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TESTING REGRESSION DISTURBANCES FOR NORMALITY WITH STABLE ALTERNATIVES: FURTHER MONTE CARLO EVIDENCE

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This paper extends previous Monte Carlo work on testing for normality of ordinary least squares regression residuals. In addition to considering variation in sample size, this study also considers the affects of variation in the number of regressors and misspecifying the degree of serial correlation in the population residuals. Based on simulations using stably distributed alternatives with varying degrees of skewness, evidence is provided that: increasing the column dimension of the matrix of exogenous variables leads to a reduction in the power of parametric normality tests in small samples; and, misspecification of the degree of serial correlation has a significant affect on parametric normality test results, across all sample sizes.

KEY WORDS: Normality testing, Monte Carlo study, stable distributions, serial correlation, projection matrix, simulation.

1. INTRODUCTION

Despite the widespread use of the normality assumption in empirical studies, in many practical situations this assumption is not strongly supported by the data (e.g., Clark, 1973; Fielitz and Roselle, 1983; Tucker and Pond, 1988). In turn, residual non-normality has been shown to have a significant impact on statistical inference in regression studies (e.g., Knight, 1986). The importance of the normality assumption in ordinary least squares (OLS) estimation has motivated Monte Carlo work on testing for normality of OLS regression residuals (Huang and Bolch, 1974; White and MacDonald, 1980; Bera and McKenzie, 1986).† Of these studies, only Bera and McKenzie (BM) have compared the powers of various normality tests with stable alternatives. Unfortunately, BM followed previous studies in concentrating primarily on the impact of varying sample size. Because sample size is only one aspect of the linear transformation from population residuals to OLS residuals, this limits the applicability of their results. In addition, BM only examined residuals which were uncorrelated by construction.

In general, the linear transformation from population to OLS residuals is

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†There have also been closely related Monte Carlo studies on normality testing for stable random variables, e.g., Smith (1975) and Baringhaus *et al.* (1989).

dependent on sample size, the column dimension of the matrix of exogenous variables (X) and, in certain cases, the generating processes for the variables contained in X. In addition, a number of studies (e.g., Pierce and Kopecky, 1979; Moore, 1982; Pierce, 1985) have shown that misspecification of the degree of serial correlation in the population residuals may have a significant impact on normality testing for OLS residuals. This study extends previous Monte Carlo work on normality testing with stable alternatives by examining variations in the regression projection matrix arising from altering both the dimensions of X. Further, the effects of projection matrix variation are examined under a number of assumptions on the degree of serial correlation in the population residuals. In all cases, the null hypothesis tested is that the population residuals are *iid* normal. Both projection matrix specification and misspecification of the degree of autocorrelation in the population residuals are shown to have a significant affect on normality testing with stable alternatives.

2. PROBLEM DESCRIPTION

The classical linear regression model for the random variables Y is typically specified as:

$$Y = Xb + u \tag{1}$$

where Y is a $T \times 1$ vector, X is a fixed, finite $T \times k$ matrix of full column rank ($k < T$), b is a $k \times 1$ vector of unknown parameters and u is a $T \times 1$ vector of random variables. u is usually assumed to be *iid*, $N(0, \sigma^2 I)$. While the assumption of normality is not essential to deriving the efficiency of the ordinary least squares (OLS) estimator, imposing normality serves at least two purposes. Firstly, it facilitates statistical inference by justifying the use of classic normality-based tests (e.g., chi-squared and t tests). Secondly, normality provides for an immediate theoretical correspondence between the OLS estimator and the maximum likelihood (ML) estimator. Violations of the normality assumption create significant problems. If u is *iid* but with non-normal skewness, then the OLS estimate of the constant will be biased; non-normal kurtosis will affect the calculation of critical values for hypothesis testing; and, dependence in u will affect the efficiency of the OLS estimates.*

As is well known, the problem of using OLS residuals ($\hat{u}$) to test for the normality of the error u in (1) is that the OLS residuals are only linear transformations of the underlying errors. Specifically:

$$\hat{u} = (I - X(X'X)^{-1}X')u = (I - M)u.$$

*Bias in the constant follows from Schmidt (1976, p. 39) and observing that the median u does not equal the expected value. For regression hypothesis testing purposes, the presence of fat (or light) tails affects the determination of confidence regions. Hence, the effect of kurtosis is most important (Knight, 1986).

White and Macdonald, Pierce and Kopecky and others demonstrate theoretically that, subject to regularity conditions on the higher moments, conventional normality tests such as the standardized sample moments of skewness and kurtosis can be modified to allow $\hat{u}$ to be used in testing the normality of u in large samples. However, the necessary regularity conditions are violated in the case of stable distributions. BM demonstrate that extending these large sample theoretical results to the stable case is "complex". As a result, Monte Carlo experiments are well-suited to provide information on convergence and other properties of the normality tests which are not readily derivable theoretically.

To data, available Monte Carlo evidence indicates that normality tests based on OLS residuals are quite robust even in samples as small as $T=20$. BM indicate that convergence may be somewhat slower in the stable cases. However, these Monte Carlo results are of restricted applicability because only T is allowed to vary (Weisberg, 1980). More specifically, *if u is uncorrelated* the relationship between u and $\hat{u}$ is dependent on both the column dimension of X, the number of regressors (k), and the row dimension of X, the sample size (T). Along with T, k may affect normality testing results. In effect, by altering the nature of the linear transformation between population and OLS residuals, variation in either dimension of X can affect the speed with which the asymptotic results are achieved. Knowledge of convergence properties of normality tests is essential for using these tests in applied work.

In addition to variation in T and k, it is also theoretically possible that the generating processes selected for the X variables will affect small sample performance of parametric normality tests for OLS residuals. In previous studies, the Monte Carlo experiments have used a fixed k with the columns of X (the X_k's) typically being generated as uncorrelated *uniform* random variables. In addition to the uniform case, BM went somewhat further to examine an additional X specification: the X matrix ($k=4$) was formed by combining a uniform, normal and chi-squared(10) distributed columns. For Monte Carlo work, uniform deviates have the desirable property that the upper bound for the off-diagonal elements of $(X'X)/T$ matrices converge to zero at a rate of order $(1/T)$. This compares with a convergence rate for the upper bound of $T^{-1/2}$ for normal deviates, the fastest converging of all the stable deviates (Johnson and Kotz, vol. 1, 2). Hence, because a uniformly generated M is likely to be closer to a diagonal matrix, the distributional specification of the X variables could theoretically affect the small sample properties of normality tests based on OLS residuals.*

The possibility of variation in small sample test performance due to X variation

*Another way in which the distributional specification of the X_k's could affect small sample performance is through the degree of so-called collinearity or dependence among the X_k's. In general, collinearity increases the relative size of off-diagonal elements in M producing a greater degree of non-diagonality in the transformation between population and OLS residuals. In particular, if the columns of the X matrix approach linear dependence, then $X'X$ approaches non-invertibility. In this case, there are real differences among the numerical routines used to compute the inverse of a symmetric matrix (e.g., Wambler, 1980). Theoretically, because the parametric normality tests are based on higher moments, the numerical implications of non-invertibility may affect these tests before being evidenced through $X'X$ non-invertibility.

is disturbing. Specifically, if the distributional specification of X variables has a significant effect on evaluating the small sample properties of regression-based estimators, then this has important implications for a wide range of Monte Carlo studies which use the *iid* uniform X's assumption to generate the transformation matrix (e.g., Raj, 1980). These implications are practical as well as theoretical because, in many types of applied regression studies, uniformly distributed right-hand-side variables are not typically observed. Fortunately, while there is limited information in this area, evidence that is available indicates, at most, a small impact (e.g., BM). This was, generally, confirmed in Monte Carlo experiments conducted in conjunction with the present study. However, because the theoretical apparatus required to examine distributional variation in X is somewhat involved, these results have been included in a companion paper.

In addition to the specification of X, the time series properties of u have considerable importance in applied OLS studies because correlated residuals are often observed. If u is correlated the normality testing problem is complicated substantially: conventional normality tests based on the null hypothesis of uncorrelated OLS residuals are asymptotically biased because the covariance matrix of the u's is no longer a scalar identity matrix. However, unlike the theoretical bias in variance estimation arising from serially correlated residuals (Neudecker, 1977; Kiviet, 1980), the bias in conventional normality tests due to correlated residuals is not readily derivable. Monte Carlo experiments are better suited to provide information on how the normality tests would be affected by the correlation in the residuals—particularly as T, k and M vary. While generalized least squares (GLS) techniques can be used to correct for the bias due to correlated residuals, information on the degree of robustness of normality tests when the OLS residuals are correlated is useful for a number of reasons. For example, if OLS is reasonably robust to the degree of serial correlation in u, then normality tests can be done with OLS without the need to repeat the estimations in GLS if residual serial correlation is (unexpectedly) observed.

3. TEST STATISTICS

In order to test for the normality of regression residuals, the estimations in Section 4 report results for a select number of parametric normality tests: the standardized sample moments of skewness ($\sqrt{b_1}$) and kurtosis (b_2), the studentized range and the Lagrange Multiplier (LM) test combining b_1 and b_2 examined by BM. Because the objectives of the present study are to demonstrate the affects of serial correlation and variation in T and k, no attempt is made to provide power comparisons for a large number of distributional tests. (See Singal *et al.*, 1989 or Baringhaus *et al.*, 1989 for recent reviews of available tests.) These particular statistics were chosen for reasons of power and practicality. As discussed below, there is considerable evidence demonstrating that the power of these tests compares favourably with other available parametric and nonparametric alternatives. In addition, these particular statistics are readily computable from the OLS output produced by standard regression packages, making the tests results useful to a wide range of empirical researchers.

Under the null hypothesis of *independently, identically, normally distributed random variables*, the standardized sample moments are defined as:

$$\sqrt{b_1} = m_3/(m_2)^{3/2}$$

$$b_2 = m_4/(m_2)^2$$

where: $m_r = \sum_{t=1}^{T} (X_t - m_1)^r/T$ and m_1 is the sample mean.

Available evidence indicates that the standardized moments are more powerful than "omnibus" tests when there is prior knowledge indicating that the residual distributions are either leptokurtic or skewed (Pearson, D'Agostino and Bowman, 1977). For example, if the distribution is suspected of being heavy tailed (as is the case with a wide range of financial variables), Hogg (1972), Smith (1975), Saniga and Hayya (1977), and Franck (1982) all find evidence supporting the use of b_2 in testing for distributional specification. For testing the normality of OLS regression residuals against both specific and general alternatives, White and Macdonald (1980) find that the third and fourth standardized sample moments perform "acceptably well" over a wide range of sample sizes. For the normal against stable case, BM present similar results.

While the third and fourth standardized sample moments are often desirable tests in identifying specific departures from normality, neither statistic performs as well in detecting departures from normality when the residual distributions are of unknown shape. Further, the standardized sample moments may be sensitive to outliers (Kendall and Stuart, vol. 2, pp. 547–549. For the purpose of identifying outliers, the studentized range (SR) has been found to have desirable properties.* The studentized range is defined as the range divided by the standard deviation, i.e.:

$$SR = (\max(\hat{u}_t) - \min(\hat{u}_t))/\sigma_u.$$

For a wide range of distributional alternatives, the SR is not as powerful as tests based on combining the standardized sample moments. In particular, the SR should not be used when trying to detect if the distribution is light tailed or not (Hogg). Other drawbacks to the SR are that it does not make full use of all the sample observations and that there is an indeterminate range in the confidence levels. However, in certain cases, the SR gives acceptable, even superior, results. For example, Uthoff (1970) finds SR is the optimal test for the normal

*Bibliographical material on the studentized range can be found in Johnson and Kotz (1970). This statistic was not considered by BM.

distribution versus uniform alternatives. Fama and Roll (1971) recommend the SR as the teting method of choice for detecting symmetric stable distributions.

Results are also reported for the LM (asymptotic chi-squared) test based on the standardized sample moments. It has long been recognized that the drawbacks associated with using the standardized sample moments as omnibus tests can be overcome if the information from both skewness and kurtosis estimates can be combined.* The LM test is based on the asymptotic normality of the distributions for $\sqrt{b_1}$ and b_2, i.e.:

$$\text{chi-squared}(2) = T\left(\frac{(b_1)}{6} + \frac{(b_2-3)^2}{24}\right) = \text{LM}$$

where: T is the number of observations in the sample.

The primary difficulty with combining the two statistics in this manner is that the sampling distributions are *not* independent when the sample size is small. In addition, there is evidence that the speed of convergence to the asymptotic distribution is somewhat slow (e.g., Bowman and Shelton, 1986; Pearson *et al.*, 1977). Fortunately, appropriately adjusted confidence regions for calculating the significance of the chi-squared tests have been provided by BM. Using these adjusted confidence regions, BM found that the LM typically had the highest power amongst the large number of normality tests they considered for the normal against stable cases.

3. MONTE CARLO EXPERIMENT DESIGN†

The stable distribution is characterized by four parameters: the characteristic exponent (α: $0<\alpha\leqq 2$), a skewness parameter (δ: $-1\leqq\delta\leqq 1$), a scale parameter and a location parameter. In keeping with the parametric specification contained in BM, four characteristic exponent (α) values are examined: 2.0, 1.9, 1.6, and 1.0. In addition, three skewness parameter (δ) values are evaluated: zero, one-half and one, i.e., both symmetric and two forms of positively skewed stable distributions are considered. In total, nine variations of the stable distributions are considered together with the normal ($\alpha=2$, $\delta=0$) case. Another key element in the Monte Carlo experiments is the specification of the X matrix which determines the linear transformation between u and $\hat{u}$. In all cases the first column of X is a $T\times 1$ vector of ones. In order to contain the number of results reported, only two column dimensions of X are examined $k\in\{5, 10\}$. Variations in T are accounted for by examining three sample sizes $T\in\{25, 50, 100\}$.

To generate the projection matrix, the X_k's for each sample size are generated

*This follows because when one of the standardized sample moments for skewness or kurtosis has weak power against a specific alternative, the other measure usually has acceptable power (e.g., if a distribution is heavy-tailed and symmetric then skewness has low power but kurtosis has high power).

†Details on the specific random number generating routines used are given in the Appendix.

separately (and then held constant over all replications). This method differs somewhat from previous studies. In examining sample variation between $T=20$ and $T=100$, WM and BM generate the columns of X by, firstly, generating an X for $T=100$. The X matrix for smaller sample sizes is then produced by using the first T rows of the original ($T=100$) X matrix (holding the resulting X matrix constant over all replications). While this approach highlights convergence properties as T varies, there is considerable overlap in the elements contained in X from one sample size to the next. In practice, however, there is little difference between the two methods. The selected process for generating the X_k's follows other studies by taking *iid* X_k's from a *uniform* distribution and uses these X_k's to define M which is then held constant over all replications. In all cases, the number of experiment replications is five hundred.

The final feature to be considered is the affect of misspecifying serial correlation in the population residuals by assuming the residuals are *iid*. Of the many possible ARMA (p, d, q) time series models for u, for present purposes attention centers on the AR(1) residual process, i.e., the chosen specification for u is:

$$u_t = \beta u_{t-1} + e_t \tag{2}$$

where $t \in \{1, 2, \ldots, T\}$, $|\beta| < 1$ and e is an *iid* random variable. In order to examine the impact that varying degrees of first order serial correlation in u have on normality tests based on $\hat{u}$, six different specifications of the autoregressive parameter β in (2) are examined: $\beta \in \{0, 0.25, 0.5, 0.9, -0.25, -0.9$. The serially correlated values for the variables are generated by applying (2) to the same u vector for each individual replication, using the six β parameter values specified.

4. EMPIRICAL RESULTS

The empirical results have been arranged in three sub-sections dealing with various features of the normality testing problem. Because of the considerable number of results to be reported, each section has been abbreviated to include only representative results. All results are given in terms of test power where power is obtained by dividing the number of times H_0 is rejected by the number of replications in the experiment, in this case 500. In all cases, the 10% critical values were used and, where appropriate, two-tailed testing invoked. To provide a basis for comparison with previous studies, the first subsection examines test convergence as sample size increases using X with $k=5$. The next subsection examines the affect of differences in column dimensionality, using *iid* uniform X_k's ($k=10$), over the range of sample sizes. The third subsection examines the variation in test results when the population residuals are serially correlated, again using *iid* uniform X_k's ($k=5$).

A. Benchmark Comparisons

Table 1 contains results for normality tests using *iid* uniform X_k's ($k=5$) to test for

Table 1 Estimated test powers against stable alternatives: Uniform X_k $(k=5)$[a]

	α	δ	$\sqrt{b_1}$	b_2	LM	SR
$T=25$	2.0	0	0.080	0.098	0.082	0.100
	1.9	0	0.178	0.178	0.176	0.174
	1.9	0.5	0.176	0.172	0.180	0.164
	1.9	1.0	0.182	0.172	0.186	0.160
	1.6	0	0.416	0.390	0.440	0.400
	1.6	0.5	0.440	0.392	0.460	0.368
	1.6	1.0	0.518	0.372	0.490	0.298
	1.0	0	0.756	0.794	0.840	0.750
	1.0	0.5	0.830	0.820	0.870	0.724
	1.0	1.0	0.920	0.778	0.906	0.524
$T=50$	2.0	0	0.102	0.118	0.058	0.092
	1.9	0	0.270	0.302	0.278	0.288
	1.9	0.5	0.288	0.288	0.278	0.266
	1.9	1.0	0.336	0.294	0.306	0.246
	1.6	0	0.622	0.722	0.724	0.712
	1.6	0.5	0.698	0.718	0.742	0.652
	1.6	1.0	0.860	0.706	0.824	0.540
	1.0	0	0.910	0.992	0.994	0.978
	1.0	0.5	0.922	0.984	0.990	0.936
	1.0	1.0	1.000	0.990	1.000	0.778
$T=100$	2.0	0	0.120	0.120	0.100	0.110
	1.9	0	0.356	0.448	0.448	0.420
	1.9	0.5	0.395	0.390	0.420	0.370
	1.9	1.0	0.528	0.476	0.512	0.388
	1.6	0	0.748	0.946	0.942	0.894
	1.6	0.5	0.889	0.918	0.938	0.848
	1.6	1.0	0.992	0.916	0.988	0.740
	1.0	0	0.940	1.000	1.000	1.000
	1.0	0.5	0.980	1.000	1.000	1.000
	1.0	1.0	1.000	1.000	1.000	0.944

[a]Critical values for LM calculated using BM's adjusted confidence regions.

the normality of a selected range of distributional assumptions for the u's. With some exceptions, these results conform with those from previous studies: test power increases considerably as sample size increases and α decreases, OLS residuals tend to be "super normal" when the null hypothesis is true, and the power of the standardized sample moments is comparable to the power of the omnibus tests. Of the tests considered, the studentized range typically exhibited the least power while the power of the LM test was typically the highest. In particular, the relative performance of the studentized range deteriorated considerably as the skewness parameter increased from 0 to 1 indicating that this test is inappropriate when testing for stable distributions which are not symmetric.

While, in general, the estimated powers of the tests correspond closely with those of previous studies, there were exceptions. In particular, a significantly greater skewness effect was identified than was the case in BM's study where only limited power differences were observed between the $\delta=0.5$ and $\delta=1$ cases. The

Table 2 Estimated test powers against stable alternatives: Uniform X_k $(k=10)$[a]

	α	δ	$\sqrt{b_1}$	b_2	LM	SR
$T=25$	2.0	0	0.070	0.096	0.058	0.098
	1.9	0	0.138	0.148	0.148	0.148
	1.9	0.5	0.124	0.126	0.118	0.130
	1.9	1.0	0.152	0.128	0.128	0.124
	1.6	0	0.298	0.284	0.288	0.240
	1.6	0.5	0.350	0.290	0.320	0.245
	1.6	1.0	0.352	0.268	0.356	0.244
	1.0	0	0.626	0.602	0.676	0.562
	1.0	0.5	0.656	0.614	0.678	0.578
	1.0	1.0	0.736	0.602	0.726	0.478
$T=50$	2.0	0	0.076	0.108	0.056	0.108
	1.9	0	0.242	0.268	0.252	0.242
	1.9	0.5	0.262	0.258	0.276	0.224
	1.9	1.0	0.308	0.270	0.324	0.222
	1.6	0	0.604	0.640	0.636	0.630
	1.6	0.5	0.654	0.626	0.656	0.570
	1.6	1.0	0.682	0.614	0.784	0.750
	1.0	0	0.852	0.978	0.990	0.992
	1.0	0.5	0.902	0.978	0.946	0.984
	1.0	1.0	1.000	0.972	0.996	0.778
$T=100$	2.0	0	0.108	0.100	0.092	0.108
	1.9	0	0.372	0.424	0.432	0.424
	1.9	0.5	0.444	0.444	0.504	0.416
	1.9	1.0	0.520	0.460	0.572	0.392
	1.6	0	0.726	0.916	0.938	0.906
	1.6	0.5	0.862	0.894	0.916	0.842
	1.6	1.0	0.984	0.904	0.972	0.728
	1.0	0	0.956	1.000	1.000	1.000
	1.0	0.5	0.968	1.000	1.000	1.000
	1.0	1.0	1.000	1.000	1.000	0.968

[a]Critical values for LM calculated using BM's adjusted confidence regions.

present results, which exhibit increasing power for the skewness estimates as δ increases, are more theoretically plausible. In addition, another potentially disturbing feature of the present results is the difference between the estimated test power and the actual critical values for the normal cases, particularly for the LM. It is possible that the adjusted critical values provided by BM may give somewhat misleading results in small samples when the null hypothesis is true. Further work on this point is required.

B. Column Dimensionality

Table 2 extends the results of Table 1 to account for higher column dimension of X, i.e., the test results have been generated using *iid* uniform X_k's with $k=10$. These results reveal *a definite reduction in test power for the $T=25$ sample when the column dimension of X is increased.* This reduction in power increases somewhat as

α decreases. For the $T=50$ results, the power reduction is considerably less than for the smaller ($T=25$) sample, though still noticeable. There was no noticeable power reduction for the $T=100$ sample. Hence, this leads to a useful rule of thumb in normality testing against stable alternatives: in small samples, test power will likely be reduced as the number of variables in the regression is increased. The power reduction will diminish as the sample size increases. Further research is required to verify whether this result generalizes to normality testing against other types of distributional alternatives than the stable.

C. Serial Correlation in the Population Residuals

Tables 3–5 provide estimated normality test powers for a range of serial correlation values. Each table gives results for a specific sample size with some abbreviation of the results for the larger sample sizes. Comparing these results with the Table 1 $\beta=0$ case reveals that *low* β *values* (*i.e.*, $|\beta|=0.25$) *have little affect on either the test power level or on the convergence speed while the higher* β *values* ($\beta=0.5$, 0.9, -0.9) *do have a substantial effect*. Test powers for the $\beta=-0.9$ case are particularly unusual. Of the tests considered, kurtosis was generally the least affected with skewness being especially sensitive to high levels of negative serial correlation. The power of the studentized range was noticeably affected by high levels of both positive and negative serial correlation. Convergence of test power as sample size increased is evident for all degrees of serial correlation.

Following Maeshiro (1987), some caution should be used in interpreting these test power results too broadly. Both the population residuals and the X_k's have been generated as stationary random variables with the degree of serial correlation defining a stationary AR(1) process. However, in applied studies, serially correlated OLS residuals often arise from regressions involving trended dependent and independent variables. The resulting residuals may not have the same properties as the "serially correlated residuals" examined here. On a more positive note, the results support the general thrust of theoretical propositions in Sharma (1987). Specifically, estimator convergence rates may be affected by both the degree of serial correlation in the residuals as well as whether the serial correlation is positive or negative.

5. CONCLUSION

In general, the linear transformation from population to OLS residuals is dependent on sample size, the column dimension of the matrix of exogenous variables and the generating process for the variables contained in X. Previous Monte Carlo studies of normality testing for OLS regression residuals have concentrated on varying sample size. In addition, previous studies have been based on *iid* population residuals, neglecting the possibility of misspecifying of the degree of serial correlation in the population residuals. This study extends previous Monte Carlo work on normality testing for OLS residuals by examining a number of previously unstudied features. It was demonstrated that, in small samples,

Table 3 Estimated test powers against stable alternatives: Uniform X_k ($k=5$, $T=25$)[a]

	α	δ	$\sqrt{b_1}$	b_2	LM	SR
$\beta=0.25$	2.0	0	0.104	0.102	0.080	0.118
	1.9	0	0.164	0.176	0.150	0.164
	1.9	1.0	0.198	0.182	0.170	0.166
	1.6	0	0.376	0.394	0.406	0.402
	1.6	1.0	0.504	0.404	0.482	0.338
	1.0	0	0.766	0.792	0.804	0.736
	1.0	1.0	0.904	0.754	0.878	0.526
$\beta=0.50$	2.0	0	0.102	0.100	0.068	0.120
	1.9	0	0.162	0.160	0.120	0.160
	1.9	1.0	0.196	0.134	0.116	0.150
	1.6	0	0.374	0.314	0.340	0.306
	1.6	1.0	0.502	0.320	0.394	0.260
	1.0	0	0.764	0.664	0.720	0.584
	1.0	1.0	0.904	0.652	0.778	0.436
$\beta=0.90$	2.0	0	0.044	0.144	0.026	0.120
	1.9	0	0.062	0.152	0.040	0.142
	1.9	1.0	0.074	0.130	0.048	0.122
	1.6	0	0.126	0.178	0.108	0.138
	1.6	1.0	0.126	0.182	0.096	0.158
	1.0	0	0.210	0.250	0.176	0.196
	1.0	1.0	0.204	0.268	0.172	0.138
$\beta=-0.25$	2.0	0	0.086	0.100	0.064	0.112
	1.9	0	0.142	0.172	0.130	0.168
	1.9	1.0	0.168	0.160	0.148	0.174
	1.6	0	0.372	0.400	0.402	0.414
	1.6	1.0	0.448	0.378	0.432	0.322
	1.0	0	0.748	0.780	0.810	0.752
	1.0	1.0	0.876	0.752	0.858	0.624
$\beta=-0.90$	2.0	0	0.008	0.340	0.030	0.240
	1.9	0	0.006	0.328	0.036	0.274
	1.9	1.0	0.012	0.306	0.030	0.248
	1.6	0	0.020	0.326	0.054	0.270
	1.6	1.0	0.022	0.312	0.030	0.222
	1.0	0	0.030	0.242	0.060	0.204
	1.0	1.0	0.034	0.278	0.074	0.232

[a]Critical values for LM calculated using BM's adjusted confidence regions.

increasing the column dimension of the matrix of exogenous variables leads to a reduction in the power of standard parametric normality tests. In addition, misspecification of the degree of serial correlation in the population residuals was found to significantly alter normality test results across all sample sizes.

Table 4 Estimated test powers against stable alternatives: Uniform X_k ($k=5$, $T=50$)[a]

	α	δ	$\sqrt{b_1}$	b_2	LM	SR
$\beta=0.25$	2.0	0	0.082	0.110	0.054	0.108
	1.9	0	0.262	0.294	0.280	0.284
	1.9	1.0	0.322	0.278	0.288	0.260
	1.6	0	0.630	0.696	0.720	0.686
	1.6	1.0	0.836	0.696	0.792	0.546
	1.0	0	0.902	0.992	0.990	0.976
	1.0	1.0	1.000	0.984	1.000	0.790
$\beta=0.50$	2.0	0	0.082	0.140	0.072	0.108
	1.9	0	0.262	0.282	0.252	0.272
	1.9	1.0	0.322	0.272	0.260	0.250
	1.6	0	0.630	0.612	0.636	0.574
	1.6	1.0	0.836	0.620	0.714	0.468
	1.0	0	0.902	0.960	0.964	0.934
	1.0	1.0	1.000	0.932	0.984	0.706
$\beta=0.90$	2.0	0	0.134	0.190	0.076	0.120
	1.9	0	0.194	0.218	0.150	0.176
	1.9	1.0	0.222	0.208	0.170	0.158
	1.6	0	0.348	0.302	0.292	0.168
	1.6	1.0	0.340	0.294	0.290	0.148
	1.0	0	0.552	0.416	0.512	0.228
	1.0	1.0	0.520	0.346	0.466	0.168
$\beta=-0.25$	2.0	0	0.082	0.108	0.040	0.090
	1.9	0	0.252	0.296	0.244	0.270
	1.9	1.0	0.294	0.276	0.270	0.238
	1.6	0	0.608	0.710	0.712	0.684
	1.6	1.0	0.812	0.682	0.770	0.574
	1.0	0	0.898	0.990	0.994	0.968
	1.0	1.0	0.998	0.988	1.000	0.872
$\beta=-0.90$	2.0	0	0.002	0.428	0.056	0.308
	1.9	0	0.010	0.422	0.078	0.302
	1.9	1.0	0.010	0.420	0.066	0.310
	1.6	0	0.012	0.428	0.164	0.346
	1.6	1.0	0.024	0.454	0.162	0.334
	1.0	0	0.042	0.532	0.384	0.496
	1.0	1.0	0.044	0.510	0.376	0.476

[a]Critical values for LM calculated using BM's adjusted confidence regions.

Table 5 Estimated test powers against stable alternatives: Uniform X_k ($k=5$, $T=100$)[a]

	α	δ	$\sqrt{b_1}$	b_2	LM	SR
$\beta=0.25$	2.0	0	0.115	0.100	0.090	0.100
	1.9	0	0.360	0.408	0.416	0.408
	1.9	1.0	0.515	0.425	0.525	0.350
	1.6	0	0.755	0.885	0.910	0.880
	1.6	1.0	0.984	0.908	0.972	0.708
	1.0	0	0.950	1.000	1.000	1.000
	1.0	1.0	1.000	1.000	1.000	0.924
$\beta=0.50$	2.0	0	0.110	0.110	0.850	0.115
	1.9	0	0.360	0.360	0.372	0.332
	1.9	1.0	0.510	0.320	0.380	0.290
	1.6	0	0.755	0.860	0.845	0.825
	1.6	1.0	0.984	0.848	0.932	0.624
	1.0	0	0.950	1.000	0.995	0.990
	1.0	1.0	1.000	1.000	1.000	0.876
$\beta=0.90$	2.0	0	0.215	0.205	0.195	0.176
	1.9	0	0.324	0.268	0.284	0.180
	1.9	1.0	0.325	0.300	0.315	0.200
	1.6	0	0.475	0.435	0.500	0.250
	1.6	1.0	0.560	0.404	0.536	0.224
	1.0	0	0.800	0.725	0.820	0.345
	1.0	1.0	0.924	0.688	0.884	0.304
$\beta=-0.25$	2.0	0	0.005	0.095	0.040	0.135
	1.9	0	0.008	0.416	0.372	0.416
	1.9	1.0	0.005	0.405	0.340	0.355
	1.6	0	0.025	0.885	0.870	0.860
	1.6	1.0	0.028	0.896	0.872	0.752
	1.0	0	0.065	1.000	1.000	1.000
	1.0	1.0	0.100	1.000	1.000	0.972
$\beta=-0.90$	2.0	0	0.005	0.460	0.130	0.340
	1.9	0	0.008	0.432	0.172	0.328
	1.9	1.0	0.005	0.475	0.225	0.390
	1.6	0	0.025	0.565	0.330	0.445
	1.6	1.0	0.028	0.520	0.376	0.448
	1.0	0	0.065	0.790	0.740	0.740
	1.0	1.0	0.100	0.760	0.736	0.712

[a]Critical values for LM calculated using BM's adjusted confidence regions.

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APPENDIX

The random number generating routines used were taken from IMSL release 9.0. For the stable, the routine used was GGSTA and is described in Chambers *et al.*

(1976). This routine calculates stable variables by forming a ratio of a uniform and an exponential random variable. The normal deviates were generated using IMSL GGNPM which generates $N(0, 1)$ variables using the polar method algorithm due to Box, Muller and Marsaglia (see Knuth, 1969). The stable distribution employed yields $N(0, 2)$ if the characteristic exponent (alpha) equals 2. The GGUBS routine was used to produce uniform deviates which are continuous on $[0, 1]$. All simulations were done on a Sun 4/280.