longer maturity (e.g., two weeks to delivery) WI trade may emerge.

3 While there is some WI trade on Thursday, in the period between auction and settlement the market is primarily concerned with sorting out of cash positions arising from the auction. Hence, the market does not usually start focusing on WI trading for the next auction until Friday.

4 In screen-oriented trading, bids and offers are listed on computer terminal screens in the offices of subscribers to the service. Bids, offers, and amounts for WI trades are entered directly onto the screen by the trader. Trades are executed by accepting one of the positions listed on the screen, again by making the appropriate terminal data input. Once a trade is executed, the broker is responsible for recording the relevant transaction information. Off-screen brokered trading of WIS is conducted like conventional OTC trading. Usage of WI brokers is significantly affected by the ability of investment dealers to do WI trade directly with their accounts. Typical broker commissions for WIS are one-half a basis point each way.

TABLE 1

Average three-month WI bid/offer spread, Friday–Wednesday: January 1986–May 1987

Day	Bid minus offer (basis points)	
	Full sample	Last 26 weeks
Friday	3.9	2.6
Monday	2.5	2.0
Tuesday	2.3	1.9
Wednesday	2.1	1.6

SOURCE: Securities Department, Bank of Canada.

In providing tbill market participants with a relatively inexpensive method for forward trading of tbills, the WI market is much like a tbill futures market. In the Canadian context, this is significant because the only currently listed futures market for Canadian tbills – traded on the Toronto Futures Exchange – is at present inactive.[5] However, the forward market for WIs does differ in a number of substantive ways from a typical futures market: contracts are not standardized, a smaller performance deposit (margin) is required,[6] there is no 'marking to market' and trading is not on a centralized exchange. Most importantly, the one week trading horizon for WIs differs markedly from the maturity dates typically associated with futures. Finally, while there is some enhancement of WI market liquidity from small players in the cash market (e.g., from small banks seeking to make a rate play without the cost of acquiring tbills), the nature of trading and settlement in the WI market effectively limits important participants to the large traders in the cash tbill market, eliminating the participation of the general public.

Regarding the liquidity of the market, there is at best only limited quantitative information. It is generally the case that the market is quite thin on Fridays – with the bid/offer spread being as wide as ten basis points on occasion. Mondays are usually a transition day, with volumes picking up substantially on Tuesdays and Wednesdays. As outlined in table 1, this is reflected in the average WI bid/offer spread for each of these days. Table 1 establishes that there is a considerable narrowing of the bid/offer for a WI contract as the auction date is approached. In addition, bid/offer spreads narrowed significantly for the last twenty-six weeks of the sample. While there may be other secondary factors at work such as the degree of uncertainty associated with the forecast horizon, liquidity is usually the dominating factor driving bid/offer spreads in the WI market.

5 In 1989, the Montreal Exchange launched a new Government of Canada bond contract. For more on other available hedging instruments see Poitras (1988a). By draining off liquidity, the growth of the WI market has undoubtedly contributed to the lack of success of the tbill futures contract.

6 The margin requirements for WIs arise out of the Investment Dealers Association's Regulations. For IDA purposes, WIs are treated as contingent liabilities, much as futures contracts are. Current margin requirements are capital sufficient to cover 1 per cent of the outstanding contingent liability.

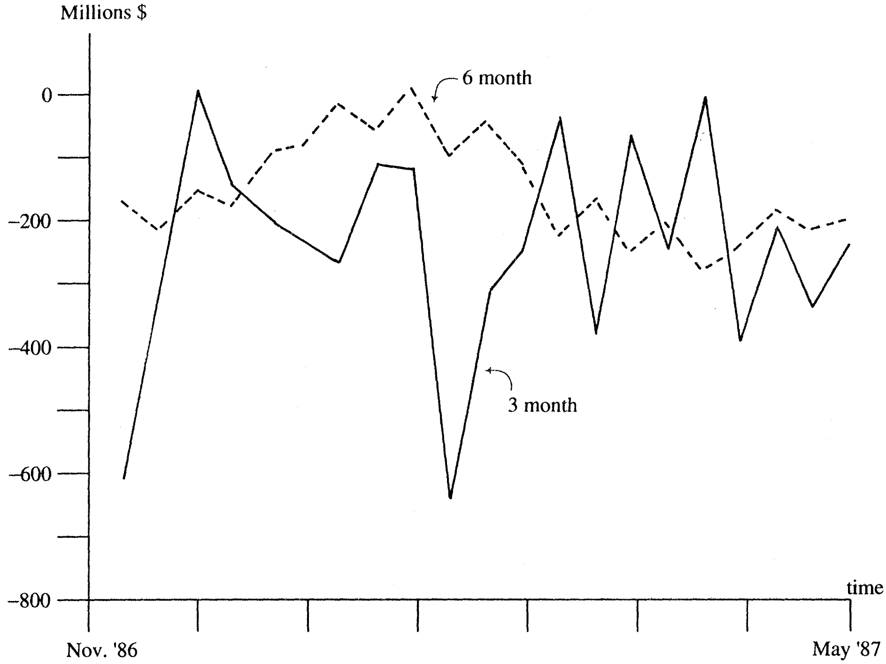

FIGURE 1 Net when-issued positions of major dealers

In terms of the net position of traders and the absolute size of outstanding contracts, there are only limited data. However, data provided on the Wednesday closing WI positions of the major dealers taken over the November 1986–April 1987 period (figure 1) do provide some information.[7] While these data do not account for all of the trade in the market, they are useful for indicating trend and other qualitative factors. Figure 1 demonstrates that the major dealers were generally net short during the period. Theoretically, net short positions by dealers arise primarily from speculating on increasing yields, hedging of inventory against increasing yields, and hedging of anticipated auction winnings. In practice, given that the available sample was, generally, a period of stable to declining rates, it would appear as though the use of WIS to hedge anticipated auction winnings was an important part of the dealer's use of WIS during the sample period under consideration. Available information on the absolute size of dealer positions indicates that the net position information given in figure 1 does not appear to be particularly sensitive to the volume of trade.

7 These statistics were provided to the Bank of Canada by the major dealers. (The IDA does not collect explicit statistics on WIS). Five weeks during the sample were omitted because of unavailability of data.

II. ARBITRAGE RELATIONSHIPS IN THE WI MARKET

The concept of market efficiency has evolved considerably since Fama's (1970) approach based on information sets. Harrison and Kreps (1979), Duffie (1986), Duffie and Huang (1985), for example, relate market efficiency to absence of arbitrage opportunities in the securities price system. Other researchers (e.g., Gregory and McCurdy 1984; Bilson 1981) impose zero expected profit to speculation as a requirement of efficiency. Working directly with the profit function for the relevant activity, this paper develops two concepts of efficiency relevant to the WI market: arbitrage efficiency and speculative efficiency. Arbitrage efficiency requires the absence of profitable arbitrage trading opportunities, that is, trades capable of generating 'riskless' profit opportunities involving no net investment of funds. Accurate construction of this concept for the WI market requires specification of the exact trading mechanics required to execute the relevant WI and cash trades.[8] On the other hand, 'no expected financial gains to speculating' involves risky trading activities, that is, purchase for later resale, requiring a net investment of funds in present or future states.

Fundamental to the arbitrage efficiency of forward markets are the cash and carry arbitrage relationships that connect the forward and spot rates (e.g., Kolb 1988). The upper and lower boundaries provided by the short and long cash versions of the arbitrage constrain the ability of forward rates to deviate from spot (cash) rates. For arbitrage efficiency, deviations of forward rates from cash rates must fall within the arbitrage bounds in order for prices to exclude the profitable execution of cash and carry arbitrages. In general, the profit function of a cash and carry arbitrage can be specified:[9]

$$\pi(0,T) = F(0,T) - [S(0)(1 + r(0,T))], \tag{1}$$

where $\pi(0,T)$ is the profit function for the cash and carry arbitrage between $t = 0$ and $t = T$; $F(0,T)$ is the forward rate observed and settled at $t = 0$ for delivery at $t = T$; $S(0)$ is the spot rate at $t = 0$; and $r(0,T)$ is the actual (not annualized) net cost of carrying the commodity between $t = 0$ and $t = T$. In frictionless markets, the no arbitrage profit condition requires $\pi = 0$; that is, if $F(0,T) > [S(0)(1+r(0,T))]$, the long arbitrage is profitable, and, if $F(0,T) < [S(0)(1 + r(0,T))]$, the short arbitrage is profitable. Hence, if there are no restrictions on the executability of either the short or the long arbitrage, $F(0,T) = [S(0)(1 + r(0,T))]$ is required for

8 For example, precise specification of the underlying arbitrage helped to resolve the seeming paradoxes that arose in earlier research on tbill futures prices (e.g., Jacobs and Jones 1980; Capozza and Cornell 1978; Rendelman and Carabini 1979) which found significant deviations between tbill futures rates and the associated implied forward rates imbedded in the term structure of interest rates. In an investigation of the cash and carry arbitrages for U.S. tbill futures, Gendreau (1985) found that cost of carry was capable of explaining the deviations between tbill futures prices and the implied forward rates imbedded in the term structure.

9 In the case of the foreign exchange market, (1) is the profit function for covered interest arbitrage; that is, $(1 + r(0,T)) = (1 + r(0,T))/(1 + r^*(0,T))$, where r^* is the foreign rate of interest and the spot and forward exchange rates are measured in domestic direct terms.

'arbitrage efficiency.' In terms of mean variance expected utility maximization, the no arbitrage restriction on prices corresponds to the optimality condition for a cash and carry 'hedger' (e.g., Danthine 1978; Poitras 1989).[10]

In order to test for arbitrage efficiency in a specific case such as the WI market, it is necessary to examine the actual arbitrage mechanics and verify what types of restrictions affect these trades in practice. In particular, transactions costs are a necessary, but not usually sufficient, adjustment. The introduction of transactions costs effectively transforms the arbitrage equality restriction into upper and lower arbitrage boundary restrictions. In effect, there is an arbitrage band within which some variation from the frictionless arbitrage condition is permitted. However, even the construction of transactions cost boundaries requires careful consideration of the underlying trading mechanics. For example, the precise calculation of transactions costs for arbitrage trades is not always apparent (e.g., Clinton 1988). In other cases, transactions costs are not sufficient to define an upper or lower boundary, because either the short or long arbitrage cannot be executed, resulting in a 'one-sided' arbitrage relationship (e.g., Poitras 1988b).

To derive the long cash and carry arbitrage condition for three month WIs, the profit function at $t = 0$ involves the *purchase* of a (3-month + N-day) tbill financed at the term call loan rate. Term financing is required for the borrowing rate to be riskless. The call loan rate is used because this market typically provides the best combination of availability of funds and competitive rates. This purchase is simultaneously covered with the sale of a three-month WI maturing in N days. In other words, the tbill being purchased is the 'old' six-month tbill which is to be reopened at the next three-month auction, $N < 7$. This cash tbill purchase is simultaneously covered with a WI short. Schematically, the long arbitrage trade can be described:

	Cash	*When-issued*
$t = 0$	Buy \$$Q$ par value of (3-month + N-day) tbills at $P(0)$ and carry at the term call loan rate	Sell \$$Q$ par value WI at price WI(0) maturing at $t = N$
$t = N$	Use the cash tbill to deliver on the short WI position	

By construction, the positions in cash and futures are equal, allowing the tbill purchased at $t = 0$ to be used for delivery on the WI position. In terms of prices:

$$\pi(0, N) = \text{WI}(0) - [P(0)(1 + r(0, N))] \leqq 0 \qquad (2)$$

10 Further, the absence of profitable arbitrage opportunities is a requirement of capital equilibrium and will hold regardless of the form of the underlying utility function.

While (2) is consistent with the form given in (1), it is somewhat misleading in the case of tbills in that $P(0)$ will increase over time, owing to the effect of maturity reduction. This is reflected in the $r(0, N)$ term which incorporates the return on the underlying tbill. In terms of yields, it can be shown (see appendix) that the profit function can be approximated as

$$\pi(0, N) = i^* - [i + (N/91)(i - R)] \leqq 0, \tag{3a}$$

where i^* is the promised yield at $t = 0$ on the 91-day WI tbill, i is the secondary market yield on the (to-be-reopened) 91 + N day tbill, and R is the term call loan rate. In other words, the WI rate is bounded from above by the tbill rate and the cost of carrying the tbill between time 0 and N.[11]

The profit function for the frictionless short arbitrage is more than (2) with the weak inequality reversed. Specifically, in order for the short arbitrage to be riskless, the (3-month + N-day) tbill has to be 'shorted.' The implied acquisition of the appropriate tbill can be done by doing a term *reverse* repo at annualized rate RR using the correctly dated tbill as underlying collateral. This tbill is then sold and the funds acquired are used to cover the cost of doing the reverse. Simultaneously, the short tbill position is covered by taking an equal par value amount of long WI contracts. At delivery, the tbills acquired from the long WI position are used to cover the short position created by the reverse; that is, the profit function is

$$\pi(0, N) = i^* - [i + (N/91)(i - RR)] \geqq 0. \tag{3b}$$

In words, the short cash arbitrage provides a lower bound on the WI rate. Hence, in addition to transactions costs, the upper and lower boundaries use different borrowing/lending rates.

In order to test empirically for 'arbitrage efficiency' using (3a) and (3b), exact estimates are required for transactions costs. Putting the trading complications aside for the moment, the WI arbitrages described above require cash and WI transactions in which the bid/offer spread[12] must be absorbed as well as broker commissions on the WI. Assuming WI bid/offer spreads of two basis points, cash tbill spreads of one basis point, one basis point in WI broker commissions, and one basis point for the bid/offer on the financing rate, transactions costs provide a five-basis point buffer on either side of (1) and (2). Unfortunately, the effect of the spread between R and RR is more difficult to estimate. In addition, from the analysis of the trading mechanics,

11 Extending this result to implied forward rates, as is done in some studies of tbill futures (e.g., Dym 1988; Capozza and Cornell 1980) is not possible, because the one-week horizon makes getting accurate cash market quotes difficult. The additional term which causes inequality between the WI and cash tbill rate is generally small. For a WI trade done on Friday for delivery on the next Friday, the factor (N/91) is equal to 0.077 and 0.044 for Monday.

12 In market parlance, 'the edge.' While the trader does not have to give up the edge on each trade, the required passive trading tactics (e.g., putting out bids instead of hitting offers) would increase the execution time (risk). Active execution of the arbitrage would necessitate that the transactions costs be paid.

it is not clear how the reverse rate is determined. Given this, it is appropriate to treat the short arbitrage as a 'weak side' boundary. Hence, while significant deviations in the direction of the short arbitrage may not represent actual violations of arbitrage efficiency, strict adherence is expected at the long arbitrage boundary.

Given this background, figures 2 and 3 provide representative plots of the three-month arbitrage differential, $i^* - [i + (N/91)(i - R)]$.[13] Summary data on all the differentials are provided in table 2. The when-issued, topical tbill and 'most' call loan rates used are mid-market rates based on Bank of Canada market closing data. This specific data configuration is of some importance. Examining figures 2 and 3 provides considerable evidence that, when account is taken of transactions costs, WI rates have on a number of occasions deviated substantially outside the arbitrage boundaries which, for present purposes, can be set at five basis points in the direction of the long arbitrage – that is, negative – and possibly as much as ten basis points in the short arbitrage direction – that is, positive. However, in addition to the transactions boundaries, there are other qualitative factors that should be taken into account before one concludes that arbitrage conditions were violated.

While (3a) and (3b) provide theoretical upper and lower bounds on WI rates, the adherence of WI rates to the boundary restrictions depends on the ability of market participants to execute the arbitrages. From a trading perspective, there are considerably more impediments to executing the WI arbitrages than there are on similar arbitrages for, say, U.S. tbill futures. In particular, unlike the term repo market in the United States, the term call (or repo) market in Canada is quite thin. This is especially so for reverses. In practice, the long WI arbitrage would likely have to be financed in the overnight call market. This would require the cash position to be refinanced on a daily basis.[14] Given the short trading horizon, this will not severely restrict execution, only the level of riskiness. On the other hand, the reverse would likely require the participation of an institutional tbill account. This would require some form of fee. By implication, as is the case in a number of other financial markets, the short arbitrage provides a significantly weaker restriction than the long arbitrage. This means that large deviations from zero arbitrage would be tend to be positive (as specified in equations (3)).

In addition to financing restrictions, the supply of the 'old' six-month tbill which is to be reopened at the next auction is often limited. In general, market activity centres on the most recently issued tbill maturities because of availability, maturity matching, and other factors. The 'old' bills are locked up in accounts. Hence, for liquidity reasons, a tbill that matures a week earlier may have to be substituted for the arbitrage tbill. This substantially complicates the arbitrage in a number of ways. For example, consider the short WI position in the long arbitrage described above. Delivery would have to be executed by winning the appropriate number of tbills at auction, simultaneously selling the (3-month less 1-week) tbill, which

13 Results for the six-month differential, $i^* - [i + (N/182)(i - R)]$, are similar.

14 Recently, the introduction of auctions of Receiver General Balances in 1986 has increased the supply of term call money, especially from the near banks. Hence, there may be scope for the development of the term financing in the future.

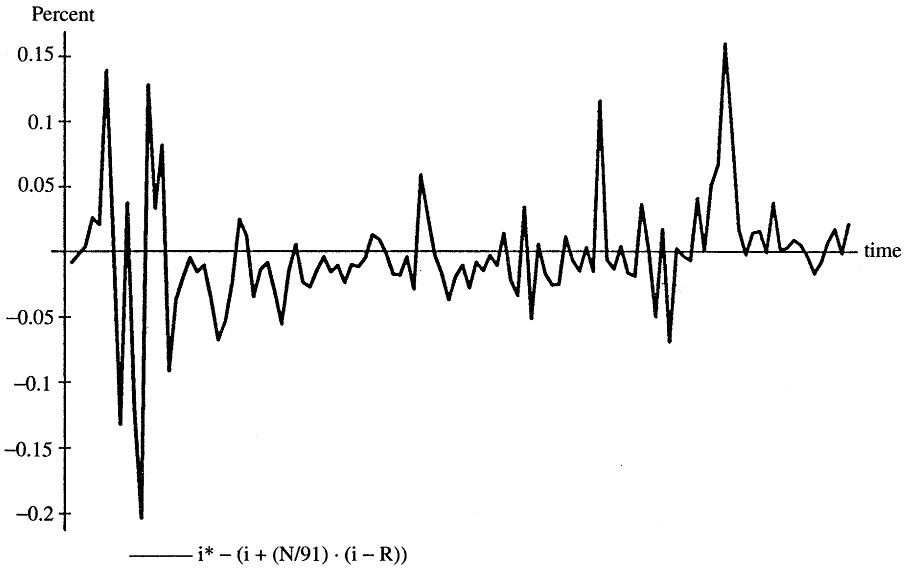

FIGURE 2 Three-month when-issued arbitrage, Fridays, January 1986 – April 1988

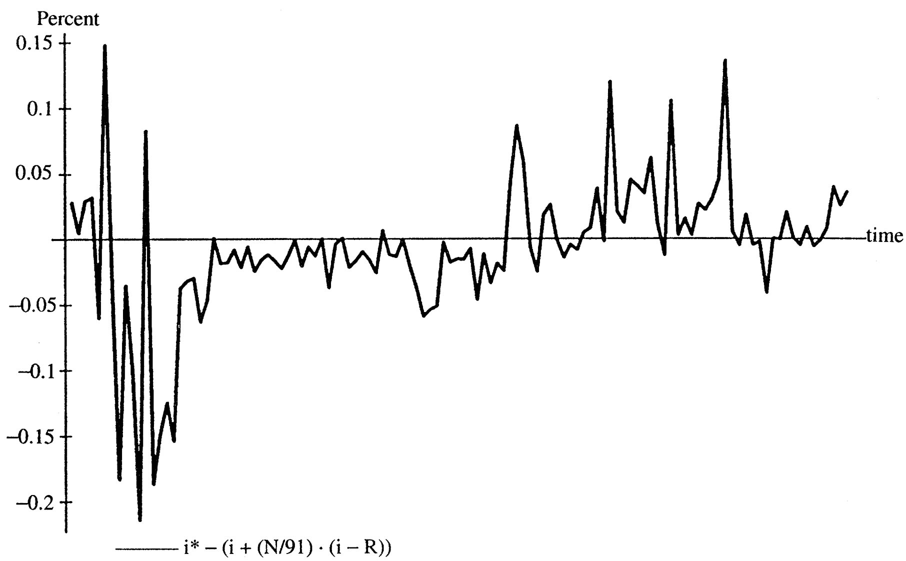

FIGURE 3 Three-month when-issued arbitrage, Wednesdays, January 1986 – April 1988

would now be less liquid because the 'new' bill had just come out. In addition to increasing the number of transactions, it is likely that the spread between the (3-month less 1-week) tbill rate and the (3-month) tbill rate will widen, further

TABLE 2*

Some distributional properties of the arbitrage differential: $i^* - [i + (N/M)(i - R)]$

Sample: Weekly, January 1986 – April 1988

Variable	Mean	Std. Dev.	Min.	Max.	SUM
PROFIT6F	−0.019	0.035	−0.27	0.08	−2.19
PROFIT3F	−0.003	0.046	−0.20	0.16	−0.35
PROFIT6M	−0.020	0.036	−0.22	0.04	−2.32
PROFIT3M	−0.007	0.054	−0.24	0.16	−0.77
PROFIT6T	−0.006	0.091	−0.19	0.92	−0.72
PROFIT3T	−0.009	0.053	−0.27	0.12	−1.14
PROFIT6W	−0.020	0.060	−0.41	0.13	−2.34
PROFIT3W	−0.007	0.054	−0.21	0.15	−0.86

* PROFIT$m, n = i^* - [i + (N/M)(i - R)]$, where i^* is the WI tbill rate, i is the cash tbill rate, and R is the call loan rate. M is either 91 or 182 (days) when the variable m, which refers to the maturity of the underlying bill, is either three or six (months). The variable n refers to the day of the week.

affecting execution costs.

Regarding actual calculation of the arbitrage differentials, further allowance has to be made to account for the use of the 'just-issued' or 'topical' bill rate instead of the 'to-be-issued' tbill rate. This proxying was necessary because the week-to-week trade in tbills centres on the most recently issued tbill, which typically has a much greater 'floating supply' (Vignola and Baker 1979) than the to-be-issued tbill. As a result, interest rate quotes for the to-be-issued tbill are not likely to be the same as those for the topical, owing to liquidity and other considerations. Unfortunately, rate quotes for the to-be-issued tbill are not available. A final point of qualification concerns the use of closing quotes. In practice, the bulk of money market activity takes place well before the closing. Consequently, trades that take place late in the day may not be fully representative of market conditions. Potential discrepancies from this source are likely to be largest when the market is most volatile.

Given these qualifications, examination of the empirical evidence reveals a number of instances of unexploited arbitrage opportunities. Most of these opportunities arose during unsettled market conditions, in particular, the exchange crisis of early 1986 and the market break in October 1987. However, it is unlikely that the results are due exclusively to unrepresentative quotes. Based on the prevalence, timing, and size of the differentials, it is more likely that the WI market is underarbitraged. Especially when markets are unsettled, speculative traders use the WI market as an inexpensive vehicle to make an interest rate play. Because of the limited amount of arbitraging activity, speculative trade is able to move the WI rate significantly outside the arbitrage bounds. If this supposition is correct, it reflects a fundamental, and potentially exploitable, weakness in the WI market for Government of Canada tbills.[15]

15 Data for secondary-market tbills, call loans, and WIS are the daily closing prices as recorded by

III. SPECULATIVE EFFICIENCY

As developed in section II, arbitrage efficiency for a given market requires the absence of riskless arbitrage trading opportunities. This requirement imposes conditions on prices and carrying costs defined at a given point in time ($t = 0$). In opposition to this concept, speculative efficiency for a given market imposes a zero expected value condition on the speculative profit function. By construction, speculative efficiency is concerned with variables defined at different points in time. The resulting speculative trading strategies are risky. Unfortunately, the introduction of risk into the concept of efficiency significantly complicates the problem of determining whether a given market is 'efficient.' In particular, the proper handling of risk requires some methodology for determining risk-adjusted profits. Unlike the arbitrage profit function, which is fully determined on the basis of contemporaneous information, the speculative profit function contains variables that are uncertain when the trading decision is initiated.

As a result of introducing risk, the concept of an 'efficient' market equilibrium is more difficult to define. For example, both the statistical properties of the random variables and the properties of the trader's objective function with respect to the relevant distributional parameters require specification. It follows that any test of 'speculative efficiency' necessarily involves a joint hypothesis because a model of market equilibrium is required to formulate testable hypotheses about market efficiency. More significantly, when applied to a forward market it is difficult to test the hypothesis empirically without using variables observed at different points in time, that is, $F(0,T) - S(T)$, the forecast error. Statistically, this can raise the problem of moving average error terms if the forecast horizon has a greater length than the sampling frequency. In the present context, WI contracts have the desirable feature of having only one-week maturities; hence overlapping data are not a problem.

1. Unbiased prediction hypothesis

In addition to being the focus of a large number of studies of forward foreign exchange market efficiency (e.g., Bilson 1981; Boothe and Longworth 1986; Gregory and McCurdy 1984), the unbiased prediction hypothesis has also been applied to test efficiency in both the WI market in the United States (Ferri et al. 1985) and to the U.S. tbill futures market (Howard 1982; Hegde and McDonald 1986; MacDonald and Hein 1989). It is possible theoretically to derive the hypothesis using a number of not mutually exclusive methods: imposing zero expected value on a specific class of speculative profit functions; in a mean-variance expected utility framework, by assuming that either that speculators are risk neutral or the second moments are unbounded; or, working directly with the properties of the conditional expectation, by assuming that there is no systematic risk in futures price forecasts.

the Securities Department, Bank of Canada. To account for the bid/offer spread, WI yields are calculated as the mid-market between the bid and the offer. All rates are used in annualized form. The auction rate is the tbill tender average for all bidders. The sample runs from January 1986 to April 1988.

While there are a number of theoretical motivations, the testable requirement for 'speculative efficiency' based on unbiased predictions, applied to the WI market, requires[16]

$$E[\text{PTB}(N)|I(N-j)] = \text{PWI}(N-j), \quad (4)$$

where: $E[\cdot|I(N-j)]$ is the conditional expectations operator with information set $I(N-j)$.
PTB(N) is the issue price of the tbill at the following auction.
PWI($N-j$) is the price of the to-be-issued tbill observed in the WI market j days before the autction settlement date.
N is the auction settlement date.

Under relatively weak conditions on the allowable functional form for the treasury bill price process, the orthogonal decomposition can also be formed:

$$\text{PTB}(N) = E[\text{PTB}(N)|I(N-j)] + U(j,N), \quad (5)$$

where $U(j,N)$ is the forecast error of the conditional expectation formed at time $N-j$. Combining (4) with (5) provides for the specification of the WI forecast residual:

$$\text{PTB}(N) - \text{PWI}(N-j) = U(j,N) \qquad \text{(under } H_0\text{)}. \quad (6)$$

It follows that, to a second-order approximation, (6) also holds for yields.[17]

Statistically, although the unbiased prediction hypothesis can be tested in a number ways, implementation of the available methods is complicated by the unobservable expectation in (4). The empirical implications are illustrated in figure 4 which plots a representative time series of the forecast errors $U(j,N)$ using Monday WI contracts. The decidedly non-normal behaviour of the forecast errors depicted in the data plot is confirmed for all four forecast horizons in table 3, which provides the relevant distributional information. Considerable research effort has been devoted to explaining the behaviour of the forecast error in various financial markets. Recognizing the need to incorporate distributional properties, recent research has concentrated on time-varying finite volatility models (e.g., McCurdy and Morgan 1988). In practice, this involves making unrealistic stationarity assumptions about the higher moments. This study uses a different approach to deal with the

16 However, despite the considerable theoretical motivation, to date little attention has been given to the trading mechanics that support the unbiased prediction hypothesis. From a trading perspective, the underlying strategies are naïve. Violation of (4) induces a long WI trade when WI prices are greater (WI rates lower) than expected tbill auction prices. A short WI trade is initiated when WI prices are lower (WI rates higher) than expected auction prices. Given the risks of these 'naked position' strategies relative to other available strategies (e.g., Yano 1989), there would have to be significant information-induced discrepancies to generate sizeable trading activity. At best, such events would be discrete.

17 To see this, observe that $P = (1/(1+r))$ can be expanded in a geometric series.

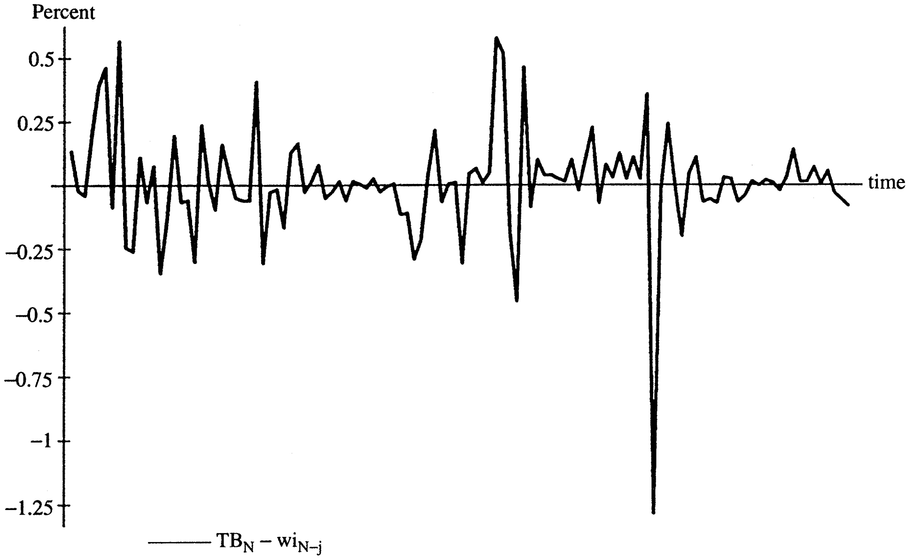

FIGURE 4 Three-month when-issued forecast error, Mondays, January 1986 – April 1988

distributional issue: censoring the sample points that indicate 'discontinuity in the information set' between the forecast and auction dates.

Given the distributional qualification, it is possible to formulate a regression specification for the unbiased prediction hypothesis. Under the null hypothesis that the unbiased prediction hypothesis is correct, then the restrictions of $a_0 = 0$ and $a_1 = 1$ can be tested in the following regression equation:

$$\text{TB}_{Nt} - \text{TB}_{jt} = a_0 + a_1(\text{WI}_{jt} - \text{TB}_{jt}) + e_{jt}, \tag{7}$$

where TB_{Nt} is the actual tbill rate observed at the following Thursday auction, WI_{it} is the WI rate at time t ($j = 1$ for Friday, = 2 for Monday, = 3 for Tuesday, and = 4 for Wednesday), and e is the error term. By differencing the cash tbill rate from both sides of (6), the WI forecast is directly compared with the forecast imbedded in the current cash tbill rate. However, because (7) involves variables observed at different points in time, the e_{jt} can become non-stationary when 'significant' amounts of information arrive in the period between the forecast date and the auction date. As a result, the critical regions of the relevant hypothesis tests must be appropriately adjusted.

With this in mind, the regression results for the three-month WI are given in table 4 and the six-month results in table 5. Three essential variations are considered including using both censored and uncensored samples. Censoring events typically occur with events like the October 1987 market break and the January–March 1986 exchange crisis. In addition to the considering the censoring of special events,

TABLE 3*

Some distributional properties of the WI forecast error: $\text{TB}_N - \text{WI}_{N-j}$

Sample: Weekly, January 1986 – April 1988

Variable	Mean	St. Dev.	Min.	Max.	SUM
DIF6F	0.023	0.291	−1.61	1.31	2.61
DIF3F	0.015	0.252	−1.48	1.07	1.73
DIF6M	0.025	0.254	−1.39	0.72	2.80
DIF3M	0.010	0.211	−1.29	0.58	1.16
DIF6T	0.009	0.159	−0.51	0.39	1.07
DIF3T	0.006	0.143	−0.54	0.46	0.69
DIF6W	0.026	0.131	−0.50	0.59	3.01
DIF3W	0.011	0.098	−0.25	0.47	1.28

	Skewness	Kurtosis	Chi	SR	AR1
DIF6F	−0.80**	13.10**	491.9**	10.0**	0.07
DIF3F	−1.13**	15.15**	718.8**	10.1**	0.09
DIF6M	−1.17**	10.67**	305.2**	8.3**	−0.32
DIF3M	−1.51**	14.74**	698.5**	8.8**	−0.09
DIF6T	−0.49**	4.39**	14.6**	5.6**	−0.01
DIF3T	0.15	5.32**	27.6**	6.9**	0.05
DIF6W	−0.19	7.35**	92.6**	8.3**	−0.22
DIF3W	0.95**	7.71**	124.8**	7.3**	−0.07

* DIF*mn* = $\text{TB}_N - \text{WI}_{N-j}$ where *m* is the maturity of the bills and *n* is the day of the week. Measures for skewness and kurtosis are the standardized sample moments. Kurtosis is not centred. Chi is a chi-squared test which combines the information in both skewness and kurtosis. SR is the studentized range. These tests are further described in Poitras (1988b). AR1 is the value of the first autocorrelation coefficient.

** Indicates that the estimated value for the distribution was significantly different from the normal at the 10% level.

tables 4 and 5 also include both OLS and seemingly unrelated regression (SUR) estimates, computed using the three- and six-month equations for a given day. Finally, to facilitate hypothesis tests on the regression coefficients, White's robust standard errors are reported in order to account for the potential non-normality of the estimated residuals. The studentized range of the regression residuals is used to test for non-normality. The corrected standard errors are used in the Wald test of the joint hypothesis that $a_0 = 0$ and $a_1 = 1$.

Examining tables 4 and 5, the most striking feature revealed is the significant contrast between the three- and six-month results. While in many cases it is not possible to reject the hypothesis that the estimated coefficients for the three-month WI case are equal to the null values ($a_0 = 0$ and $a_1 = 1$), this was not the case for the six-month WI results. A possible explanation for this discrepancy lies in the greater amount of variability in the three-month $\text{WI}_t - \text{TB}_t$ rate spread than in the six-month spread. In general, six-month WI rates tend to trade closer to the cash

TABLE 4[a]

Selected regression results for the unbiased prediction hypothesis: three-month results

Sample: weekly, January 1986 – April 1988, uncensored

OLS estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	SR	Wald
Friday	0.016 (0.02)	1.02 (0.45)	0.07	0.253	2.11	9.93*	0.43 (0.81)
Monday	0.009 (0.02)	0.91 (0.37)	0.08	0.212	2.15	8.80*	0.31 (0.86)
Tuesday	0.006 (0.01)	0.99 (0.30)	0.16	0.143	1.85	6.94*	0.19 (0.91)
Wednesday	0.008 (0.01)	0.69 (0.16)	0.15	0.097	2.06	6.71*	2.52 (0.28)

Sample: weekly, January 1986 – April 1988, twenty-two observations censored

OLS estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	SR	Wald
Friday	0.030 (0.016)	1.24 (0.38)	0.09	0.152	1.32	6.30*	3.84 (0.15)
Monday	0.009 (0.015)	1.13 (0.42)	0.10	0.146	1.99	6.36*	0.48 (0.79)
Tuesday	−0.008 (0.012)	1.30 (0.36)	0.13	0.125	1.39	7.11*	1.51 (0.47)
Wednesday	0.011 (0.007)	0.60 (0.17)	0.12	0.072	1.66	6.63*	9.74 (0.01)

Sample: weekly, January 1986 – April 1988, uncensored

SUR[b] estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	Wald
Friday	0.02 (0.02)	1.33 (0.50)	0.07	0.252	1.85	0.93 (0.63)
Monday	0.01 (0.02)	1.21 (0.38)	0.08	0.211	2.21	0.65 (0.73)
Tuesday	0.01 (0.01)	1.00 (0.30)	0.16	0.142	1.86	0.20 (0.91)
Wednesday	0.01 (0.01)	0.99 (0.30)	0.15	0.097	2.14	1.52 (0.47)

TABLE 4 (*concluded*)

Sample: weekly, January 1986 – April 1988, twenty-two observations censored

SUR[b] estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	Wald
Friday	0.03 (0.02)	1.37 (0.39)	0.09	0.150	1.33	4.19 (0.12)
Monday	0.01 (0.02)	1.38 (0.43)	0.10	0.145	2.05	1.21 (0.55)
Tuesday	0.01 (0.01)	1.22 (0.37)	0.13	0.123	1.38	1.09 (0.58)
Wednesday	0.01 (0.01)	0.89 (0.19)	0.12	0.072	1.66	3.54 (0.17)

a SEE is the standard error of the equation, DW is the Durbin-Watson statistic, SR is the studentized range and Wald is the Wald test for the joint hypothesis $a_0 = 0$ and $a_1 = 1$. The value for the Wald is the chi-squared value with two degrees of freedom. The associated probability is given in brackets. White's heteroscedastic-adjusted standard errors are given in brackets beneath the regression coefficients. For the SR, * indicates significantly different from the value for the normal distribution at the 10 per cent level.

b SUR estimates are based on combining the three- and six-month equations. See table 5 for the related six-month results.

TABLE 5[a]

Selected regression results for the unbiased prediction hypothesis: six-month results

Sample: weekly, January 1986 – April 1988, uncensored

OLS estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	SR	Wald
Friday	0.014 (0.03)	0.31 (0.72)	0.02	0.291	2.11	9.93*	1.85 (0.40)
Monday	0.005 (0.03)	−0.01 (0.47)	0.00	0.251	2.51	7.96*	6.18 (0.05)
Tuesday	0.007 (0.01)	0.47 (0.06)	0.07	0.152	2.09	5.83*	78.0 (0.06)
Wednesday	0.015 (0.01)	0.44 (0.27)	0.15	0.097	2.06	7.50*	9.07 (0.01)

Sample: weekly, January 1986 – April 1988, twenty-two observations censored

OLS estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	SR	Wald
Friday	0.039 (0.018)	1.10 (0.30)	0.04	0.177	1.81	5.40	5.18 (0.07)
Monday	0.017 (0.02)	0.27 (0.44)	0.00	0.187	2.12	6.38*	4.48 (0.11)
Tuesday	0.002 (0.013)	0.84 (0.35)	0.03	0.138	1.74	6.01*	0.24 (0.89)
Wednesday	0.011 (0.007)	0.60 (0.17)	0.12	0.072	1.66	5.38	9.74 (0.01)

TABLE 5 (*concluded*)

Sample: weekly, January 1986 – April 1988, uncensored

SUR estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	Wald
Friday	0.02 (0.03)	0.56 (0.71)	0.00	0.289	2.12	1.27 (0.53)
Monday	0.01 (0.03)	0.28 (0.47)	0.00	0.249	2.55	3.66 (0.16)
Tuesday	0.00 (0.01)	0.02 (0.33)	0.07	0.156	2.18	9.65 (0.00)
Wednesday	0.02 (0.01)	0.66 (0.24)	0.04	0.126	2.44	6.99 (0.03)

Sample: weekly, January 1986 – April 1988, twenty-two observations censored

SUR estimated equation: $TB_{Nt} - TB_{jt} = a_0 + a_1(WI_{jt} - TB_{jt}) + e_{jt}$

	a_0	a_1	R^2	SEE	DW	Wald
Friday	0.03 (0.02)	0.64 (0.33)	0.04	0.176	1.81	4.80 (0.09)
Monday	0.02 (0.02)	0.43 (0.45)	0.00	0.185	2.14	3.34 (0.19)
Tuesday	−0.00 (0.01)	0.20 (0.47)	0.03	0.138	1.67	2.98 (0.23)
Wednesday	0.03 (0.01)	0.84 (0.13)	0.19	0.094	1.72	11.3 (0.00)

NOTES: See notes to table 4. See table 4 for the related six-month results.

rates than do the three-month. For example, on Wednesday 24 August 1988 the three- and six-month WI rates were 9.72 and 10.16, while the cash rates were 9.64 and 10.16. The rates at next day's tender were 9.78 and 10.29. This discrepancy between three- and six-month results could be attributed to the larger role that the chartered banks play in the six-month tender; that is, the dealers tend to dominate the three-month tender, while the banks are more active participants in the (less liquid) six-month tender.[18]

Discrepancy between the three- and six-month results extend to the effect of censoring. Specifically, while censoring of observations led to an unambiguous reduction in the performance of the unbiased prediction hypothesis for the three-month case, there was a noticeable, albeit mixed, improvement in the six-month results. In addition, while censoring did produce specific instances of convergence

18 Regarding interesting results for specific days, the reduction in the a_1 coefficient value for the three-month Wednesday results is indirect evidence indicating that participants in Wednesday trade may have somewhat different motivations than those of participants on other trading days.

to normality for the distribution of the regression residuals, there were some instances where censoring was ineffective. On balance, the evidence provides little support for the use of censoring as a method of handling residual non-normality. A similar conclusion applies to the use of SUR versus OLS. While there was a general improvement in the six-month results, with the important exception of Wednesdays, there was a deterioration in the three-month case. However, despite the improvement, SUR estimates continued to indicate that six-month WIS consistently underestimate the auction rate. While SUR improved the three-month Wednesday results, there was deterioration in performance for other days.

IV. SUMMARY

This study examined the market for when-issued contracts on Government of Canada tbills. Arbitrage relationships underlying WI interest rates were derived, and it was demonstrated that the arbitrage boundaries have been violated on a number of occasions. Even though most of these violations were attributed to restrictions on the precise calculation of the arbitrage differentials, there was some limited evidence that the WI market is underarbitraged. Regression evidence was also presented regarding whether the WI rate is an 'unbiased predictor' of the future auction rate. This evidence indicated that, allowing for the information arrival associated with special events, the three-month WI rate typically appears to be an unbiased predictor of the tbill rate at the following auction. However, the same cannot be said about the six-month WI.[19]

APPENDIX: DERIVATION OF YIELD VERSION OF CASH AND CARRY

Assume that the tbill purchase can be financed at the term call (or term repo) rate, that the par values of the tbill and WI have been normalized to 1 and that the deliverable tbill has 91 days; that is, only the three-month WI case is being considered. Given this, then $P(0) = 1/(1 + (i(91 + N)/365))$ and $\text{WI}(0) = 1/(1 + (i^*(91/365))$ where i is the tbill rate and i^* is the WI rate at time 0. The cost of carrying the tbill from time 0 to N at the term repo rate is $P(0)R/(N/365) \equiv P(0)R(0,N)$, where R is the annualized term call rate.

With these assumptions and ignoring transactions costs, the no arbitrage profit condition is

$$\pi(0) = \text{WI}(0) - [P(0)(1 + r(0,N))] = 0,$$

where $r(0,N) = R(0,N) - i(0,N) = (R(0) - i(0))(N/365)$. Evaluating individual terms:

$$(\text{WI}(0) - P(0)) = \frac{((i^* - i)(91/365)) + (i(N/365))}{(1 + (i(91 + N)/365))(1 + (i^*(91/365))}.$$

19 While the presentation of the results in the appendix can be derived more compactly in continuous time, the present approach is used because it provides an exact value for arbitrage profit.

Similarly, the interest expense can be derived:

$$P(0)R(0,N) = (R(N/365)(1/(1 + (i(91 + N)/365)).^{20}$$

After manipulating the two expressions, evaluation of the resulting *numerator* gives[20]

$$i^* - i = i(N/91) - (R(N/91)(1 + (i^*(91/365))).$$

(3) in the text follows from ignoring the second-order term $Ri^*(N/365)$.

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20 For equality to hold, the arbitrage must be 'two-way'; that is, both the short and long cash and carry trades must be unrestricted. The example above describes the long cash and carry trade, where the cash commodity is held long. In the short cash and carry, the cash commodity is shorted at $t = 0$ and the position is covered by a purchase of WIS.

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