Alternative Derivation of the Time Value in Continuous Time

Let $N_t = A_t - L_t$. Following Shiu (1990), let the surplus function be defined:

$$S(\delta) = \sum_{t \geq 0} N_t \exp\{ -\int_0^t \delta(s)\, ds \}$$

where $\delta(s)$ is the force of interest, also referred to as the instantaneous forward rate function. Observing that N_t is fixed by the specification of the portfolio components, taking the time derivative involves terms such as:

$$\frac{d}{dt} \exp\{ -\int_0^t \delta(s)\, ds)$$

This derivative can be solved as:

$$X = \exp\{ -\int_0^t \delta(s)\, ds) \rightarrow \ln X = -\int_0^t \delta(s)\, ds$$

$$\frac{d \ln X}{dt} = -\delta(t) \rightarrow d \ln X = \frac{1}{X} dX \rightarrow -\frac{1}{X}\frac{dX}{dt} = \delta(t)$$

With this result, evaluating the derivative gives the desired result
:

$$\frac{dS(\delta)}{dt} = \sum_{t \geq 0} N_t \exp\{ -\int_0^t \delta(s)\, ds \} \delta(t)$$

Upon dividing by S, the result given in the paper is the discrete time version of this solution.
To show that the time value is equal to the force of interest in Redington's model observe that when there is only one yield the surplus function has the form:

$$S = \sum_{t \geq 0} N_t e^{-\delta t} \rightarrow \frac{dS}{dt} = -\delta \sum_{t \geq 0} N_t e^{-\delta t} \rightarrow -\frac{1}{S}\frac{dS}{dt} = \delta$$