# Hedging and Crop Insurance

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Traditional treatments of the optimal hedging problem, e.g., Rolfo (1980), Cecchetti et. al. (1988), Myers (1991), ignore the possibility that the optimizing trader can use hedging instruments other than futures. This distinction is particularly important in cases where the precise amount to be hedged is uncertain when the hedge is initiated. For example, this can happen to farmers who must determine the quantity to be hedged at planting time based on the (uncertain) potential crop at harvest time.[1] Hence, the farmer must hedge against both price and quantity uncertainty. To do this, farmers have a number of practical alternatives. In addition to being able to hedge with derivative securities such as futures contracts, farmers have access to (possibly subsidized) crop insurance plans. The primary objective of this article is to provide solutions to various specifications of the farmer's optimal hedging problem when both futures and crop insurance are available to hedge sources of uncertainty. The following section provides an overview of previous approaches to the farmer's hedging problem. A brief discussion of crop insurance is provided also. The next section examines the conventional specification and solution of the farmer's optimal hedging problems without crop insurance; the third section extends this model to allow for asymmetries in the distribution for terminal wealth. Starting from these initial results, the fourth section introduces crop insurance into the problem specification. This involves reformulating the terminal wealth function associated with the underlying optimization. The last section evaluates the impact of crop insurance on farmer hedging behavior. Unlike previous treatments of the optimal hedging problem, the asymmetric nature of the terminal wealth distribution is introduced into the specification of the expected utility function. Among other things, it is demonstrated that with both price and quantity uncertainty, futures hedging activity depends fundamentally on the type of crop insurance provided.

## BACKGROUND

Despite the obvious practical connection, analysis of the farmer's hedging problem has traditionally omitted the possibility of using crop insurance.[2] This practical omission is

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[1]The potential crop is subject to the uncertainties of weather, pests, and plant disease. These problems are not applicable to situations where the production functions are more controllable, e.g., feeder cattle.

[2]Turvey (1989) and Turvey and Baker (1990) demonstrate that higher farm debt/equity ratios will increase the (optimal) use of derivatives. Hence, subsidized government farm loan programs and other initiatives which (indirectly) increase the farmers ability to borrow, e.g., marketing boards, may also impact the optimal risk management decision.

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often compounded by assuming the size of the position to be hedged is nonstochastic, i.e., the only source of uncertainty is price variability [e.g., Danthine (1978); Lapan et al. (1991)]. Even in cases where both price and output uncertainty are admitted [e.g., Rolfo (1980); Grant (1985); Karp (1987)], optimal hedge solutions are limited in a number of ways. For example, there is often an implied market incompleteness: while two sources of risk are present, only one derivative security is provided to hedge the risk. In this vein, Myers (1988) demonstrates that market completeness affects the real supply response of farmers, i.e., with two sources of uncertainty, both futures contracts and crop insurance are required to complete the market. By implication, adding crop insurance to the hedging problem results in market completeness which changes the associated hedging optimality conditions.

In addition to market completeness, various practical and analytical issues must be addressed before specifying a solution of the farmer's risk management problem. An analytical issue arises, for example, in determining whether the amount of crop to plant is cast as a choice variable or set exogenously. If it is cast as a choice variable, the extent to which production and hedging decisions are separable becomes important. While in certain cases, uncertainty only affects the level of hedging and not the production decision [e.g., Danthine (1978)], this result does not hold in general, [e.g., Paroush and Wolf (1989); Lapan et al. (1991)]. Another analytical concern involves the conditions under which the optimal hedge ratio is independent of risk preferences. While independence can be established with a properly specified source of uncertainty and unbiased expectations [e.g., Heaney and Poitras (1991)], it is not known whether this result extends to, say, the case where two sources of uncertainty are present. The complexity of the problem defies a general solution.

Given that it is necessary to abstract from some features of the farmer's optimal risk management decision, Wolf (1987) uses a mean-variance approach to consider the optimal hedging problem using options, ignoring the production decision. Lapan et al. (1991) maintain that, because options truncate the probability distribution for prices (violating normality and monotonicity for profit or wealth), mean-variance will not necessarily provide an accurate representation of expected utility. Using a general expected utility function, Lapan et al. go on to model the farmer's production decision allowing both futures and put options to hedge a single source of diversifiable uncertainty, i.e., price risk. Assuming both that options are fairly priced and that futures prices are unbiased and linearly related to cash prices, Lapan et al. demonstrate that when both options and futures are available to hedge price risk, only futures will be used to hedge. In effect, futures make options a redundant hedging instrument.[3]

In the present context, the Lapan et al. result is relevant because put options represent one potential form of crop insurance. In general, three kinds of insurance schemes are possible:

A. *Quantity Insurance:* where the physical yield is insured from falling below some minimum amount, usually set as some percentage of historical yields. This case is consistent with many traditional crop insurance plans.
B. *Price Insurance:* where the crop delivery price is insured from falling below a minimum amount. This type of insurance can be accomplished using exchange-traded put options.

[3]In using a linear cash-futures price relationship, Lapan et al. are subject to much the same criticisms that they direct at Wolf (1987).

C. *Mixed or Revenue Insurance:* where the total revenue is insured from falling below a minimum amount; this case is consistent with farm income stabilization and, to a lesser extent, disaster relief programs.[4]

In practice, federally administered crop insurance in the U.S. is quantity-based (A) insurance, but disaster relief programs are available also. In Canada, all three types of insurance are offered as alternatives under recently introduced farm income protection legislation. Various schemes are offered in other countries [e.g., Hazell et al. (1986)].

## THE BASIC MODEL

The basic model is discrete. Farmers have access to a variety of possible risk management instruments to hedge production decisions. The representative farmer plants a crop at time $t$ and harvests it at time $t + 1$. Both the price and quantity at harvest are unknown at time $t$. In addition to choosing the hedging instrument(s), the farmer's optimization problem involves determining the amount of initial wealth to invest in crop production. Therefore, the production decision is treated in a portfolio context. The costs associated with planting the given acreage must be determined also. Starting from a given initial level of wealth, the farmer's objective is to maximize the expected utility of terminal wealth assuming that the balance (possibly negative) of initial wealth which is not allocated to planting costs will earn (pay) the riskfree rate of interest.

Given this basic structure, it is assumed initially that the only hedging instrument available is futures contracts. In this case, the underlying wealth dynamics can be specified:

$$W_{t+1} = AY_{t+1}P_{t+1} + [W_t - C(A)](1 + r) + Q_f(f_{t+1} - f_t) \tag{1}$$

where: $W_{t+1}$ is wealth at time $t + 1$ and $W_t$ is the known level of initial wealth; $A$ is the number of acres planted; $Y_{t+1}$ is the random yield per acre observed when the crop is harvested at $t + 1$; $P_{t+1}$ is the random spot price at $t + 1$; $C(A)$ is the known functional relationship between cost and the planting of $A$ acres; $r$ is the riskfree interest rate; $Q_f$ is the quantity of futures contracts sold (−) or bought (+); and $f_{t+1}$ and $f_t$ are the futures prices observed at $t + 1$ and $t$, respectively. Manipulation of (1) gives:

$$\begin{aligned} W_{t+1} &= W_t(x(1 + R) + (1 - x)(1 + r) + HR_f) \\ &= W_t((1 + r) + x(R - r) + HR_f) \\ &= W_t + \pi_{t+1} \end{aligned} \tag{2}$$

where: $\pi_{t+1}$ is the profit, or change in wealth, implied by (1) realized at time $t + 1$; $x$ is $(C(A)/W_t)$ the fraction of initial wealth invested in the crop production; $H$ is the value ($f_t$ times $Q_f$) of the hedge position divided by initial wealth *(not the value of the spot position)*; $R_f$ is $(f_{t+1} - f_t)/f_t$; and, $(1 + R)$ is $[(AY_{t+1}P_{t+1})/C(A)]$, one plus the rate of return on planting.

Given this, the traditional farmer's optimal risk management decision problem is to choose $x$ and $H$ such that the expected utility of terminal wealth is maximized. This decision problem can be modeled with a general expected utility function. To achieve analytically concise results using this approach, joint normality of $R$ and $R_f$ is invoked. This leads to the following:

[4]Total revenue is the realized income from planting a given crop. Given that various types of farm income stabilization programs are possible, e.g., where payments are made prior to planting on the condition that farmers do not plant certain crops, the revenue insurance schemes being examined here are not fully descriptive of all possible plans.

**Proposition I: The Crop Investment and Hedging Decision**

Assuming that the returns $R$ and $R_f$ are jointly normal random variables, and that the farmer chooses $x$ and $H$ so as to maximize the expected utility of terminal wealth given by (2) then:

$$H^* = \left\{-\frac{E[U']}{(E[U'']W_t)(1-\rho^2)}\right\}\left\{\frac{E[R_f]}{\sigma_f^2} - \rho\frac{E[R]-r}{\sigma_R\sigma_f}\right\} \quad (3)$$

$$x^* = \left\{-\frac{E[U']}{(E[U'']W_t)(1-\rho^2)}\right\}\left\{\frac{E[R]-r}{\sigma_R^2} - \rho\frac{E[R_f]}{\sigma_R\sigma_f}\right\} \quad (4)$$

where:

$$\rho = \frac{\sigma_{Rf}}{\sigma_R\sigma_f}$$

$$\sigma_{Rf} = Cov(R, R_f), \quad \sigma_f^2 = Var(R_f)$$

$$\sigma_R^2 = Var(R)$$

and $U$ is the farmer's utility function for wealth ($U' > 0, U'' < 0$). Significantly, while Proposition I reveals that the individual optimal solutions (denoted by *) to the farmer's risk management problem ($x^*, H^*$) depend on risk preferences, the ratio ($H^*/x^*$) *only involves statistical parameters*, i.e., the $E[U']$ and $E[U'']$ terms cancel out and only expected values, second moments, and correlations remain.

When used to analyze ($H^*/x^*$), a practical implication of Proposition I is that the fraction of the investment in crop production to be hedged ($Q_f f_t/C(A)$) is independent of the size of the crop. While this result and other implications of (3) and (4) may appear similar to traditional optimal hedging solutions, because $R$ involves the product of two random variables ($Y$ and $P$) decidedly unconventional solutions can emerge. For example, if $Y$ and $P$ exhibit sufficiently negative correlation, then $\sigma_{Rf}$ can be negative resulting in $H^* > 0$ or a long hedge position. More importantly for the case at hand, Proposition I depends fundamentally on the assumption of joint normality of $R$ and $R_f$. In the present context, this is a questionable assumption. To model the impact of introducing crop insurance poses even greater problems because, in many cases, crop insurance is for protection against the significant probability of crop disaster or price collapse. These cases imply negative asymmetry in the $R$ distribution.

## INTRODUCING SKEWNESS INTO THE PROBLEM

Given the results for the farmer's hedging problem without crop insurance, it remains to derive the associated optimality conditions when the potentially unrealistic assumption of joint normality is dropped. In the present context this is essential as the random variables induced by the introduction of crop insurance are likely to have even more substantial deviations from the normality assumption than when hedging alone is used for risk management. Analytically this creates considerable difficulties because eliminating the assumption of joint normality undermines the applicability of mean-variance-based solutions. In effect, the optimization problem requires a framework which does not suppress consideration of essential features associated with the distributional shape of the relevant random variables. For this purpose, it is necessary to work with a functional form for expected utility which can account for mean, variance, and skewness.

In practice, the value of crop insurance depends on the limitation of downside risk. In effect, to adequately model the importance of crop insurance requires direct consideration of asymmetries in the distribution of terminal wealth. While the optimal hedging literature has not directly considered the asymmetry issue, considerable work has been done on this topic in the asset pricing literature [e.g., Kraus and Litzenberger (1976); Scott and Horvath (1980); Sears and Trennepohl (1983); Lim (1989); Cotner (1991)], where the interest has centered on the impact of systematic (nondiversifiable) skewness on expected returns. However, because the farmer's hedging problem omits consideration of the effects of diversification, the portfolio-theoretic features which are the focus of the three-moment CAPM are of limited relevance. Given this, certain methodological features of the capital asset pricing approach can still be adapted to the problem at hand.

The conventional starting point for introducing moments higher than the second into the expected utility function is to take a Taylor series expansion of an admissible utility function for terminal wealth ($W$) evaluated at the expected value of terminal wealth ($\mu$):

$$U[W] = U[\mu] + U'[\mu](W - \mu) + \frac{U''[\mu]}{2!}(W - \mu)^2 + \frac{U'''[\mu]}{3!}(W - \mu)^3 + \dots$$

This expansion requires that certain technical conditions be satisfied, e.g., that the utility function be analytic (holomorphic) and convergent within the interval of interest [e.g., Kraus and Litzenberger (1976); Rudin (1987, Ch. 10)]. With some qualifications, the utility functions conventionally used in finance (e.g., the HARA class) are admissible.[5] Desirable properties for utility functions provide further restrictions on the coefficients in the expansion: $U' > 0$, nonsatiation; $U'' < 0$, risk aversion; and, $U''' > 0$, preference for positive skewness.

Transforming the Taylor series representation into an applicable expected utility function involves taking expectations and ignoring terms associated with moments higher than the third. This produces:

$$\begin{aligned} EU[W_{t+1}] &= U[\mu] + 0 + \frac{U''[\mu]}{2}\, var[W_{t+1}] + \frac{U'''[\mu]}{3!}\, skew[W_{t+1}] \\ &= U[E\{W_{t+1}\}] - b\; var[W_{t+1}] + c\; skew[W_{t+1}] \end{aligned} \tag{5}$$

where: skew[·] is the third moment about the mean for terminal wealth. In this form, the properties for utility functions allow the last two coefficients in the expected utility function to have positive signs, i.e., $b, c > 0$. Ignoring moments higher than the third can be rationalized by observing that information contained in those terms is proxied by the terms which are included [e.g., Scott and Horvath (1980)].[6] In addition, while it is possible to make the analytically attractive assumption that $U[E\{W_{t+1}\}] = E\{W_{t+1}\}$, this is basically equivalent to assuming cubic utility which has a number of unattractive practical implications [e.g., Levy (1969)].

Consideration of the general form which optimal solutions associated with (5) using (2) can take indicates a significant increase in the complexity of the solutions. It is not possible to solve closed forms for $H^*$ and $x^*$, i.e., solely in terms of statistical and preference parameters.[7] However, it is possible to solve the terms associated with mean-variance component leaving an additional term which is associated with the effect of skewness. Further simplification can be achieved by assuming that futures prices

[5]Loistl (1976) describes the types of technical issues that can arise. In particular, problems can arise due to restrictions on the limits of convergence, which can have a significant impact on the commonly used power and logarithmic utility functions. In addition, when expectations are taken over the expansion, further requirements have to be imposed on the behavior and existence of the moments of the distribution for terminal wealth.

[6]Levy (1969) questions this assumption. However, given the significant complications introduced by extending the analysis to consider the third moment, evaluating the fourth and higher moments makes the problem intractable unless other forms of simplifying assumptions are made.

[7]Attempting to reduce complexity by assuming specific distributions and utility functions also leads to intractability as indicated by Rendleman (1981) where a similar problem is considered.

are unbiased predictors, i.e., $E(R_f) = 0$. This leads to the following extension of Proposition I:[8]

**Proposition II: The Crop Investment and Hedging Decision**

Assuming that futures prices are unbiased predictors and the farmer chooses $x$ and $H$ so as to maximize the expected utility of terminal wealth given by (5) using (2) then:

$$x^* = \frac{1}{2b(1-\rho^2)}\left\{\frac{U'E(R-r)}{W_t\sigma_R^2} + 3W_t c\left[\frac{cosk_{\sigma,R}}{\sigma_R^2} - \rho\frac{cosk_{\sigma,f}}{\sigma_f\sigma_R}\right]\right\}$$

$$H^* = \frac{1}{2b(1-\rho^2)}\left\{-\rho\frac{U'E(R-r)}{W_t\sigma_R\sigma_f} + 3W_t c\left[\frac{cosk_{\sigma,f}}{\sigma_f^2} - \rho\frac{cosk_{\sigma,R}}{\sigma_f\sigma_R}\right]\right\}$$

While the mean-variance parts of the $H^*$ and $x^*$ in Proposition II are basically the same as in Proposition I, the substantive difference is in the cosk or co-skewness related terms which are defined as:

$$\begin{aligned} cosk_{\sigma,R} &\equiv E\{\{x^2(R - E[R])^2 + H^2(R_f - E[R_f])^2 \\ &\quad + 2Hx(R - E[R])(R_f - E[R_f])\}(R - E[R])\} \\ &= x^2\, skew[R] + H^2\sigma_{f^2R} + 2Hx\sigma_{R^2f} \\ cosk_{\sigma,f} &\equiv E\{\{x^2(R - E[R])^2 + H^2(R_f - E[R_f])^2 \\ &\quad + 2Hx(R - E[R])(R_f - E[R_f])\}(R_f - E[R_f])\} \\ &= H^2\, skew[R_f] + x^2\sigma_{R^2f} + 2Hx\sigma_{f^2R} \end{aligned}$$

Evaluation of these expressions (specified ignoring the assumption that $E[R_f] = 0$) depends on the properties of the $R_f$ and $R$ distributions.[9]

While a number of distributional specifications are possible, the problem of crop insurance is involved with the limitation of downside risk. Hence, for expositionary purposes, $R$ can be assumed to be negatively skewed while $R_f$ can be taken as symmetric. This leaves two scenarios to consider where $H$ is nonzero: (a) $\sigma_{Rf} < 0$; and (b) $\sigma_{Rf} > 0$. Consistent with the negative asymmetry in $R$, in both cases $\sigma_{f^2R} < 0$ while $\sigma_{R^2f}$ will be $> 0$ in case (a) and $< 0$ in case (b). The size of these parameters will depend on the strength of the correlation between $R$ and $f$, e.g., as the $R, f$ correlation tends to zero, so will the relevant cross product, co-skewness parameter values. This is consistent with the behavior of $H^*$ which also goes to zero in this case. Given this, the relative sizes of $\sigma_R$ and $\sigma_f$ must be considered also. In opposition to Proposition I where only $\sigma_R$ affected $(H^*/x^*)$ when $E[R_f] = 0$, $\sigma_f$ can now affect the optimal hedge ratio.

Further analysis of Proposition II reveals the cross-product terms in cosk[·] are of different sign, i.e., tending to offset. For example, in the term:

$$H^2\sigma_{f^2R} + 2Hx\sigma_{R^2f}$$

the first part will be negative when $R$ is negatively skewed. The second part has $\sigma_{R^2f} > 0$ (or $< 0$) when, due to the assumption about the correlation between $R$ and $R_f$, $H$ will

[8]While the statistical parameter definitions carry over from Proposition I, the definition of $U'$ is now $\partial U/\partial E[\circ]$.

[9]Combining the distributional information with a preference assumption, exact solutions for $H^*$ and $x^*$ can be achieved by iteration of the Proposition II conditions starting from the mean-variance result. In other words, solve the mean-variance part; substitute the resulting $H^*$, $x^*$ into the formula with skewness to get a second step solution; substitute the second step solution and resolve; proceed with the iterative substitution process until the changes in $H^*$ and $x^*$ from one step to the next are less than some desired criterion. A formal proof of the convergence of this algorithm is beyond the scope of this article.

almost surely be $> 0$ (or $< 0$). While the amount of offset and the ultimate sign of the skewness effect will depend on parameter values selected, when $R_f$ is symmetric there is a decided tendency for the negative skewness in $R$ to *reduce* the investment in crop production ($x^*$), with the offsetting impact on $H^*$ depending on whether a long or short futures position is indicated. If a short futures position is indicated ($H^* < 0$, $\rho > 0$) then negative skewness in $R$ will favor increasing $H^*$ because positive payoffs on the futures hedge will tend to occur when disaster states occur.

## INTRODUCING CROP INSURANCE[10]

In the absence of crop insurance, the farmer's terminal wealth function with hedging is given in (2). While the terminal wealth functions which include crop insurance follow appropriately, some motivation is useful. In particular, in the absence of crop insurance and hedging, there is a natural minimum on $R$. Either a complete crop loss where $Y_{t+1} = 0$, or a spot price of zero at time $t + 1$ corresponds to the case $(1 + R) = 0$. Significantly, unlike revenue insurance, neither yield insurance nor price insurance by itself can guarantee a higher minimum return when $(1 + R)$ equals zero. For example, price insurance guaranteeing \$K a bushel ($P_{t+1} > K$) cannot prevent a 100% crop loss due to drought, nor can quantity insurance providing for, say, $\underline{Y}$ bushels an acre ($Y_{t+1} > \underline{Y}$) prevent the future spot price falling to zero. However, both price and quantity insurance do reduce the probability of the total return attaining low values and, as a result, alter the shape of the distribution for terminal wealth. As it turns out, there are substantive differences in how price and yield insurance accomplish this result.

### Revenue Insurance

To see the formal implications of admitting insurance, it is necessary to derive the terminal wealth functions for the price, yield, and revenue crop insurance cases. For example, if in addition to hedging with futures the farmer is assumed to buy revenue insurance against the full value of the crop ($AP_{t+1}Y_{t+1}$), the terminal wealth function can be specified:

$$\begin{aligned} W^i_{t+1} &= W_t\{(1 + r) + x[\max\{\underline{R}, R\} - s - r] + HR_f\} \\ &= W_t\{(1 + r) + x[(R - r) + \max\{0, \underline{R} - R\} - s] + HR_f\} \\ &= W_t + \pi_{t+1} + W_t\{x[\max\{0, \underline{R} - R\} - s]\} \end{aligned} \tag{6}$$

where: $s$ equals $(SA)/C(A)$ with $S$ being the price (insurance premium) per acre for the revenue insurance and $\underline{R}$ is the income floor specified in the insurance plan. It can be seen from (6) that the effect of adding revenue insurance to the risk management problem depends on assumptions made about both the pricing of the insurance premium ($S$) and the requirement that the full value of the crop be insured.

In general, evaluating expected wealth from (6) reveals that revenue insurance will increase terminal wealth by $x$ times the purchase price adjusted payout on a put option written on the return $R$, with exercise price $\underline{R}$. If the insurance (put option) is priced as an actuarially sound expected indemnity, then the last term in (6) will be zero and insurance will not change expected wealth for the selected production level. All insurance does is limit downside risk. Whether this affects the production level is one of the analytical questions to be addressed. The conclusion about the effect on expected wealth does not change if the farmer is permitted to choose the fraction of acreage insured in (6) which

[10]In the following discussion, the problem of setting the deductible is ignored. To account for this, the floor set by the insurance scheme can be viewed as net of the deductible, where applicable.

leads to the terminal wealth function:

$$W^{i}_{t+1} = W_t\{(1 + r) + x[R - r] + HR_f + x\theta\{\max[0, \underline{R} - R] - s\}\} \qquad (6')$$

where $\theta$ is the fraction of the total planted acreage insured under the revenue insurance scheme.

### Yield Insurance

In many respects, the yield and revenue insurance cases are identical. As in the revenue insurance case, it is the number of acres to insure which is the decision variable:

$$\begin{aligned} W^{Y}_{t+1} &= AY_{t+1}P_{t+1} + (W_t - C(A))(1 + r) \\ &\quad + Q_f(f_{t+1} - f_t) + Q_y(P_{t+1}\max[0, \underline{Y} - Y_{t+1}] - L) \end{aligned} \qquad (7)$$

where $L$ is the price (insurance premium) per acre for the crop insurance, $Q_y$ is the number of planted acres covered by physical yield insurance, and $\underline{Y}$ is the yield floor provided by the insurance plan. In practice, $\underline{Y}$ is set based on a percentage (<100%) of relevant historical physical yield averages. While the price used would actually depend on a specific method of price election selected by the farmer, taking the price elected to be the harvest period cash price ($P_{t+1}$) is not unrealistic [e.g., FCIC (1989)].

Assuming that $Q_y = A$ in (7), i.e., all planted acres are insured, leads to the following:

$$W^{Y}_{t+1} = W_t\left\{(1 + r) + x\left[(R - r) + \max\left[0, \frac{P_{t+1}\overline{Y}A - P_{t+1}Y_{t+1}A}{C(A)}\right] - 1\right] + HR_f\right\} \qquad (8)$$

where 1 equals $(LA/C(A))$. Observing that the expression inside the max function involves the difference between two random variables illustrates the primary analytical difference between (8) and (6). This distinction depends crucially on assuming that both price and quantity are uncertain. Observing that it may be unrealistic to assume that all acreage is insured, allowing the farmer to choose the acreage insured in (8) leads to:

$$W^{Y}_{t+1} = W_t\{(1 + r) + x[R - r] + HR_f + x\lambda[\max[0, \underline{RR} - R] - 1]\} \qquad (8')$$

where $\lambda$ is the fraction of the total planted acreage insured under the physical yield crop insurance scheme and $\underline{RR} = \{P_{t+1}\underline{Y}A\}/C(A)$. As with yield insurance, fair pricing requires insurance to impact the decision problem through its effect on downside risk.

### Price Insurance

The specification of terminal wealth where price insurance is used stands in contrast with that for physical yield and revenue insurance. Specifically, instead of the number of acres to insure, the price insurance case involves introducing put options (written on the futures price). This leads to:[11]

$$\begin{aligned} W^{z}_{t+1} &= AY_{t+1}P_{t+1} + (W_t - C(A))(1 + r) \\ &\quad + Q_f(f_{t+1} - f_t) + Q_z(\max[0, K - f_{t+1}] - z) \end{aligned}$$

[11]Given the ability to replicate call positions by combining puts and futures, little is gained by introducing a term for a call option. Similarly, while a straddle position would have an interesting random variable distribution, this would add little when futures are present.

$$= W_t\left\{x(1+R) + (1-x)(1+r) + HR_f + \frac{Q_z f_t}{W_t}\left[\max\left[0, \frac{K - f_{t+1}}{f_t}\right] - \frac{z}{f_t}\right]\right\} \quad (9)$$

$$= W_t\left\{x(1+R) + (1-x)(1+r) + HR_f + \gamma\left(\max[0, -R_f] - \frac{z}{f_t}\right)\right\} \quad (9')$$

where $K$ is exercise price on the put option *which in going from (9) to (9′) is assumed to be at the money (i.e., $K = f_t$),* $z$ is the price per unit of output of the put, $Q_z$ is the number (in output units) of puts purchased, with the ratio $\gamma$ being the value of the option position divided by initial wealth.[12] As in the yield and revenue insurance cases, if the put option is fairly priced then the expected value of the last term in (9′) is zero and insurance only has relevance for maximizing the expected utility of wealth insofar as it limits downside risk. However, price insurance has a direct impact on the distribution for $R_f$ and not $R$ as in the other two cases.

## THE EFFECT OF CROP INSURANCE ON HEDGING

This section assumes that the farmer optimizes an expected utility function of the form: $EU = U[E[W_{t+1}]] - b \text{ var}[W_{t+1}] + c \text{ skew}[W_{t+1}]$, where $b$ and $c$ (both $>0$) measure the sensitivity of $EU$ to changes in the variance and skewness of terminal wealth, respectively; that all options and insurance premiums are fairly priced; that futures (not cash) prices are used to specify the price insurance option as in (9′); and, that futures prices are unbiased predictors. The following extension of Proposition II then applies:[13]

### Proposition III: Constrained Yield Insurance and Hedging

Assuming the farmer chooses $x$ and $H$ to maximize (5) using (8), i.e., where the farmer is constrained to insure all planted acreage, then:

$$x^* = \frac{1}{2b(1-\rho_{yf}^2)}\left\{\frac{U'E(R-r)}{W_t\sigma_y^2} + 3W_t c\left[\frac{cosk_{\sigma,y}}{\sigma_y^2} - \rho_{yf}\frac{cosk_{\sigma,f}}{\sigma_f\sigma_y}\right]\right\}$$

$$H^* = \frac{1}{2b(1-\rho_{yf}^2)}\left\{-\rho_{yf}\frac{U'E(R-r)}{W_t\sigma_y\sigma_f} + 3W_t c\left[\frac{cosk_{\sigma,f}}{\sigma_f^2} - \rho_{yf}\frac{cosk_{\sigma,y}}{\sigma_y\sigma_f}\right]\right\}$$

[12]It is also possible to specify the put option using cash prices. However, this significantly complicates the analysis. In addition, exchange traded options are usually written using futures prices, so the present construction is potentially more realistic.

[13]In the following, if the option is not fairly priced it will probably be underpriced, due to government subsidies. In the present context, underpriced insurance will produce an increase in $x^*$. This result is well-known and does not add substantively to the following analysis. Similarly, if either futures prices are biased predictors or price options are written on cash instead of futures prices, this introduces additional terms into the optimality conditions which only serve to qualify the results in ways which are not significantly revealing.

where: $\sigma_{yf} = \text{cov}\{R_f, \max[\underline{\text{RR}}, R]\}$, $\sigma_y^2 = \text{var}\{\max[\underline{\text{RR}}, R]\}$ and $\rho_{yf} = \sigma_{yf}/\sigma_y \sigma_f$. The definitions of the cosk[·] terms follow appropriately, observing that the subscript $y$ refers to the random variable $\max[\underline{\text{RR}} - R]$:

$$
\begin{aligned}
cosk_{\sigma,y} &\equiv E\{\{x^2(\max[\underline{\text{RR}}, R] - E[\max[\underline{\text{RR}}, R]])^2 + H^2(R_f)^2 \\
&\quad + 2Hx(\max[\underline{\text{RR}}, R] - E[\max[\underline{\text{RR}}, R]])(R_f)\} \\
&\quad \times (\max[\underline{\text{RR}}, R] - E[\max[\underline{\text{RR}}, R]])\} \\
&= x^2\, skew[y] + H^2 \sigma_{f^2 y} + 2Hx\sigma_{y^2 f} \\
cosk_{\sigma,f} &\equiv E\{\{x^2(\max[\underline{\text{RR}}, R] - E[\max[\underline{\text{RR}}, R]])^2 + H^2(R_f)^2 \\
&\quad + 2Hx(\max[\underline{\text{RR}}, R] - E[\max[\underline{\text{RR}}, R]])(R_f)\}(R_f)\} \\
&= H^2\, skew[R_f] + x^2 \sigma_{y^2 f} + 2Hx\sigma_{f^2 y}
\end{aligned}
$$

Comparing these results to those in Propositions II, using $E[R_f] = 0$, again involves the evaluation of the statistical parameters.

Under reasonable assumptions, the introduction of physical yield insurance almost certainly will increase the investment in crop production due to the smaller size of $\sigma_y^2$ versus $\sigma_R^2$. In addition, other factors, such as the positive sign for skew[$y$] favor a higher $x^*$ when all planted acreage is insured. The impact of introducing crop insurance on $H^*$ is less clearcut due to a number of potential offsetting effects. These include: a *possible* reduction in the hedge ratio due to a change in the covariance of (insured) crop and futures price returns ($\rho_{yf}^2 < \rho^2$) and an indirect increase in $H^*$ arising from the reduction in $\sigma_y$. Again, no support is provided for the necessity of a short hedge position. Given this, it is useful to consider the restriction that all acreage be insured may have significant implications for $H^*$.

With this in mind, consider the solution to the yield insurance problem, invoking the same assumptions as in Propositions II and III, where both the fraction of total acreage to insure and the amount to hedge have to be determined:

**Proposition IV: Physical Yield Insurance and Hedging**[14]

Assuming that the farmer chooses $x$, $H$, and $\lambda$ to maximize (5) using (8′), then the optimal values for the futures and physical yield insurance positions are:

$$
\lambda^* = \frac{1}{1 - \rho_{Yf}^2} \left\langle \left\{ \rho_{Yf} \frac{\sigma_{Rf}}{\sigma_Y \sigma_f} - \frac{\sigma_{YR}}{\sigma_Y^2} \right\} + \frac{3W_t c}{2bx^*} \left\{ \frac{cosk_{\sigma,Y}}{\sigma_Y^2} - \rho_{Yf} \frac{cosk_{\sigma,f}}{\sigma_f \sigma_Y} \right\} \right\rangle
$$

$$
\frac{H^*}{x^*} = \frac{1}{1 - \rho_{Yf}^2} \left\langle \left\{ \rho_{Yf} \frac{\sigma_{YR}}{\sigma_Y \sigma_f} - \frac{\sigma_{Rf}}{\sigma_{f^2}} \right\} + \frac{3W_t c}{2bx^*} \left\{ \frac{cosk_{\sigma,f}}{\sigma_f^2} - \rho_{Yf} \frac{cosk_{\sigma,Y}}{\sigma_f \sigma_Y} \right\} \right\rangle
$$

[14]In Propositions IV and V the optimal solution for $x^*$ is not provided because the solutions are not found to have a revealing representation. However, the almost certain increase in $x^*$ again appears to apply for much the same reasons as provided in the discussion of Proposition III.

where: $\sigma_{YR} = \text{cov}\{R, \max[0, \underline{RR} - R]\}$, $\sigma_{Yf} = \text{cov}\{R_f, \max[0, \underline{RR} - R]\}$, $\sigma_Y^2 = \text{var}\{\max[0, \underline{RR} - R\}$ and, observing that the subscript $Y$ refers to the random variable $\max[0, \underline{RR} - R]$:

$$\begin{aligned} cosk_{\sigma,Y} &\equiv \lambda^2 x^2\ skew[Y] + H^2 \sigma_{Yf^2} + x^2 \sigma_{YR^2} \\ &\quad + 2\{\lambda x H \sigma_{Y^2 f} + \lambda x^2 \sigma_{Y^2 R} + H x \sigma_{RYf}\} \\ cosk_{\sigma,f} &\equiv H^2\ skew[R_f] + \lambda^2 x^2 \sigma_{Y^2 f} + x^2 \sigma_{fR^2} \\ &\quad + 2\{\lambda x H \sigma_{Yf^2} + H x \sigma_{Rf^2} + \lambda x^2 \sigma_{RYf}\} \end{aligned}$$

While exact evaluation of the optimality conditions in Proposition IV requires specific distributional assumptions, the dependence of both the size and sign of $H^*$ on $\rho_{Yf}$ is apparent. In turn, this depends on the behavior of futures prices when the disaster state occurs. As for the fraction of acreage insured, the cosk[·] term almost certainly has a positive impact.

Given the results for yield insurance, little change is required to specify the optimality conditions for revenue insurance. Comparing the relevant profit functions reveals that the revenue and physical yield insurance cases differ only in how the max[·] function is specified. In physical yield insurance, the max function depends on random price behavior while in revenue insurance the direct behavior of $R$ is the determinant. It follows that the optimality conditions for the revenue insurance case will be virtually identical to those given in Propositions III and IV, allowing for the difference in how max function is specified (together with the resulting covariances and variances). Given this, insofar as yields as opposed to prices drive $R$, in the revenue insurance case $\text{cov}[\max\{0, \underline{R} - R\}, R]$ will be higher than $\text{cov}[\max\{0, \underline{RR} - R\}, R]$ with $\text{cov}[\max\{0, \underline{R} - R\}, R_f]$ lower than $\text{cov}[\max\{0, \underline{RR} - R\}, R_f]$. However, while there will be a general tendency toward higher fractions of the crop insured and lower levels of hedging with revenue insurance, specific outcomes will depend on the type of situation under consideration.

Solving for the optimal hedge ratios in yield and revenue insurance cases poses similar analytical problems to those arising in the price insurance case; though, in this case, the primary impact is on $R_f$ not $R$. With this in mind, the following result applies:

**Proposition V: Hedging with Price Insurance**

Assuming that the farmer chooses $x$, $H$, and $\gamma$ to maximize (5) using (9′), then the optimal values for the futures and options positions are:

$$\frac{H^*}{x^*} = \frac{1}{1 - \rho_{fz}^2}\left\langle\left\{\frac{\sigma_{Rz}}{\sigma_f \sigma_z}\rho_{fz} - \frac{\sigma_{Rf}}{\sigma_f^2}\right\} + \frac{3W_t c}{2bx^*}\left\{\frac{cosk_{\sigma,f}}{\sigma_f^2} - \rho_{fz}\frac{cosk_{\sigma,z}}{\sigma_z \sigma_f}\right\}\right\rangle$$

$$\frac{\gamma^*}{x^*} = \frac{1}{1 - \rho_{fz}^2}\left\langle\left\{\frac{\sigma_{Rf}}{\sigma_f \sigma_z}\rho_{fz} - \frac{\sigma_{Rz}}{\sigma_z^2}\right\} + \frac{3W_t c}{2bx^*}\left\{\frac{cosk_{\sigma,z}}{\sigma_z^2} - \rho_{fz}\frac{cosk_{\sigma,f}}{\sigma_z \sigma_f}\right\}\right\rangle$$

where:

$$\rho_{fz} = \frac{cov\{R_f, \max[0, -R_f]\}}{\sigma_f \sigma_z} \qquad \sigma_{Rz} = cov\{R, \max[0, -R_f]\}$$

$$\sigma_z^2 = var\{\max[0, -R_f]\} \qquad \sigma_{fz} = cov\{R_f, \max[0, -R_f]\}$$

and, observing that the subscript $z$ refers to $\max[0, -R_f]$:

$$cosk_{\sigma,z} = \gamma^2\, skew[z] + H^2\sigma_{zf^2} + x^2\sigma_{zR^2} + 2\{\gamma H\sigma_{fz^2} + \gamma x\sigma_{Rz^2} + xH\sigma_{Rfz}\}$$

$$cosk_{\sigma,f} = H^2\, skew[R_f] + \gamma^2\sigma_{fz^2} + x^2\sigma_{fR^2} + 2\{\gamma H\sigma_{zf^2} + Hx\sigma_{Rf^2} + x\gamma\sigma_{Rfz}\}$$

Using Proposition V, it is now possible to analyze the relative importance of futures and options by evaluating the sign and size of the relevant parameters. Without referring to special cases, it is possible to draw some conclusions. Specifically, the negative value for $\rho_{fz}$ combined with the opposite signs for $\sigma_{Rz}$ and $\sigma_{Rf}$ implies that the introduction of put options tends to reduce the use of futures. In addition, insofar as $\sigma_{Rf}$ and $\sigma_{Rz}$ are nonzero, the Lapan et al. (1991) result that futures dominate options does not in general extend to cases involving more complicated forms of uncertainty.

## SUMMARY

This article examines the implications of admitting both crop insurance and futures hedging into the farmer's risk management problem under the condition that both prices and yields are allowed to vary. It is demonstrated that making farm income the product of two random variables significantly changes the nature of the solutions to the traditional mean-variance optimal hedging problem. Initially the farmer is assumed to have access only to futures hedging. In this case, a combination of joint normality of the rates of return to planting and the futures hedge together with unbiased futures prices is sufficient for the fraction of the crop position hedged to be independent of the scale of investment in crop production. In other words, the optimal amount of hedging can be determined on the basis of statistical parameters. This is consistent with conventional mean-variance results derived for a number of different optimization settings.

In the many practical applications, the difficulty with the mean-variance approach is that the distribution for the returns involved in the farmer's hedging problem are not likely to be jointly normal. Specifically, in the presence of negatively asymmetric returns to crop production, i.e., possible crop failures or price collapses, the mean-variance results are not likely to apply. In the no crop insurance case, when the optimization problem accounts for positive skewness preference by farmers and negatively asymmetric crop investment returns, a potential reduction of investment in crop production is indicated. While the impact on hedging activity will depend on specific distributional assumptions, when a short futures position is indicated this is likely to increase the amount of hedging activity because positive payoffs on the hedge will tend to coincide with farm income failures.

For the case where farmers have access to both crop insurance and futures hedging, optimality conditions are derived for a farmer possessing an expected utility function which incorporates concern for skewness, under the assumptions that insurance is fairly priced and that futures prices are unbiased. Again, specific distributional assumptions are required to evaluate exact implications across various types of yield and price outcomes for the optimal crop insurance and hedging positions. However, in general, the availability of crop insurance is found to have a significant impact on both the nature of the optimal futures hedging solutions and on investment in crop production. Most importantly, by permitting a change in the crop return distribution from negative to positive asymmetry, "fairly priced" crop insurance generally will increase the investment in crop production.

In addition, when put options are admitted, no analytical support is found for the general proposition that futures dominate options.

The analytical results of this article provide significant opportunities for future research. Specifically, the main results of this study indicate an absence of solvable closed forms when crop insurance is included in the farmer's mean-variance-skewness optimization problem. In addition, specific distributional assumptions are required to interpret the conditions that are provided. A useful next step would be to develop exact solutions over a range of statistical scenarios. Simulation analysis could then be used to solve for a range of hedging, crop insurance, and crop investment solutions. Among other things, such results could provide useful information to policy makers in explaining the reluctance of farmers, for certain crops, to either take out crop insurance or to hedge.

# Appendix

### PROOF OF PROPOSITION I

The farmer's problem is to maximize the expected utility of terminal wealth, $E[U[W_t + \pi_{t+1})]$. Using Eq. (2) for the terminal wealth function, this involves solving for $H$ and $x$ in the problem:

$$\max_{H,x} EU\{W_{t+1}\} = \max_{H,x} EU\{W_t[x(1 + R) + (1 - x)(1 + r) + HR_f]$$
$$= \max_{H,x} E\{U[W_t + W_t((r) + x(R - r) + HR_f)]\}$$

where $W_t$ is assumed constant. The first order conditions are:

$$H\text{: } E[U'[\cdot]R_f] = 0$$
$$x\text{: } E[U'[\cdot](R - r)] = 0$$

Using the definition of the covariance the first order conditions can be rewritten as:

$$E[U']E[R_f] + Cov[U', R_f] = 0$$
$$E[U']E[(R - r)] + Cov[U', R - r] = 0 \qquad \text{(A1)}$$

The assumption of bivariate normality permits the use of the Stein–Rubinstein Lemma [Rubinstein (1976)], i.e., if $X$ and $Y$ are bivariate Normal and $f(X)$ is a function of $X$ then under mild regularity conditions on $f$:

$$Cov[f(X), Y] = E[f'(X)]\, Cov[X, Y]$$

Applying this result to (A1):

$$E[U']E[R_f] + E[U'']W_t\, Cov[((1 + r) + x(R - r) + HR_f), R_f] = 0$$
$$E[U']E[R - r] + E[U'']W_t\, Cov[((1 + r) + x(R - r) + HR_f), R - r] = 0 \quad \text{(A2)}$$

Solving the eqs. (A2) and using the definitions given in Proposition I gives:

$$cov[U', R_f] = E[U'']W_t(x\sigma_{Rf} + H\sigma_f^2)$$
$$cov[U', (R - r)] = E[U'']W_t(x\sigma_R^2 + H\sigma_{Rf})$$

Substituting in the covariances and manipulating gives:

$$x^* = -\left\{H^* \frac{\sigma_{Rf}}{\sigma_R^2} + \frac{E[U']}{E[U'']} \frac{E[R - r]}{W_t \sigma_R^2}\right\}$$

$$H^* = -\left\{x^* \frac{\sigma_{Rf}}{\sigma_f^2} + \frac{E[U']}{E[U'']} \frac{E[R_f]}{W_t \sigma_f^2}\right\}$$

Substituting $x^*$ and $H^*$ where appropriate and using the definition for $\rho$ gives the solutions stated in the Proposition.

## PROOF OF PROPOSITION II

The objective is to maximize the expected utility of terminal wealth as specified in (5), in this case using the assumed fair pricing of options and the unbiased prediction property of futures prices. This leads to the following mean, variance, and skewness functions:

$$E[W_{t+1}] = W_t\{(1 + r) + xE[R - r]\}$$

$$var[W_{t+1}] = W_t^2\{x^2\sigma_R^2 + H^2\sigma_f^2 + 2Hx\sigma_{Rf}\}$$

$$skew[W_{t+1}] = W_t^3\{x^3\, skew[R] + 3[H^2 x\sigma_{f^2R} + Hx^2\sigma_{R^2f}] + H^3\, skew[R_f]\}$$

where the variances and covariances are as defined in the Proposition. Evaluating the first order conditions for $x$ and $H$ and solving gives the results stated in the Proposition.

## PROOF OF PROPOSITION III

As in the Proof of Proposition II, the objective is to maximize the expected utility of terminal wealth as specified in (5), in this case using the assumed "fair pricing" of options and the unbiased prediction property of futures prices. This leads to the following mean, variance, and skewness functions:

$$E[W_{t+1}^y] = W_t\{(1 + r) + xE[R - r]\}$$

$$var[W_{t+1}^y] = W_t^2\{x^2\sigma_y^2 + H^2\sigma_f^2 + 2Hx\sigma_{yf}\}$$

$$skew[W_{t+1}^y] = W_t^3\{x^3\, skew[y] + 3[H^2 x\sigma_{f^2y} + Hx^2\sigma_{y^2f}] + H^3\, skew[R_f]\}$$

where the variances and covariances are as defined in the Proposition. Evaluating the first order conditions for $x$ and $H$ and solving gives the results stated in the Proposition.

## PROOF OF PROPOSITION IV

The proof is similar to that for Proposition III. Invoking the fair pricing and unbiased expectations assumptions, using (8′) the expected value, variance and skewness of terminal wealth are evaluated as:

$$E[W_{t+1}^Y] = W_t\{(1 + r) + xE[R - r]\}$$

$$var[W_{t+1}^Y] = W_t^2\{x^2\sigma_R^2 + H^2\sigma_f^2 + \lambda^2x^2\sigma_Y^2 + 2\{Hx\sigma_{Rf} + H\lambda x\sigma_{Yf} + \lambda x^2\sigma_{YR}\}\}$$

$$skew[W_{t+1}^Y] = W_t^3\{x^3\, skew[R] + H^3\, skew[R_f] + \lambda^3x^3\, skew[Y] + 3[Hx^2\sigma_{R^2f} + H\lambda^2x^2\sigma_{Y^2f} + H^2\lambda x\sigma_{Yf^2} + H^2x\sigma_{Rf^2} + \lambda x^3\sigma_{YR^2} + \lambda^2x^3\sigma_{Y^2R}] + 6H\lambda x^2\sigma_{RYf}\}$$

Evaluating the first order conditions for $\lambda$ and $H$, the first order conditions for $H$ and $\lambda$ are derived and solved for $H^*$ in terms of $\lambda^*x^*$ and $x^*$ and for $\lambda^*x^*$ in terms of $H^*$ and $x^*$. Substituting for $\lambda^*x^*$ and $H^*$ where appropriate, the optimality conditions in terms of $x^*$ alone are derived. The solution of $\lambda^*$ follows when $\lambda^*x^*$ is expressed solely as a function of $x^*$ alone, the $x^*$ in the mean-variance part cancels out.

**PROOF OF PROPOSITION V**

Given the objective is to maximize the expected utility of terminal wealth as specified in (5) using (9′), the assumed fair pricing of options and the unbiased prediction property of futures prices leads to the following mean, variance, and skewness functions:

$$\begin{aligned}
E[W_{t+1}^z] &= W_t\{(1 + r) + xE[R - r]\} \\
var[W_{t+1}^z] &= W_t^2\{x^2\sigma_R^2 + H^2\sigma_f^2 + \gamma^2\sigma_z^2 + 2\{X\gamma\sigma_{Rz} + Hx\sigma_{Rf} + H\gamma\sigma_{fz}\}\} \\
skew[W_{t+1}^z] &= W_t^3\{x^3\ skew[R] + H^3\ skew[R_f] + \gamma^3\ skew[z] \\
&\quad + 3[Hx^2\sigma_{R^2f} + H\gamma^2\sigma_{z^2f} + H^2\gamma\sigma_{zf^2} + H^2x\sigma_{Rf^2} \\
&\quad + \gamma^2x\sigma_{z^2R} + \gamma x^2\sigma_{zR^2}] + 6H\gamma x\sigma_{Rzf}\}
\end{aligned}$$

Using these functions, deriving the first order conditions for $\gamma$ and $H$, manipulating and using the definition for $\rho_{fz}$ provided in the Proposition gives the required results.

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