Figure 4.1: **Profit Function for a Long Gold Cash-and-Carry Arbitrage**

DATE	***Cash Position***	***Futures Position***
$t=0$	Borrow $\$[Q_G S(0)]$ at interest rate $r(0,T)$ and buy Q_G ounces of gold at $S(0)$ for storage until $t=T$	Short Q_G units at $F(0,T)$

-- The cash gold position provide no pecuniary return between $t=0$ and $t=T$

$t=T$	Deliver the Q_G units against the maturing futures contract and use the proceeds to repay the maturity value of the loan, $\$[Q_G S(0)]\{1 + r(0,T)\}$	

In this case, the profit function can be specified:

$$\pi(0) \leq \{F(0,T) - S(0)(1 + r(0,T))\} Q_G$$

Consider the gold futures price data for Aug. 8, 1994 given in Figure 4.3. Examining the Dec 1994/Dec 1995 price relationship, for the associated closing prices of \$382.30 and \$403.20, the interest rate implied in the gold futures prices is 5.467%, while Jun95/Jun 96 contracts give an implied interest rate of 5.75% This is consistent with the rates offered on Euro-US deposits (LIBOR) for one year that, on Aug. 8, 1994 were trading at 5 13/16%. The recent relationship between these two rates is discussed further in Sec. 4.2. Examination of the futures price structures for other commodities reveals a range of different relationships. Silver, for example, exhibits a carry rate of 6.45% for the Dec 94/Dec 95. Soybeans exhibit 5.22% for the same Nov 94/Nov 95 differential. The price structure of copper is inverted, with prices for deferred delivery being lower than the nearby contracts. Most of the agricultural commodities exhibit some form of kinking or reversal in the direction of futures prices as delivery dates get more distant, e.g., the CBT wheat contract. The diversity of futures price spread behavior should be apparent. Full carry relationships are the exception, usually applying to the precious metals futures complex.

Because each futures contract involves both a long and a short position, the arbitrage relationship between cash and futures prices involves two trading strategies. In addition to the long-the-cash strategy described above which involves combining a fully leveraged purchase of the spot commodity with a short futures position, it is also possible to combine a short position in the cash commodity with a long futures position, a ***short cash-and-carry arbitrage*** trade. Hence, cash-and-carry arbitrage strategies for futures contracts are said to be *two-sided*, having both a long and a short arbitrage trade to be satisfied. While there are differences across commodities, in practice, due to restrictions on the ability to short the cash commodity, execution of the short *or reverse* cash-and-carry arbitrage can be substantially more difficult than the long arbitrage, that only involves purchase of the spot commodity.[3] In these cases, the cash-futures basis will be determined by what Siegel and Siegel (1990) call *quasi-arbitrages*, typically involving trades with natural hedges in the spot commodity. A classic example would be a jewelry manufacturer holding a gold inventory. If the futures price falls "too far" below full carry, then gold inventories would be sold (or purchases deferred), the funds invested, and the required gold inventory would be hedged using a long futures position. As will be illustrated in Sec. 6.1, banking activities provide a number of excellent examples of natural hedges in currency and other financial commodities.

In the case of gold, numerous mining companies engage in forward selling activities, particularly in North America (where approx. 30% of mine output is sold forward), Australia and, more recently, in South Africa (15% of mine output).[4] Forward sales by mines are combined with deferred delivery of spot transactions, that enable the forward selling mining company to defer or roll forward the delivery date, thereby benefitting from upward price movements. In addition to quasi-arbitrage, in recent years short selling requirements have been facilitated by the use of central bank gold stocks as a source of gold for loan. In addition to other charges, shorting costs involve make a leasing payment to the central bank or other market participant supplying the physical gold for the short. In addition to arbitrage activities, the gold leasing market is also affected by companies involved in gold loans seeking to cover a short term deficiency in physical supplies. Compared to the long arbitrage, the gold lease payment is an offset to the non-financing costs of storage that are associated with carrying a long commodity position through time.

Figure 4.2: **Profit Function for a Short Gold Cash-and-Carry Arbitrage**

DATE	***Cash Position***	***Futures Position***
$t=0$	Borrow Q_G ounces and sell at $S(0)$. Invest the funds received at interest rate $i(0,T)$	Long Q_G ounces at $F(0,T)$
$t=T$	Take delivery of the Q_G units against the maturing futures contract, pay with the proceeds of the investment, $\$[Q_G S(0)]\ \{1 + i(0,T)\}$, returning the Q_G units to settle the short position	

In this case, the profit function can be specified:

$$\pi(0) = \{S(0)(1 + i(0,T)) - F(0,T)\}\ Q_G$$

To execute the *short cash-and-carry arbitrage* for gold, the funds received from the sale of the borrowed gold will be invested at a different, probably lower, rate of interest than the long arbitrage. Taking $i(0,T)$ to be the all-in lending rate and again ignoring incidental costs, the short arbitrage is described in Figure 4.2. Recalling that absence of arbitrage implies $\pi \leq 0$ and observing that $Q > 0$ gives the short cash-and-carry arbitrage restriction on the gold futures price: $F(0,T) \geq S(0)\ \{1 + i(0,T)\}$. If this inequality is violated the short arbitrage can be profitably executed.