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# Last Lecture

- Trade Results and Exam Review
  - Portfolio Insurance (RSD, p.511-24)
  - Executive Stock Options (Class Notes)
  - Exotic Options (Class Notes)
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# Basics of Portfolio Insurance

- The basic idea behind portfolio insurance is to provide a rate of return on the portfolio which will not fall below a given floor. (Recall that a put is a type of insurance).
- The basic mechanics of portfolio insurance can be isolated from the put-call parity arbitrage condition for a non-dividend paying stock:

$$S + P = C + X e^{-rt^*}$$

For portfolio insurance, instead of an individual stock  $S$  now refers to a portfolio of stocks (dividends have been ignored for simplicity of exposition).

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# Basics of Portfolio Insurance (cont'd)

- ❑ Put-call parity provides two ***path independent*** insurance strategies.
  - ❑ One strategy is  $S + P$ , buy puts against the portfolio. If  $S$  is an index portfolio, relevant exchange traded puts may be available.
  - ❑ Another strategy is  $C + X e^{-rt^*}$ , buy calls and invest the remainder in appropriately dated bonds. Again, if the portfolio is an index portfolio, exchange traded calls may be available
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# Properties of Insured Portfolios

## ■ ***Path Independence***

- "A strategy that is not path independent gives an uncertain payoff, and therefore violates the very premise of portfolio insurance: giving a known payoff."

## ■ ***Time invariance***

- This requires that the insurance strategy does not depend on the time remaining in the program (or on the use of a fixed time horizon).
  - The primary practical difficulty arises because of the option price convexity with respect to time, e.g., an option with six months to expiration will cost less than twice what a three month option costs.
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# Greeks for Insured Portfolios

- A delta neutral portfolio can be contrasted with portfolio insurance plans (which are delta positive).
- One possible method of insuring a stock position is to form a portfolio which combines purchased puts with a long stock position. In this case:

$$V = n_1 S + n_2 P \quad \rightarrow \quad \Delta_V = n_1 + n_2 \Delta_P$$

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# Solving Greeks for Insured Portfolios

- For full insurance, the number of units of stock underlying the put equal the size of the long stock position, such that  $n_1 = n_2$ :
- Unlike hedge portfolios and other positions which are delta neutral, portfolio insurance requires the position delta to be greater than zero (V exhibits a positive response to stock price increases)
  - The position is not rebalanced over time in order to maintain the starting delta value
  - Insured long positions do require the delta of the portfolio to be positive and the gamma to be equal to zero. Over sufficiently long time periods, this may require rebalancing the derivative positions.

$$\Delta_V = n_1 \{1 + \Delta_P\} > 0 \quad \text{where} \quad \Delta_P > -1$$

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# More on Dynamic Portfolio Insurance

- Dynamic portfolio insurance strategies can be illustrated by substituting the Black-Scholes formula into the put-call parity condition:

$$\begin{aligned} S + P &= S N[d_1] - X e^{-rt^*} N[d_2] + X e^{-rt^*} \\ &= S N[d_1] + X e^{-rt^*} (1 - N[d_2]) \\ &= w_1 S + w_2 X e^{-rt^*} \end{aligned}$$

where the weights  $w_1$  and  $w_2$  indicate the proportions of the portfolio held in stock and bonds in order to achieve insurance with an exercise price of  $X$  and time to maturity of  $t^*$ .

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# Dynamic Portfolio Insurance

- Dynamic portfolio insurance strategies involve actively trading portfolios composed of stocks and bonds in order to replicate the payoff on an insured stock portfolio.
  - Dynamic portfolio insurance is a *path dependent* technique that can be constructed to be approximately time invariant
  - An advantage of dynamic strategies is that the term to maturity and exercise price can be set precisely.
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# Comparison of Basic Strategies

Insured stock portfolio value at alternative stock index levels, using static portfolio insurance.\*

Index Level $S$	Put Option $P$	Portfolio Value $S + P$
59.87	36.21	96.08
63.02	33.06	96.08
66.34	29.75	96.09
69.83	26.29	96.13
73.51	22.70	96.21
77.38	19.03	96.41
81.45	15.38	96.83
85.74	11.87	97.61
90.25	8.67	98.92
95.00	5.94	100.94
100.00	3.79	103.79
105.00	2.29	107.29
110.25	1.27	111.52
115.76	0.65	116.42
121.55	0.31	121.86
127.63	0.13	127.76
134.01	0.05	134.06
140.71	0.02	140.73
147.75	0.01	147.75
155.13	0.00	155.13
162.89	0.00	162.89

Insured stock portfolio value at alternative index levels, using dynamic portfolio insurance with continuous rebalancing.\*

Index Level $S$	T-Bill Price $Xe^{-rT}$	Stock Portfolio Weight $w_1$	T-Bill Weight $w_2$	Portfolio Value
59.87	96.08	0.001	1.000	96.08
63.02	96.08	0.002	0.999	96.08
66.34	96.08	0.005	0.996	96.09
69.83	96.08	0.014	0.990	96.13
73.51	96.08	0.034	0.975	96.21
77.38	96.08	0.072	0.945	96.41
81.45	96.08	0.136	0.892	96.83
85.74	96.08	0.231	0.809	97.61
90.25	96.08	0.355	0.696	98.92
95.00	96.08	0.496	0.560	100.94
100.00	96.08	0.638	0.416	103.79
105.00	96.08	0.758	0.289	107.29
110.25	96.08	0.852	0.183	111.52
115.76	96.08	0.918	0.106	116.42
121.55	96.08	0.959	0.056	121.86
127.63	96.08	0.981	0.026	127.76
134.01	96.08	0.992	0.011	134.06
140.71	96.08	0.997	0.004	140.73
147.75	96.08	0.999	0.001	147.75
155.13	96.08	1.000	0.000	155.13
162.89	96.08	1.000	0.000	162.89

# Discrete Rebalancing and No Rebalance

Insured stock portfolio value at alternative stock index levels, using dynamic portfolio insurance with no rebalancing.\*

Index Level $S$	T-Bill Price $Xe^{-rT}$	Stock Portfolio Weight $w_1$	T-Bill Weight $w_2$	Portfolio Value
59.87	96.08	0.638	0.416	78.18
63.02	96.08	0.638	0.416	80.19
66.34	96.08	0.638	0.416	82.31
69.83	96.08	0.638	0.416	84.53
73.51	96.08	0.638	0.416	86.88
77.38	96.08	0.638	0.416	89.35
81.45	96.08	0.638	0.416	91.95
85.74	96.08	0.638	0.416	94.68
90.25	96.08	0.638	0.416	97.56
95.00	96.08	0.638	0.416	100.59
100.00	96.08	0.638	0.416	103.79
105.00	96.08	0.638	0.416	106.98
110.25	96.08	0.638	0.416	110.33
115.76	96.08	0.638	0.416	113.84
121.55	96.08	0.638	0.416	117.54
127.63	96.08	0.638	0.416	121.42
134.01	96.08	0.638	0.416	125.49
140.71	96.08	0.638	0.416	129.76
147.75	96.08	0.638	0.416	134.25
155.13	96.08	0.638	0.416	138.97
162.89	96.08	0.638	0.416	143.92

Insured stock portfolio value at alternative stock index levels, using dynamic portfolio insurance with discrete rebalancing.\*

Index Level $S$	T-Bill Price $Xe^{-rT}$	Stock Portfolio Weight $w_1$	T-Bill Weight $w_2$	Portfolio Value
59.87	96.08	0.001	0.985	94.65
63.02	96.08	0.002	0.984	94.66
66.34	96.08	0.005	0.982	94.67
69.83	96.08	0.014	0.976	94.72
73.51	96.08	0.034	0.961	94.85
77.38	96.08	0.071	0.933	95.12
81.45	96.08	0.135	0.882	95.67
85.74	96.08	0.229	0.802	96.65
90.25	96.08	0.353	0.691	98.24
95.00	96.08	0.495	0.558	100.59
100.00	96.08	0.638	0.416	103.79
105.00	96.08	0.755	0.288	106.98
110.25	96.08	0.847	0.183	110.94
115.76	96.08	0.911	0.105	115.61
121.55	96.08	0.951	0.055	120.89
127.63	96.08	0.973	0.026	126.67
134.01	96.08	0.984	0.011	132.87
140.71	96.08	0.988	0.004	139.46
147.75	96.08	0.990	0.001	146.42
155.13	96.08	0.991	0.000	153.73
162.89	96.08	0.991	0.000	161.41

# Different Types of Portfolio Insurance

Different Forms of  
Equity Portfolio Insurance(a)

<u>Strategy</u>	<u>Advantages</u>	<u>Disadvantages</u>
Buying index puts against a portfolio	Insurance cost determined in advance. Investor captures portfolio nonmarket return.	Listed puts do not trade with expirations greater than 4 months. Must accept the pricing risk of subsequent options purchases.
Buying puts on individual stocks	Portfolio positions protected against decline on a stock-by-stock basis.	Premiums greater than for index puts. Not every stock has a listed put.
Buying index calls and money market securities	Can vary fixed income strategy around the call position; call performance tied to a diversified index.	Cannot capture nonmarket return on a portfolio of stocks. Must accept the pricing risk of subsequent options purchases.
Buying calls on individual stocks and money-market securities	Full participation in all gains from individual stock movement.	Premiums greater than for index calls. Not every stock has a listed call.
Selling stock index futures to create a synthetic put	Can create strike price and expiration date. Will capture portfolio alpha.	Actual cost cannot be predetermined. Must accept pricing risk of the futures contract.
Raising cash by selling stocks to create a synthetic put	No futures pricing risk.	Higher transaction costs and market impact costs in most instances.
Buying stock index futures to own a synthetic call	Can vary fixed income investment. Equity performance tied to a common index such as the S&P 500.	Cost cannot be predetermined. Position is exposed to index futures pricing risk.

(a) In addition to the insurance strategies using listed options and stock index futures, it is possible to create an over-the-counter European or American index option with a longer life than that available in the listed markets.

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# Portfolio Insurance and the Crash of 1987

- Reading (RSD, p.52-58)
  - Dynamic Insurance programs assumed widespread importance starting around the early mid-1980's due to the lack of available exchange traded derivatives for large institutional investors to use in managing portfolios
  - Both specialist firms and large investment banks marketed these strategies to clients, e.g., Black at Goldman-Sachs; Leland, O'Brien, Rubinstein and Associates.
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## What Happened in October 1987? The Prelim's

- As measured by the DJIA, the US equity market had achieved a peak of 2722 in 8/87. P/E ratios for the S&P 500 were averaging 23.
  - Wed. Oct. 15, 1987: news release reporting an unexpectedly large US trade deficit, banks raised prime rates and there was considerable downward pressure on equity prices. The S&P 500 fell from over 314 to below 306. Despite a calming statement by Treasury Secretary Baker on the Thursday, the S&P 500 fell again to 298.
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# Events of October 1987

- Friday Oct. 17, 1987. In the face of the somewhat negative sentiment, the DJIA fell a record 108 points. The S&P 500 started the day at 298 and fell to around 282.
  - Monday Oct 20. At the open the DJIA was down 67 points. The S&P 500 futures contract on the CME fell 18 points at the open. At a time when 100 million share volumes were uncommon events, the NYSE processed 50 million shares in the first half hour.
  - By 2 pm volume had reached 400 million. The final numbers for Oct. 19 were 603 million shares traded, with a drop of 508 points (23%) on the Dow and 80.75 points on the S&P 500, a loss of nearly 30%.
  - Crash of the DOT
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# More Events of October 1987

- As the US market collapse spread overseas, there was complete or almost complete trading halts on Tokyo and Hong Kong. There was an unprecedented drop on the London FT Index.
  - Opening in New York preceded by reassuring statements and actions from the FRB, major banks were lowering prime rates and the NYSE shut down the DOT system to prevent the execution of program trades. A temporary and partial trading halt just after 11 am as the market approached 180 on the S&P futures, while the cash market was trading just below 220. This seemed to spell the end of the crash.
  - There were various reforms introduced as a result of the crash: market circuit breakers, upgrades to computerized trading.
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# Portfolio Insurance for Other Situations

- There is nothing unique about a portfolio of domestic stocks. The notions of portfolio insurance can be applied to any commodity, e.g., insuring the domestic currency value of a foreign bond position.
  - Dynamic portfolio insurance for foreign bonds can be derived using put-call parity for currency options.
  - The objective is to dynamically trade a portfolio composed of domestic bonds and foreign bonds in order to achieve the same payout as a path independent portfolio composed of a foreign bond plus a currency put option.
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# Dynamic Insurance for Foreign Bonds

- If the exchange rate increases, then the dynamic strategy involves selling foreign bonds and buying domestic bonds. If the exchange rate deteriorates, the domestic bond is sold in favour of buying the foreign bond. As before, the Black-Scholes formula for a call can be substituted into the put-call parity condition to derive the appropriate portfolio weights.

$$\begin{aligned} V &= S \exp\{-r_f t^*\} + P = C + X \exp\{-r t^*\} \\ &= S \exp\{-r_f t^*\} N[d_1] - X \exp\{-r t^*\} N[d_2] + X \exp\{-r t^*\} \\ &= S \exp\{-r_f t^*\} N[d_1] + X \exp\{-r t^*\} (1 - N[d_2]) \end{aligned}$$

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# Employee/Executive Stock Options

*“You issue stock options to reduce compensation expense and therefore increase your profitability.”*

Jeffrey Skilling, former CEO of Enron Co.

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# Exotic Options

## ■ Types of Exotic Options

- ❑ Asian Options— two types: Average Strike, Average Rate – either the payout price or the strike price is set by taking the average over the time path
  - ❑ Barrier Options— Knock Out, Knock In
  - ❑ Chooser Option— can choose to be put or call
  - ❑ Compound Option— Option on an option
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# More on Exotic Options

- ❑ Forward Start– Option starts at a future time
  - ❑ Lookback Option– Payout is on Max. (Min.) Price over the path
  - ❑ Shout Option– Can ‘Shout’ an intrinsic value (usually once) during the option life
  - ❑ Russian Option– Perpetual on Max. Price over the path
  - ❑ Others: Spread Options, Exchange Options, Basket Options, Digital Options, Quanto Options.
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