On the origin and interpretation of OAS
Paul Kupiec; Adama Kah
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On the Origin and Interpretation of OAS

Paul Kupiec and Adama Kah

Paul Kupiec
is a director in the financial research department at the Federal Home Loan Mortgage Corporation in McLean, Virginia.

Adama Kah
is a senior financial analyst in the financial research department at the Federal Home Loan Mortgage Corporation.

The option-adjusted spread is a common measure in the market for mortgage-backed securities. The OAS is a by-product of the mortgage pricing process. A common approach to mortgage pricing uses a risk-neutral interest rate model in conjunction with a prepayment model to simulate potential interest rate paths and associated cash flows over the life of a mortgage security. In this approach, prepayment estimates are driven by the potential path of interest rates in a Monte Carlo simulation. As the simulated interest rate paths are consistent with a risk-neutral interest rate process, risk-neutral pricing techniques are used to value the mortgage securities' potential cash flows. The securities' model price is the expected present value of its potential cash flows in accord with a probability distribution implied by the risk-neutral interest rate process.[1]

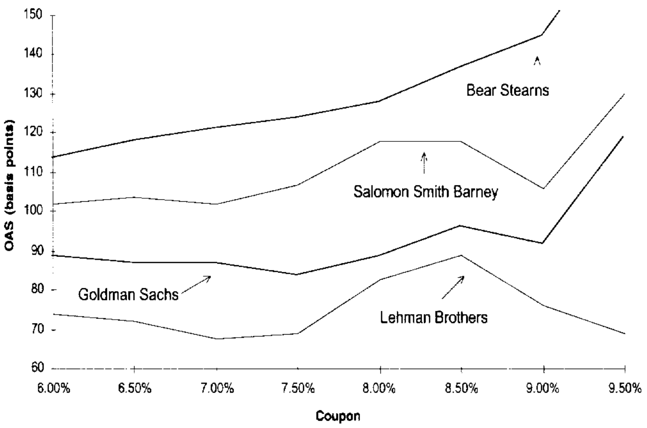
From a computational viewpoint, an OAS arises because the risk-neutral pricing approach generates model prices that differ from the observed market price quotes for mortgage securities. In the case of standard agency mortgage pass-through securities, model prices are significantly higher than market price quotes. The OAS is defined as the constant spread that, when added to all the risk-neutral spot interest rates in the Monte Carlo simulation, equates the mortgage securities' model price with the observed market price.

OAS are security-specific and specific to a set of prepayment function assumptions. For the same prepayment model, mortgage security OAS will vary over time and vary with changes in the level of interest rates. Exhibit 1 plots the OAS estimates of four broker-dealers for an identical set of TBA Freddie Mac thirty-year gold PCs on August 16, 1999.[2] While differences in interest rate models may explain some of the OAS variation, much of the observed variation is likely attributable to alternative prepayment model assumptions.

It is generally not possible to calculate the OAS on a current-coupon agency pass-through security and use that OAS to accurately price a discount or a premium pass-through security issued by the same agency. The time, the prepayment model, the interest rate environment, and the security-specific nature of the OAS all limit its usefulness in the actual pricing of mortgage securities. Indeed, given a prepayment function, OAS is more appropriately interpreted as a spread-based measure of the unexplained portion of the mortgage securities' market price.

While OAS magnitudes differ among prepayment models, the OAS on virtually all agency mortgage pass-through securities are consistently positive, substantial in magnitude, and persistent over time. In an absence-of-arbitrage pricing framework, on average (in a cross-section), securities should sell for prices very close to their model-

Exhibit 1
8/16/99 Dealer OAS



determined price. The model pricing errors that give rise to the OAS do not identify arbitrage opportunities. OAS are not distributed about zero, and positive OAS are not expected to revert to a zero OAS measure over time. Indeed, no experienced mortgage investor/trader would likely interpret an OAS as a measure of a potential arbitrage opportunity.

A common view is that positive OAS represent compensation for prepayment risk. OAS are often interpreted as the average yield spread over Treasuries that will be earned by the mortgage security investor, should the mortgage security be held to maturity.[3]

In our view, positive OAS measures on mortgage pass-through securities are the result of the omission of important prepayment factors in the risk-neutral Monte Carlo simulation process. This omission "contaminates" the prepayment cash flow simulation estimates, and thereby biases equilibrium mortgage price estimates. The positive and persistent nature of OAS is an indication that the pricing model simulation has not been calibrated correctly.

This statement may not be particularly controversial, as it is well known that economic factors other than interest rates drive mortgage prepayments, and these factors are typically omitted in the Monte Carlo mortgage pricing approach. What may be controversial is our claim that the OAS is not a measure of the expected return premium for bearing prepayment risk.[4]

A defining characteristic of the equivalent martingale or risk-neutral approach to pricing assets is that all traded assets are expected to earn the risk-free rate. If an empirical risk-neutral model is calibrated correctly, on average the traded assets used to calibrate the model should be priced correctly by the model. Assets that are not priced correctly represent riskless arbitrage opportunities. In this framework, there is no scope for attributing asset mispricing to the existence of an economic risk premium, as all economic risk premiums will have been removed under the risk-neutral measure, and all traded assets are expected to earn the risk-free rate.

As mortgage pricing models generally predict that all mortgage pass-through securities are persistently and significantly underpriced in the market (large positive OAS), there would appear to be little doubt that by the traditional standards used to judge the fit of an absence of arbitrage model, the Monte Carlo mortgage pricing models at issue are poorly calibrated.

The interpretation of OAS as a prepayment risk premium is equivalent to saying that the magnitude of the calibration errors from a pricing simulation is an accurate guide to the magnitude of investors' expectations of the equilibrium excess return for bearing prepayment risk. As an OAS is the result of model misspecification, it is difficult to imagine how it could be an accurate measure of an underlying risk premium; there are infinitely many ways to misspecify a mortgage prepayment function and presumably a unique underlying prepayment risk premium.

Moreover, any expected risk premium for prepayment risk would necessarily include compensation for the non-diversifiable risk of prepayments that are driven by interest rates as well as the non-diversifiable risk of prepayments that are driven by non-interest rate factors. As a consequence, the risk premium must include a component for the risk premium that is presumably still embedded in the simulated mortgage cash flow distribution (the components that give rise to the positive OAS) as well as an interest rate-related prepayment risk premium that will have been removed through the partial risk neutralization of the cash flows.

Viewed in these terms, it is difficult to see how OAS, a measure of the mortgage model's pricing residual, could provide an accurate indication of investor expectations for the total expected prepayment risk premium.

I. RISK-NEUTRAL PRICING AND THE OAS CALCULATION

The OAS is calculated using Monte Carlo simulation. The most common approach simulates many interest rate paths using an equivalent martingale (risk-neutral) interest rate process.[5] Under the equivalent martingale approach, a stochastic process specification for the path of interest rates is calibrated so that there are no (or minimal) arbitrage opportunities in currently observed Treasury market prices. Once a risk-neutral interest rate model is calibrated, the risk-free risk-neutralized rate process is used to generate a set or "path" of monthly interest rates for the remaining life of a mortgage.

Using an auxiliary function, a so-called prepayment function, the interest rate path is mapped into a set of anticipated payments to the mortgage security-holders that include coupon, planned amortization, and the return of principal in case of default (for agency mortgage-backed securities) and voluntary prepayments. Prepayment function parameter settings differ according to the characteristics of the mortgage pool.

Many independent interest rate paths are simulated from the risk-neutral interest rate model. Along each simulated path, mortgage security cash flows are projected, and the present value of these anticipated cash flows is calculated. The average of the path-specific cash flow present values is an estimate of the equilibrium market price of the mortgage.

More formally, let $[(r_{it}), t = 1, 2, 3, ..., 360]$ represent the set of monthly interest rates along path i simulated using the risk-neutral interest rate process. Let $(r_{it} \mid t < k) = (r_{i1}, r_{i2}, r_{i3}, ..., r_{ik})$ be the subset of interest rates along path i that consists of rates from the initial month up until month k into the life of the mortgage simulation. Let $C_{ikm} = [C([r_{it} \mid t < k]), k = 1, 2, 3, ..., 360]$ represent the anticipated cash flows for each month in the potential life of mortgage security m along interest rate path i.[7]

The present value of the cash flows on mortgage security m along interest rate path i can be represented by PV_{im}:

$$PV_{im} = \sum_{k=1}^{360} \frac{C_{ikm}}{\prod_{j=1}^{k} (1 + r_{ij})}$$

Let P_m represent the observed market price and P_m^e the equilibrium price of mortgage security m. The equivalent martingale approach to pricing requires

$$P_m^e = \Sigma_{i=1}^{N} \frac{PV_{im}}{N}$$

$P_m^e - P_m$ is defined as the model error.

In a typical non-mortgage application that models all relevant risks in the simulation, a model error that differs from zero represents an estimate of the under- or overvaluation of a security. If $P_m^e - P_m > 0$, the security is estimated to be undervalued in the market; if $P_m^e - P_m < 0$, the security is estimated to be overvalued in the market; if $P_m^e - P_m = 0$, the security is estimated to be fairly priced. In the case of mortgage-backed securities analysis, the risk-neutral pricing model error is given an interpretation that differs from the common meaning in martingale pricing applications.

It is well-known that the exercise of the mortgage prepayment option is dependent on factors other than the level of interest rates. Seasonal factors, employment levels, economic growth, trends in home prices, the initial loan-to-value ratio, and demographic variables determine, at least in part, the frequency with which homeowners exercise their contractual right to prepay or default on a mortgage.[8]

Notwithstanding the importance of non-interest rate factors, most mortgage pricing models account only for the interest rate sensitivity of prepayments. While practitioners may implicitly recognize the importance of the omitted prepayment factors in generating the risk-neutral model pricing error, it has become common to transform the risk-neutral model pricing error into a constant yield spread measure or an OAS. This OAS yield spread frequently is interpreted as "risk premium" compensation for the prepayment risk factors omitted from the pricing model simulation.

More formally, the OAS is defined as the specific value of θ such that when θ is added to every monthly interest rate, the risk-neutral model price is equal to the observed market price:

$$\theta \ni P_m = \frac{1}{N} \Sigma_{i=1}^{N} \left[\sum_{k=1}^{360} \frac{C_{ikm}}{\prod_{j=1}^{k} (1 + r_{ij} + \theta)} \right]$$

The OAS, θ, is found by iteration.

Further intuition into the OAS calculation can be acquired by expressing the risk-neutral mortgage pricing equation in a non-traditional form. It is straightforward to show that:

$$P^e_{mt} = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{k=1}^{360}\frac{C_{ikm}}{\prod_{j=1}^{k}(1+r_{ij})}\right]$$

$$= \sum_{t=1}^{360}[E^{\eta}(C_{tm})\, E^{\eta}(D_t) + Cov^{\eta}(C_{tm}, D_t)]$$

where:

$$E^{\eta}(C_{tm}) = \frac{1}{N}\sum_{i=1}^{N} C_{itm}$$

$$E^{\eta}(D_t) = \frac{1}{N}\sum_{i=1}^{N} D_{it}$$

$$D_{it} = \left[\frac{1}{\prod_{j=1}^{t}(1+r_{ij})}\right]$$

and:

$$Cov^{\eta}(C_{tm}, D_t) =$$

$$\frac{1}{N}\sum_{i=1}^{N}[C_{itm} - E^{\eta}(C_{tm})][D_{it} - E^{\eta}(D_t)]$$

where the η is used to designate that the moments are taken with respect to the risk-neutral probability measure. While it may not be immediately transparent, it can be shown that the covariance term has negligible magnitude compared to the magnitude of the product of the expectations.[9]

Define the mortgage discount function to be the collection of values, ($E^{\eta}(D_t)$, t = 1, 2, ..., 360). The OAS transformation leaves the mortgage's expected cash flows unchanged, but alters the cash flow discount function. Because the OAS affects the discount function non-linearly (i.e., OAS is not a simple mean shift), the OAS calculation does have an effect on the covariance between the values of the discount function and cash flows. As the relative magnitude of the covariance terms is negligible compared to the discount function effect, there is little cost in terms of descriptive accuracy if we ignore the effect of the OAS on the covariance terms to simplify the discussion.[10]

By adding a constant spread to all simulated short rates, the OAS calculation equates the model value estimate to the mortgage security's market value and applies a higher discount rate to all expected mortgage cash flows. Exhibit 2 illustrates the effect of a 100 basis point OAS on a typical mortgage security discount function. Note that, because the OAS is a constant across all months in the mortgage life, the OAS calculation reduces the present value of distant expected future cash flows by much more than it affects the present value of expected cash flows received more immediately.

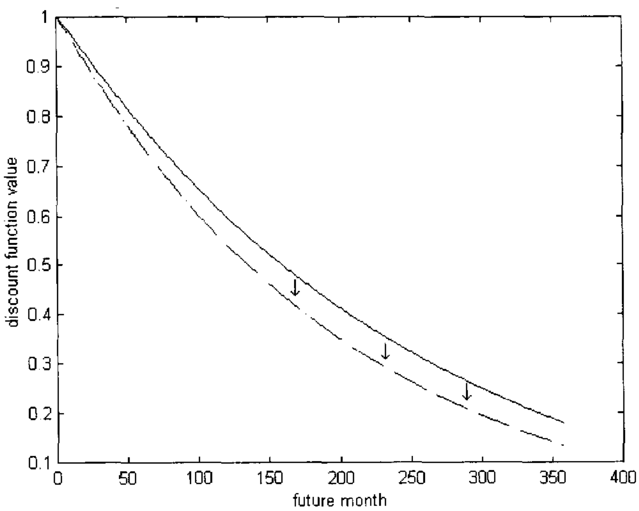
II. ORIGINS OF THE OAS

Before considering whether OAS is an estimate of an expected risk premium, it is useful to investigate the reason for the existence of an OAS. To understand how an OAS comes about, consider a simplified example of a phenomenon that ultimately leads to an OAS.

Omitted Variables

Assume there exists a function f(x, y), where x and y and z are random variables with a joint density function given by h(x, y, z). Let the marginal density functions be represented by the notational convention in which h(x, ◦, ◦) represents the marginal density function for $\tilde{x}$ and h(x, ◦, z) the marginal distribution for the ($\tilde{x}$, $\tilde{z}$) pair. Assume that we are interested in knowing the

EXHIBIT 2
Effects of 100 bp OAS on Discount Function



values: $E[f(\tilde{x}, \tilde{y})]$, and $Cov[f(\tilde{x}, \tilde{y}), \tilde{z}]$.

Instead of calculating $E[f(\tilde{x}, \tilde{y})]$ and $Cov[f(\tilde{x}, \tilde{y}), \tilde{z}]$ directly, assume that we attempt a shortcut and calculate a different, computationally easier, expectation and covariance:

$$E[f(\tilde{x}, y_0)] = \int_{\forall x} f(\tilde{x}, \tilde{y} = y_0)\, h(x, \circ, \circ)\delta x$$

$$Cov[f(\tilde{x}, y_o), \tilde{z}] =$$

$$\iint_{\forall x,z} f(\tilde{x}, \tilde{y} = y_0)[\tilde{z} - E(\tilde{z})]\, h(x, \circ, z)\delta x\, \delta z$$

Note that the shortcut expectations are not conditional expectations, just the expectation of the function evaluated at a fixed value for y taken with respect to the unconditional marginal distribution of x, and the joint marginal distributions of x and z.

Is there a transformation of the "f" function that will make, simultaneously, the shortcut expectation equal to the true expected value of interest and the simplified covariance estimate equal to the true covariance of interest? That is, is there a function, g(.), such that:

$$E[f(\tilde{x}, \tilde{y})] = E[g(f[\tilde{x}, y_0])]$$

and $$Cov[f(\tilde{x}, \tilde{y}), \tilde{z}] =$$

$$\iint_{\forall x,z} g(f[\tilde{x}, y_0])[\tilde{z} - E(\tilde{z})]h(x, \circ, z)\delta x\, \delta z$$

If such a function exists, it is possible to arrive at an accurate estimate of the value of $E[f(\tilde{x}, \tilde{y})]$ and $Cov[f(\tilde{x}, \tilde{y}), \tilde{z}]$ using the shortcut approach of eliminating the random variable $\tilde{y}$ from the calculations. A simple example will serve to make the issue more concrete.

Let $f(\tilde{x}, \tilde{y}) = a\tilde{x} + b\tilde{y}$. Note that:

$$E[f(\tilde{x}, \tilde{y})] = a\mu_x + b\mu_y$$

$$E[f(\tilde{x}, y_0)] = a\mu_x + by_0$$

$$Cov[f(\tilde{x}, \tilde{y}), \tilde{z}] = aCov(\tilde{x}, \tilde{z}) + bCov(\tilde{y}, \tilde{z})$$

$$Cov[f(\tilde{x}, y_0), \tilde{z}] = aCov(\tilde{x}, \tilde{z})$$

where $[\mu_x, \mu_y]$ denotes the means of the respective random variables.

If the required transformation function exists, the Weierstrass approximation theorem guarantees that there is a polynomial function with real coefficients that will approximate the true transformation function with an arbitrarily small amount of error.[11] In this instance, the first-order polynomial function, $g[f(\tilde{x}, y_0)] = f(\tilde{x}, y_0) + b(\mu_y - y_0)$ will ensure that $E[f(\tilde{x}, \tilde{y})] = E[g(f[\tilde{x}, y_0])]$.

It is also possible to approximate the function with any higher-order polynomial. If a second-order polynomial is selected, the transformation function:

$$g[f(x, y_0)] = f(x, y_0) + \alpha_m[f(x, y_0)]^2$$

with $$\alpha_m = \frac{b(\mu_y - y_0)}{a^2 E(\tilde{x}^2) + b^2 y_0^2 + 2aby_0 E(\tilde{x})}$$

will ensure that $E[f(\tilde{x}, \tilde{y})] = E[g(f[\tilde{x}, y_0])]$.

Unlike the equal expectations condition, the covariance equality condition requires a second-order (or higher) polynomial to represent the transformation. An example of a transformation that satisfies the covariance equality is

$$g[f(x, y_0)] = f(x, y_0) + \alpha_C[f(x, y_0)]^2$$

where $$\alpha_C = \frac{bCov(\tilde{y}, \tilde{z})}{a^2 Cov(\tilde{x}^2, \tilde{z}) + 2aby_0 Cov(\tilde{x}, \tilde{z})}$$

Notice that, unless $y_0 = \mu_y$, and $Cov(\tilde{y}, \tilde{z}) = 0$ (implying $\alpha_m = \alpha_C = 0$), different transformations are required to meet each moment-matching condition.[12] In other words, unless special conditions are satisfied, it is not generally possible in the linear case to find a single transformation $g[f(\tilde{x}, y_0)]$ that will simultaneously satisfy the conditions:

$$E[f(\tilde{x}, \tilde{y})] = E[g(f[\tilde{x}, y_0])]$$

and

$$Cov(g[f(\tilde{x}, y_0)], \tilde{z}) = Cov(f(\tilde{x}, \tilde{y}), \tilde{z})$$

While the linear example constitutes a rigorous proof that the necessary transformation function does not in general exist, it is even less likely that such a transformation can exist, even under special moment restrictions, when $f(\tilde{x}, \tilde{y})$ is a non-linear function of $\tilde{x}$ and $\tilde{y}$. While we cannot offer proof of non-existence for general non-linear functions $f(\tilde{x}, \tilde{y})$, in the appendix we consider the case when $f(\tilde{x}, \tilde{y})$ is quadratic and demonstrate the difficulties associated with establishing the existence of the "g" function. We conjecture that there are few if any non-linear functions that are used in the context of mortgage pricing that will admit the required transformation function.

What does all this have to do with OAS? Notice that, assuming that the risk-neutral interest rate model is correctly calibrated, the magnitude of the model error, $P_m^e - P_m$, depends on the accuracy of the estimates of the terms $E^{\eta}(C_{tm})$ and $Cov^{\eta}(C_{tm}, D_t)$. Assuming that market prices approximate true equilibrium prices on average across the range of traded mortgage securities, a sizable model error implies that the market's assessment of the cash flow's risk-neutral expected values and covariances differs from the model's estimates of these parameters. Considering how these parameters are estimated in practice, the usefulness of the "g" function analogy should be apparent.

Recall that it is common practice to estimate the risk-neutral expected cash flows using a Monte Carlo simulation in which prepayments are driven by interest rates alone, notwithstanding the fact that other factors are known to be important drivers of prepayments. This is akin to estimating $E[f(\tilde{x}, y_0)]$ instead of estimating $E[f(\tilde{x}, \tilde{y})]$. One may conjecture that it is possible to alter the structure of the prepayment function so that, instead of estimating the true economic specification, holding constant the value of omitted factors, it is possible to take the shortcut expected value approach using an altered prepayment function specification, and still arrive at the correct expected cash flow value.

In our simplified example, the prepayment function is represented by a linear function of two random variables ($\tilde{z}$ plays the role of the cash flow discount factor). In this example, the existence of the "g" transformation is the key to identifying whether it is possible to take the shortcut method for pricing. The results show that, in general, the "g" transformation function does not exist. There may be special prepayment function specifications for which it can be demonstrated that "g" exists, but in general, the presumption must be that "g" does not exist.[13]

This suggests that in general it is not possible to accurately estimate a mortgage security's expected cash flows and cash flow covariances under the risk-neutral measure using Monte Carlo simulation based upon interest rates alone. Thus the omission of non-interest rate factors in the Monte Carlo simulation will induce biases in the mortgage cash flow estimates that ultimately will be a source of the OAS.

Prepayment Model Error

While omitted prepayment factors are the most likely source of an OAS, any prepayment model specification that results in a predictable component in prepayment model errors will induce an OAS. Any prepayment model specification that results in a bias in the Monte Carlo estimates of the risk-neutral expected cash flows and covariance terms will result in a model price that differs from the market price, thereby creating an OAS.

The character of the prepayment model error structure is critical as to whether prepayment modeling error induces an OAS. For example, an OAS is sometimes attributed to the fact that prepayments are not perfectly predictable, and it is alleged that the inherent randomness in prepayment behavior — so-called prepayment model error — is the source of the OAS. We define prepayment model error to be the true stochastic component of prepayments that cannot be predicted, given a correctly specified structural prepayment function and complete information on the underlying economic variables that drive prepayments.

Does prepayment model error induce an OAS? To answer this question, it is useful to revisit the example with random variables $(\tilde{x}, \tilde{y}, \tilde{z})$ and a true structural prepayment function $f(\tilde{x}, \tilde{y})$. Suppose that the random variable $\tilde{x}$ represents interest rates, $\tilde{y}$ represents the true unpredictable component of mortgage cash flows, and $\tilde{z}$ represents the random cash flow discount function value $(\tilde{D}_t)$ that is a non-linear function of interest rates. Because the distribution of $\tilde{y}$ is not specified, without loss of generality we can write the structural prepayment function as $f(\tilde{x}, \tilde{y}) = f(\tilde{x}) + \tilde{y}$.

Now, if prepayment function forecasts are efficient, $\tilde{y}$, the stochastic portion of prepayments, should have a mean 0 and be uncorrelated with $(\tilde{x}, \tilde{z})$, the information set used in the forecast. Forecast efficiency implies $Cov(\tilde{y}, \tilde{z}) = 0$.

Notice that the two conditions that must be satisfied if prepayment forecasts are to be unbiased and efficient are the conditions that guarantee the existence of the "g" transformation. Our analysis demonstrates that, if $y_0 = 0$, and $\text{Cov}(\tilde{y}, \tilde{z}) = 0$, then $\alpha_m = \alpha_C = 0$, and $g[f(\tilde{x}, y_0)] = f(\tilde{x}, y_0)$. While this result is established in the context of a linear model, it is independent of the functional form through which $\tilde{x}$ affects expected prepayments.

The implication is that the mortgage's risk-neutral cash flow expectations and covariances can in this instance be estimated using the shortcut method of omitting the prepayment model error from the Monte Carlo simulation without compromising the model's accuracy. Thus, prepayment model error — a mean zero unforecastable random innovation in prepayments — is not a source of an OAS.

III. IS THE OAS A RISK PREMIUM?

In a traditional arbitrage-free pricing model application, there is no OAS. Differences between model prices and market prices signal arbitrage possibilities. One does not expect to find all securities to be simultaneously out of equilibrium, so a finding that many or perhaps even all securities are underpriced by the market relative to the model's estimated price is generally taken as strong evidence that the model is misspecified. Why, then, in mortgage pricing applications, is it common to find that all pass-through securities have positive OAS, indicating under the traditional absence of arbitrage interpretation that all mortgage securities are underpriced in the market?

The reconciliation of this counter-intuitive practice lies in the interpretation that many mortgage practitioners give to the OAS. In the practitioner world of mortgage pricing, OAS are not taken to be indications of mispricing (necessarily); instead they are interpreted as estimates of the expected risk premium the mortgage offers as compensation for bearing prepayment risk.

Our analysis shows that the omission of relevant non-interest rate factors induces an OAS, but does not address the issue of the interpretation of the OAS. Is the magnitude of a mortgage security's OAS a measure of the security's expected risk premium? The short answer is no.

The relationship between OAS and an asset's expected risk premium can be illustrated in the context of the valuation of a random cash flow at the end of a single period. Cox, Ingersoll, and Ross [1985] establish the equivalence of two alternative approaches to valuation: 1) discounting the expected end-of-period cash flow using an equilibrium risk-adjusted discount rate where the expectation is taken with respect to the physical cash flow density function; and 2) discounting the expected end-of-period cash flow using the risk-free rate, where the cash flow expectation is taken relative to the equivalent martingale measure.

In a simple setting, this equivalence can be written:

$$V = \frac{E^{\eta}(\tilde{C})}{1 + rf} = \frac{E(\tilde{C})}{1 + rf + \beta}$$

where V is the current equilibrium market value of the asset, $\tilde{C}$ is the random end-of-period cash flow, rf is the one-period spot risk-free interest rate, and β is the equilibrium risk premium associated with the cash flow's risk characteristics. Notice that the risk-neutral valuation approach reproduces the asset's market value without an OAS, assuming that the asset's expected cash flow under the risk-neutral measure is measured accurately.

Assume that the asset's expected cash flow under the risk-neutral measure is measured with error. If $\hat{E}^{\eta}(\tilde{C})$ is the risk-neutral expected cash flow estimate, assume that:

$$\hat{E}^{\eta}(\tilde{C}) = E^{\eta}(\tilde{C}) + e$$

where e is a measurement error of unknown magnitude. In terms of pricing model estimates, the risk-neutral valuation approach will require an OAS to satisfy the pricing condition:

$$V = \frac{\hat{E}^{\eta}(\tilde{C})}{1 + rf + \theta} = \frac{E(\tilde{C})}{1 + rf + \beta}$$

where θ is the OAS.

The issue of interest is the relationship between the OAS and the asset's equilibrium risk premium β. Provided that end-of-period cash flows and the risk-free rate are positive, it is simple to demonstrate that the OAS and the measurement error have the same sign. A positive OAS implies a positive cash flow measurement error, $E^{\eta}(\tilde{C}) < \hat{E}^{\eta}(\tilde{C})$; conversely, a negative OAS implies a negative cash flow measurement error, $E^{\eta}(\tilde{C}) > \hat{E}^{\eta}(\tilde{C})$. This is true regardless of the magnitude of the asset's equilibrium risk premium.

In this simple setting, it is straightforward to relate an asset's OAS to its equilibrium risk premium and its pricing model measurement error:

$$\theta = \left[\frac{[E^{\eta}(\tilde{C}) - E(\tilde{C})]}{E(\tilde{C})}(1 + rf) + \frac{e}{E(\tilde{C})}(1 + rf + \beta) \right]$$

The first term in the expression for θ is a term that is proportional to the discount or premium on the risk-neutral expected cash flows relative to the expected cash flows measured under the physical probability distribution. Note that $E^{\eta}(\tilde{C}) < E(\tilde{C}) \Leftrightarrow \beta > 0$, and $E^{\eta}(\tilde{C}) > E(\tilde{C}) \Leftrightarrow \beta < 0$. The second term in the expression for θ is a term that is proportional to the magnitude of the measurement error of the risk-neutral expected cash flow relative to the true expected cash flow under the physical distribution. The coefficient on the second term depends on the risk-free rate as well as on the asset's equilibrium expected risk premium.

If the asset has a zero risk premium in equilibrium, $\beta = 0$, $E^{\eta}(\tilde{C}) = E(\tilde{C})$, and:

$$\theta = (1 + rf)\frac{e}{E(\tilde{C})}$$

or the OAS is proportional to the relative magnitude of the measurement error, $e/[E(\tilde{C})]$. Thus a positive cash flow measurement error, $E^{\eta}(\tilde{C}) < \hat{E}^{\eta}(\tilde{C})$, will result in a positive OAS estimate even if the true risk premium is 0. Moreover, when risk premiums are non-zero, the sign of the OAS depends only on the sign of the measurement error. If measurement error is positive, the OAS will be positive even if the equilibrium risk premium is negative. Thus, not only is OAS not an accurate measure of an asset's risk premium, but the sign of the OAS is also not even an accurate estimator of the sign of the asset's equilibrium risk premium.

IV. CASH FLOW TIMING AND OAS DIRECTIONALITY

The interaction of the mechanics of the OAS calculation and the prepayment behavior of consumers induces a relationship between OAS and interest rates. For a given coupon rate mortgage security, other things equal, there will be a tendency for a mortgage security's OAS to widen as rates fall and tighten as rates increase. This tendency has nothing to do with changing prepayment risk premiums, nor is it related to shifts in consumer prepayment behavior, although it is common to find articles that claim otherwise.

To illustrate the inherent relationship between a mortgage's OAS and the level of interest rates, consider a mortgage with a given coupon rate, and fix the parameter values of the mortgage's interest rate-sensitive prepayment function. Suppose the mortgage model pricing error, $P_m - P^e_m$, is held fixed, and the mortgage's OAS is calculated for various levels of initial interest rates.[14]

Exhibit 3 illustrates the results of this conceptual experiment for the current-coupon TBA Freddie Mac gold PC as of August 16, 1999.[15] It shows an inherent relationship between a mortgage security's OAS and the level of interest rates. Holding the mortgage model pricing error constant, OAS will widen as interest rates fall.

Holding other things constant, there is a negative relationship between OAS and the level of interest rates as a consequence of the interest rate sensitivity of prepayments. Exhibit 4A represents the equivalent martingale expected cash flow profile of a current-coupon Freddie Mac gold PC. Changes in the level of interest rates will alter the time profile of these expected cash flows.

As interest rates fall, prepayments speed up. The acceleration of prepayments under alternative declines in the level of interest rates is illustrated by the heavy line in Exhibits 4B, 4C, and 4D. In an up-rate environment, expected payments extend as is illustrated for selected interest rate increases by the thinner line in B, C, and D.

Recall that Exhibit 2 shows that a given OAS has a much greater effect on the present value of distant cash flows. Thus, as interest rates fall, prepayments are accelerated, and the OAS must be wider to eliminate a fixed pricing error. As rates rise, a mortgage extends, and a narrower OAS is required to eliminate a fixed error.

EXHIBIT 3
OAS Directionality of Current-Coupon Freddie Mac Gold PC on 8/16/99

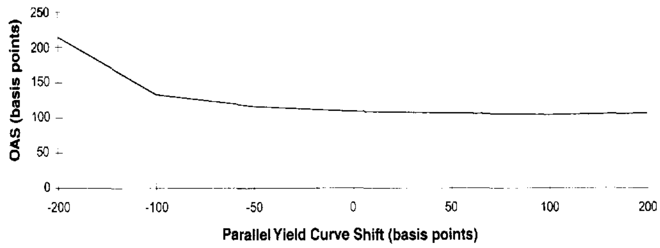


EXHIBIT 4A

Projected Principal Payments of $100 Gold 30yr, 7.5%

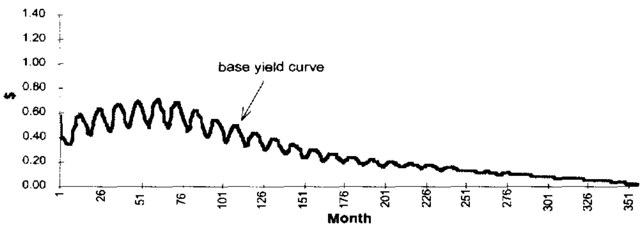


EXHIBIT 4B

Projected Principal Payments of $100 Gold 30yr, 7.5%

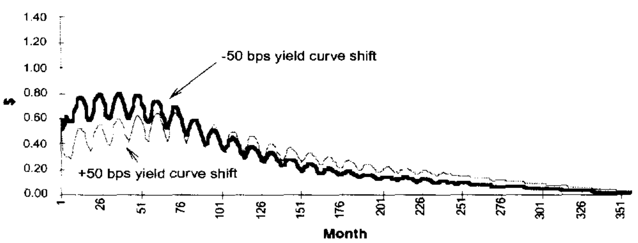


EXHIBIT 4C

Projected Principal Payments of $100 Gold 30yr, 7.5%

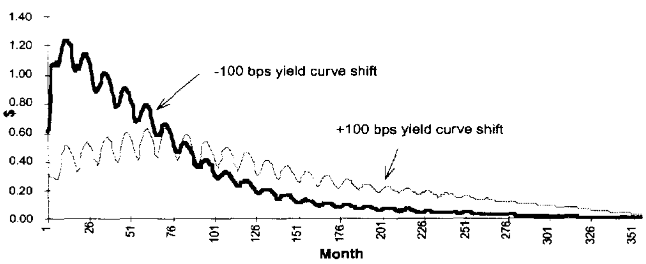
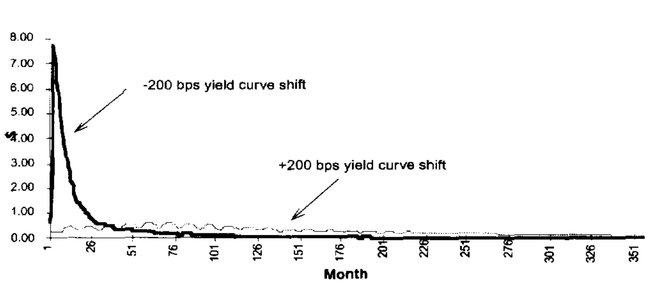


EXHIBIT 4D

Projected Principal Payments of $100 Gold 30yr, 7.5%



While pricing model errors will almost certainly change as the level of interest rates changes and market prices react, this does not diminish the accuracy of our claim that, other things equal, OAS should be directional.

OAS directionality is also visible in the typical upward-sloping shape of the OAS curve for the coupon stack of agency TBA issues. For example, except perhaps for the Lehman Brothers quotes, the OAS plots in Exhibit 1 show a positive relationship between OAS and the coupon rate on TBA issues. A positively sloped OAS-coupon profile is typical over time. While the OAS values plotted in Exhibit 1 almost certainly correspond with varying values for mortgage pricing model error, a similar upward-sloped OAS profile is generated if model error is held constant.

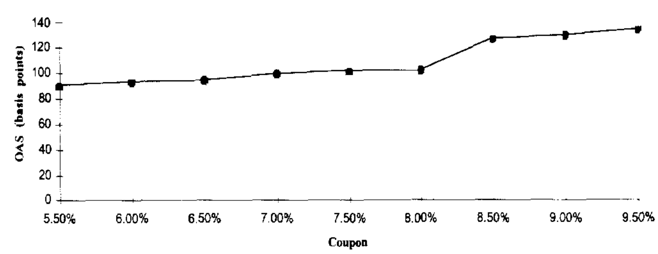
Exhibit 5 plots the relationship between the OAS and the coupon rate for a "stack" of Freddie Mac TBA gold PCs when OAS is calculated to satisfy a hypothetical market price that is artificially set to be $5.50 lower than the estimated model price for a mortgage pool with $100 unpaid principal balance. Notice that the OAS increases monotonically through the coupon stack from discount to premium mortgages. This positive slope is attributable, at least in part, to the same forces that make OAS inherently "directional" when rates change.

The relationship between OAS and the level of interest rates is well known among practitioners and has been recognized in the literature, but such a relationship is often attributed to a phenomenon extraneous to the basic mortgage pricing model. OAS directionality has been attributed to alleged changes in the required risk premium for bearing prepayment risk or to shifts in consumers' prepayment behavior that alter prepayment risk.[16]

For example, the so-called PORC model of Cohler, Feldman, and Lancaster [1997] is a multiple-factor mortgage pricing model that is specifically designed

EXHIBIT 5

OAS by Coupon for Freddie Mac Gold PC



to produce the phenomenon known as OAS directionality.[17] In the PORC model, a separate prepayment model error factor is introduced to explain the observed correlation between OAS changes and changes in interest rates.

A more recent example can be found in the May 14, 1999, Salomon Smith Barney (SSB) *Bond Market Roundup: Strategy*. SSB's analysts note that: "OASs tend to become directional during periods of heightened prepayment fears (in other words, when the media effect is strong)" (p. 24). They later say: "OASs should not be directional in a perfect model. Therefore, the directionality may indicate that the market thinks current prepayment models understate the true risk" (p. 27).

While our purpose is not to comment specifically on any explanations for the empirical relationships that are observed between changes in OAS and changes in the level of interest rates, our analysis highlights the fact that there is a natural relationship between changes in OAS and changes in the level of interest rates, irrespective of the existence of any risk premium for bearing prepayment model risk, or any changes in such risk or risk premium. OAS will naturally exhibit "directionality" even when there is no risk premium.

It is therefore important for those attempting to model OAS directionality to separate the effects of their predictive model from the natural directionality that would arise if model pricing errors remain constant in the face of a changing interest rate environment. Unless the underlying tendency for OAS directionality is controlled for, it is impossible to assess the importance of alleged behavioral changes and changes in investor risk tolerance for changes in OAS.

Indeed our arguments should suggest that neither the level of, nor changes in the level of, investors' required premium for bearing prepayment risk has any power to predict a mortgage's OAS or the relationship between changes in interest rates and changes in a mortgage's OAS.

V. CONCLUSIONS

That there is an OAS in mortgage-backed securities pricing at all is a consequence of misspecification of the prepayment function. Mortgage pricing models are misspecified because they typically ignore the effect of non-interest rate factors that have been shown to be important determinants of residential mortgage prepayment behavior. In general, it is not possible to modify prepayment functional forms or parameter values to fully compensate for the omission of factors that influence prepayment behavior.

As the OAS is caused by model misspecification, it is unrelated to an expected risk premium for prepayment risk. While prepayment model misspecification induces an OAS, prepayment model forecast error — a mean zero unforecastable random component of prepayment behavior — does not generate an OAS.

APPENDIX

If $f(x, y) = (a\tilde{x} + b\tilde{y})^2$, a second-order polynomial (or greater) is needed to match each moment condition. The coefficients of the "g" transformation are:

$$\alpha_m = \frac{b^2\left[E(\tilde{y}^2) - y_0^2\right] + 2abE(\tilde{x})\left[E(\tilde{y}) - y_0\right]}{\left[a^2E(\tilde{x}^2) + b^2y_0^2 + 2aby_0E(\tilde{x})\right]^2}$$

and:

$$\alpha_C = \frac{b^2\text{Cov}(\tilde{y}^2, \tilde{z}) + 2ab\left[\text{Cov}(\tilde{x}, \tilde{y}, \tilde{z}) - y_0\text{Cov}(\tilde{x}, \tilde{z})\right]}{\left[a^2\text{Cov}(\tilde{x}^2, \tilde{z}) + 2aby_0\text{Cov}(\tilde{x}, \tilde{z})\right]^2}$$

The restrictions on the moments of $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are necessary to ensure that $\alpha_m = \alpha_C$ can be worked out. It is unlikely that the conditions will be satisfied; if not, the "g" function does not exist.

ENDNOTES

The authors are indebted to Mark Fisher and Gary TeSelle for useful discussions.

[1]See, for example, the discussion in Fabozzi and Yuen [1998, Chapter 11].

[2]The dealer OAS estimates are taken from the particular dealer's fixed-income mortgage research publications and are based on 8/13/99 (Friday) closing Treasury market prices.

[3]For example, the discussions in Belton [1988], Cohler, Feldman, and Lancaster [1997], Finnerty and Rose [1991], Hayre [1990], and Selvaggio [1996] all refer to OAS as a risk premium.

[4]Some claim that the OAS represents compensation for prepayment model error risk. Our analysis will show that this is not the case.

[5]See, for example, Belton [1988], Fabozzi [1997], Finnerty and Rose [1991], or Fabozzi and Modigliani [1992].

[6]The simplest class of prepayment models are based upon multiples of the PSA Standard Prepayment benchmark model. See Fabozzi [1997, Chapter 19] for discussion.

[7]The notation allows for the cash flow at future date k on path i to depend on all risk-free rates up to and including the rate on date k .

[8]Fabozzi and Modigliani [1992, Chapter 10] provides a general discussion. Deng, Quigley, and Van Order [forthcoming] and Kau et al. [1992] provide more detailed analysis.

[9]The contemporaneous covariation between the i-th period short rate and the i-th period cash flow is small because rate changes influence prepayment behavior with a lag in excess of thirty days.

[10]We are not suggesting that the covariance terms be ignored in mortgage pricing calculations, but rather that it is easier to see how the OAS transformation "solves" the pricing error problem if the covariance terms are ignored and the discussion focuses on the discount function effects of OAS.

[11]See, for example, Simmons [1983, p. 154] for a formal statement of the Weierstrass theorem.

[12]In the mortgage applications that follow, if $\tilde{y}$ is an omitted economic factor that simultaneously drives prepayments, in general it is anticipated that $\mathrm{Cov}(\tilde{y}, \tilde{z}) \neq 0$.

[13]For example, one would expect omitted pricing factors to have non-zero drifts whether or not they are correlated with the risk-free discount function.

[14]That is, the mortgage's OAS is calculated at initial rates, and then the initial term structure is shifted up and down by various amounts.

[15]The calculations that underlie Exhibits 3, 4, and 5 were performed using Salomon Smith Barney's *Yield Book* fixed-income software package.

[16]Kon and Polk [1998, p. 8] write: "The directionality intuition is that as rates decline, prepayment uncertainty increases. If this is a priced risk factor in the mortgage market, then OASs will widen...."

[17]Cohler, Feldman, and Lancaster [1997, p. 9] characterize their model as "intuitively appealing and better reflecting the reality of what every market participant knows (OAS widens in rallies and tightens in backups...)."

REFERENCES

Belton, Terrence. "The New Breed: Option Adjusted Spreads." *Secondary Mortgage Markets*, Winter 1988-1989, pp. 6-11.

Cohler, Gene, Mark Feldman, and Brian Lancaster. "Price of Risk Constant (PORC): Going Beyond OAS." *Journal of Fixed Income*, March 1997, pp. 6-15.

Cox, John, Jonathan Ingersoll, and Stephen Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, Vol. 53, No. 2 (1985), pp. 363-384.

Deng, Yongheng, John Quigley, and Robert Van Order. "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options." *Econometrica*, forthcoming.

Fabozzi, Frank. *Fixed Income Mathematics*, 3rd ed. New York: McGraw-Hill, 1997.

Fabozzi, F., and F. Modigliani. *Mortgage and Mortgage-Backed Securities Markets*. Boston: Harvard Business School Press, 1992.

Fabozzi, F., and D. Yuen. *Managing MBS Portfolios*. New Hope, PA: Frank J. Fabozzi Associates, 1998.

Finnerty, John, and Michael Rose. "Arbitrage-Free Spread: A Consistent Measure of Relative Value." *Journal of Portfolio Management*, Spring 1991, pp. 65-81.

Hayre, Lakhbir. "Understanding Option-Adjusted Spreads and their Use." *Journal of Portfolio Management*, Summer 1990, pp. 68-69.

Kau, James, Donald Keenan, Walter Muller III, and James Epperson. "A Generalized Valuation Model for Fixed-Rate Residential Mortgages." *Journal of Money, Credit and Banking*, Vol. 24, No. 3 (1992), pp. 279-299.

Kon, Stanley, and Christine Polk. "Time-Varying Empirical Duration and Slope Estimates for Mortgage-Backed Securities." *Journal of Fixed Income*, September 1998, pp. 7-28.

Selvaggio, Robert D. "An Options Approach to Valuing and Hedging Mortgage Servicing Rights." In Frank J. Fabozzi, ed., *The Handbook of Fixed Income Options: Strategies, Pricing and Applications*, Chapter 14. Chicago: Irwin, 1996.

Simmons, George F. *Introduction to Topology and Modern Analysis* (Reprint). Melbourne, FL: Robert E. Krieger Publishing Company, Inc., 1983.