PUT-CALL PARITY AND THE EARLY EXERCISE PREMIUM FOR CURRENCY OPTIONS

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Put-call parity is used to study the early exercise premium for currency options traded on the Philadelphia Stock Exchange. Using 564 pairs of call and put options evidence is provided that the early exercise premiums are on average 5.71% for put options and 6.88% for call options. The premiums for both call and put options are strongly related to time to maturity and the interest rate differential. These results are important when using a European option pricing model for the valuation of American options.

Since the collapse of the Bretton Woods system in 1973, companies have looked for ways to hedge their currency risks. This has led to the creation of a large market for currency options. Most of these options are American style, which means that they can be exercised at any time before their maturity date. This causes a problem: The most commonly used formula for the valuation of currency options — the currency option variant of the Black-Scholes model — does not take the premium for early exercise into account.[1] For this reason it is important to get an idea about the size of the early exercise premium and of the factors that determine this premium.

1. For example, see Bollen and Rasiel (2003) for a discussion of this and other currency option models.

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Jorion and Stoughton (1989) compute the early exercise premium as the difference between values of European currency options on the Chicago Board of Options Exchange (CBOE) and American currency options on the Philadelphia Stock Exchange. They find that the premiums for call options are significantly positively related to the ratio of the spot price to the exercise price, the foreign interest rate, and the volatility. Their coefficient for the time to expiration is positive, but just not significant (t-statistic of 1.79). The premium is significantly negatively related to the domestic interest rate. For the same variables for put options, they only find insignificant relationships.

Zivney (1991) provides a different methodology to derive the early exercise premium for index options using the put-call parity condition. He argues that the American option pricing models do not value the early exercise premium appropriately. Therefore, he suggests that this early exercise premium needs to be established empirically. De Roon and Veld (1996) refine Zivney's methodology for index options on an index in which dividends are reinvested. Building on the original study by Zivney (1991), the paper by Engström and Nordén (2000) refines this methodology for equity options. To the best of our knowledge, no study has yet applied this methodology to currency options. This is remarkable, because currency options are more likely to be exercised early than both equity and index options. It is well-known that rational investors only exercise equity options just before an ex-dividend date (see Merton 1973). The early exercise decision for index options is less clear, because indexes consist of different stocks that pay dividends at different times. In general, it is practical to assume a continuous dividend yield for an index (see Hull 2006). Under such assumptions, rational investors will exercise call options early if the dividend yield is higher than the risk-free interest rate. In practice, this will hardly ever be the case. Contrary to equity and index call options, currency options are a prime candidate for early exercise (see, e.g., Bodurtha and Courtadon 1995). Call options on high-interest currencies and put options on low-interest currencies are the most likely to be exercised early because a high-interest rate currency is expected to depreciate relative to the U.S. dollar and a low-interest rate currency is expected to appreciate relative to the U.S. dollar. The objective of this paper is to study the early exercise premiums for currency options using the methodology originally suggested by Zivney (1991).

The empirical results examine 564 pairs of call and put currency options traded on the Philadelphia Stock Exchange between January 2, 1992, and September 24, 1997. Over the admissible range of exchange rate/exercise pairs considered, the average early exercise premium is 5.71% for put options and 6.88% for call options. The premiums for both call and put options are strongly related to the time to maturity and the interest rate differential. These results are largely in line with the earlier results of Jorion and Stoughton (1989). The major difference between our results and theirs is that we find significant coefficients for both call and put options, while they only find significant results for call options. It is important to consider the results of our study when valuing American options with a European model.

The remainder of this paper is organized as follows. Section I presents the methodology. Section II contains the data description, followed by Section III, which covers the results. The paper is concluded in Section IV.

I. METHODOLOGY

Deviations from the European put-call parity are used in order to measure the early exercise premium for currency options. The European put-call parity has the following form:

$$c - p = Se^{-rf(T-t)} - Ke^{-r(T-t)} \tag{1}$$

where *c* is the European call price; *p* is the European put price; *T-t* is the number of years to maturity of the option; *rf* is the foreign risk-free interest rate; r is the domestic risk-free interest rate; *K* is the exercise rate (price) on the put and call options; and *S* is the spot (FX) rate at time t. Following Zivney (1991), the unobserved early exercise premium (*EEP*) can be estimated by subtracting the observed theoretical European option price differentials from the observed American option price differentials, leading to

$$(C-P)-(c-p) = EEP_C - EEP_P = (C-P)-(Se^{-rf(T-t)} - Ke^{-r(T-t)}) \tag{2}$$

where the *EEP* are defined as $C = c + EEP_C$ and $P = p + EEP_P$. Given that (2) produces an estimate for the difference of *EEP*s, properties of the *EEP* specific to the currency options are used to enhance the estimate for the individual *EEP*s. The estimated early exercise premium for the currency options is then checked for consistency with the boundaries for the early exercise premium (EEP) according to the put-call parity.[2,3]

The options data are divided into two subgroups with respect to moneyness. Moneyness is defined as the ratio of the spot price to the exercise price (S/K). We identify in-the-money puts (S<K) and in-the-money calls (S>K). We do not consider the options that are near-the-money, since the early exercise premium for the options in this group can be attributed to both call and put options. The exact definitions

2. Following Merton (1973), Poitras (2002, pp. 477-479) shows that the lower bound for a European currency call option is the maximum of zero and the difference between the spot price discounted at the foreign risk-free interest rate and the exercise price discounted at the domestic risk-free interest rate. The lower bound for a European put option is the maximum of zero and the difference between the discounted exercise price and the discounted spot price. The lower bound for an American option is the same as the lower bound for a European option. In addition, American options can never be worth less than the immediate exercise value.

3. In this context it should be noticed that another use of put-call parities is to derive implicit prices. See, for example, the study of Lung and Nishikawa (2005) on currency options.

are as follows:

Group 1: In-the-money put S/K<0.995

Group 2: In-the-money call S/K>1.005

A multiple regression model is used to test four hypotheses about the early exercise premium. The dependent variable in this model is the relative early exercise premium (REEP). The premium REEP is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity.

$$REEP = \alpha + \beta_1(r-r_f) + \beta_2(T-t) + \beta_3(S/K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

The first hypothesis is that the REEP depends on the domestic and foreign interest rates. For calls, the REEP should increase when the domestic interest rate is lower than the foreign rate. In that case the lower boundary becomes

$$Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

This will be lower than S-K. In that case there is an incentive for the call to be exercised early. This premium will rise if the difference becomes larger. The situation for the put is the reverse, because if the call is in-the-money, the put is out-of-the-money. Here, if the domestic rate is higher than the foreign rate, the REEP will be larger for the put. The second hypothesis is that the early exercise premium increases with time to maturity. This holds for both calls and puts. The third hypothesis is that the REEP should increase for calls as the ratio of the spot price to the exercise price (S/K) increases. When the spot price is higher than the exercise price, calls are in-the-money and thus are more likely to be exercised. However, if the option gets very far in-the-money, the value of the call reduces to

$$Se^{-r_f(T-t)} - Ke^{-r(T-t)}$$

which is higher than the value of early exercise: S-K.[4] In such a case, early exercise will not be optimal. The effect for puts is the opposite. The early exercise premium should decrease in absolute terms as the ratio of the spot price to the exercise price increases because puts are moving in the direction of out-of-the-money. The fourth hypothesis is that the REEP increases if the volatility increases. This hypothesis is not obvious. Jorion and Stoughton (1989) argue that a greater volatility raises the optimal exercise boundary for all maturities. However, it also increases the dispersion of future spot prices which makes it more likely that this boundary is struck before the option's maturity. According to Jorion and Stoughton (1989), the net effect is that increases in volatility also increase the value of the early exercise

4. This is the case if $r>r_f$.

premium. The volatility is estimated as the implied volatility of the call option with the same exercise price and time to maturity on the day before the estimation day ($\sigma_{call,t-1}$). Notice that we also use the implied volatility of call options for the put regressions; the implied volatility of a call option gives a better estimate of the future volatility than the implied volatility of a put option. Finally, to take into account the effects of possible heteroskedasticity and autocorrelation, Newey-West standard errors for the regression estimates are reported.

In addition to the regression specification mentioned above, we also tried two alternative specifications. Both are related to the non-linearity of the hypothesized relationship between the early exercise premium and moneyness. In the first alternative specification we use S-K instead of S/K. This is in line with Zivney (1991) who also uses S-K. The second alternative specification uses log(S/K) instead of S/K. This is in line with Jorion and Stoughton (1989). The justification for the second alternative specification is that taking a logarithm tends to reduce the skewness and makes the variable more normally distributed.

II. DATA DESCRIPTION

Closing prices for currency options traded on the Philadelphia Stock Exchange (PHLX) are used for the period from January 2, 1992, to September 24, 1997. Over this period, the currency options on the PHLX experienced active trading and relatively high volumes. We use data for the six currencies that are most actively traded, that is, the Australian dollar, the British pound, the Canadian dollar, the Deutsche (German) mark, the Japanese yen, and the Swiss franc. Data on the exercise prices, expiration dates, spot prices, and the closing prices of the options are derived from the PHLX database. The original database consists of 2,389 pairs of American call and put options that have the same trade date, underlying value, and exercise price. We first eliminate the options that are at-the-money (1,041), because in this case it is not possible to attribute the EEP solely to either puts or calls. From the remaining 1,348 options we eliminate the options with prices that are not consistent with the boundaries of the American put-call parity (325). Finally, we eliminate 459 observations because they have a negative EEP, which is likely to be caused by nonsynchronous trading of the options and/or the currencies. The remaining sample consists of 564 observations.

Three-month eurodollar interest rates, obtained from the U.S. Federal Reserve Board website, are used as the domestic interest rate.[5] The Eurodollar interest rates are applied to the Covered Interest Rate Parity to determine the foreign interest rates. For this purpose we use futures on currencies as traded on the International Money Market Division of the Chicago Mercantile Exchange. These futures have the same expiration cycle as the options traded on the PHLX. The futures prices are obtained from the Thomson Financial Datastream database.

5. El-Mekkaoui and Flood (1998) argue that eurodollar rates are more appropriate than T-bill rates, because due to regulation and market structure the domestic T-bill markets may be less efficient than the Eurodollar markets.

Table 1: Market Valuation of Early Exercise Premium.

	No. of observations	*Average US minus foreign interest rate, %*	*Average premium as % of option price*	*Median premium as % of option price*	*Standard deviation of premiums*
Group 1: Puts					
Overall					
Aus. Dollar	295	1.91	5.71	4.34	5.68
British Pound	17	-1.62	4.55	3.93	3.69
Can. Dollar	35	-1.67	2.76	1.74	3.14
Deutsche Mark	24	-0.41	4.99	3.04	5.28
Japanese Yen					
Swiss Franc	71	1.72	4.77	4.17	3.73
	101	4.14	7.94	6.43	5.87
	47	2.49	5.33	4.87	3.84
Group 2: Calls					
Overall					
Aus. Dollar	269	-1.37	6.88	4.12	7.21
British Pound	23	-1.54	6.20	5.25	4.08
Can. Dollar	75	-2.43	6.23	4.11	6.35
Deutsche Mark	19	-1.61	7.78	5.60	6.89
Japanese Yen					
Swiss Franc	79	-2.41	10.12	6.38	9.41
	26	1.32	3.88	2.59	3.94
	47	0.75	4.11	2.60	4.51

This table includes the relative early exercise premium of put and call options traded on the Philadelphia Stock Exchange between January 2, 1992 and September 24, 1997. This premium is calculated as the absolute value of (a) – (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity. In total 564 pairs of options are used with an identical exercise price and time to maturity. Puts are taken into account if the ratio of spot price (S) to exercise price (K), $S/K < 0.995$. Calls are taken into account if $S/K > 1.005$.

III. RESULTS

Table 1 summarizes the valuation of early exercise premium in the two groups. The table shows average premiums as a percentage of the average call or put price. In Table 1 we find that the average early exercise premium as a percentage of the put prices is 5.71%. The average early exercise premium as a percentage of the call prices is 6.88%. The overall results in Table 1 are somewhat different from the results of Zivney (1991) for index options in the sense that the early exercise premium is larger for calls than for puts. The large premiums for the Deutsche mark in both groups are noteworthy. These are caused by the large fluctuations in German interest rates in 1992–1993. Even apart from the Deutsche mark options, the differences within each group between the early exercise premiums for the different currencies can be substantial. For example, in Group 1 (puts) the early

Table 2. Descriptive Statistics.

Puts	Time to maturity (T-t)	Ratio of spot price to exercise price (S/K)	Spot price minus exercise price (S-K)	Log of spot price to exercise price log(S/K)	Early exercise premium as percentage of option price	Average US minus foreign interest rate, %	Call option volatility in period t-1 (σ_{t-1})	Closing price
Mean	0.28	0.982	-0.018	-0.008	5.71	1.91	0.12	2.76
Median	0.18	0.987	-0.012	-0.006	4.34	2.32	0.11	2.16
Standard deviation	0.27	0.015	0.018	0.007	4.95	2.61	0.03	1.95
Minimum	0.01	0.910	-0.129	-0.041	0.00	-5.95	0.03	0.48
Maximum	0.99	0.995	-0.003	-0.002	22.00	6.05	0.30	13.68
1st percentile	0.02	0.924	-0.091	-0.034	0.09	-3.20	0.04	0.58
99th percentile	0.93	0.995	-0.003	-0.002	20.45	5.70	0.20	9.06
Calls								
Mean	0.35	1.021	0.020	0.009	6.88	-1.37	0.12	3.18
Median	0.22	1.014	0.012	0.006	4.12	-1.43	0.12	2.48
Standard deviation	0.29	0.019	0.022	0.008	7.21	2.87	0.03	2.32
Minimum	0.01	1.005	0.003	0.002	0.10	-7.19	0.05	0.47
Maximum	1.01	1.121	0.148	0.049	34.09	5.40	0.22	15.45
1st percentile	0.02	1.005	0.004	0.002	0.15	-6.72	0.06	0.79
99th percentile	0.96	1.105	0.115	0.043	29.81	4.62	0.21	11.35

This table includes the descriptive statistics for the sample of 564 put and call options traded on the Philadelphia Stock Exchange between January 2, 1992, and September 24, 1997.

exercise premium for the Japanese yen is 7.94% while the premium for the British pound is only 2.76%. In Group 2 (calls), the premium for the Deutsche mark is 10.12%, while it is only 3.88% for the Japanese yen. These large differences are consistent with the fact that there is significant cross-sectional variation in foreign interest rates. As a result the differences in early exercise premiums for foreign currency options are expected to be greater than, for example, for stock options.

The column for the median premiums shows that in all cases the median early exercise premium is slightly below the average premium. Overall, the median premium is 4.34% for put options (compared to an average premium of 5.71%) and 4.12% for call options (compared to an average premium of 6.88%). The column for the standard deviation of the premiums shows that the standard deviations seem sufficiently high to make the regression analysis useful.

Table 2 includes descriptive statistics for our sample of 564 put and call options. The descriptive statistics show that the time to maturity of the options in our sample varies between a few days and a little more than a year. The ratio of spot price to exercise price shows that most options are not very far in-the-money or out-of-the-money, since the ratio never exceeds 1.121 (for call options) and is never below 0.910 (for put options). Table 3 shows the results of the multiple linear regressions of the relative early exercise premium (REEP) on the different parameters for groups of puts and calls.

We first discuss the results for the regressions in which moneyness is defined as S/K. The results for the interest rate differential confirm the first hypothesis. The REEP is positively related to the difference between the domestic (US) and the foreign interest rate for puts and negatively for calls. This coefficient is significant at the 5%-level for puts and at the 1%-level for calls. The results for time to maturity for both calls and puts give the hypothesized sign as well. Moreover, in both cases the coefficient is significant at the 1%-level. The results for the moneyness give the hypothesized sign for put options. This coefficient is significantly different from zero at the 5%-level. The sign for call options is negative and insignificant, whereas we hypothesized a positive sign. This result is different from Zivney (1991), who finds that the coefficient of the moneyness is highly significant. This may be caused by the fact that he considers index options, whereas we consider currency options. Another potential explanation for the unexpected result for call options is that the hypothesized relationship only holds if the options are not very far in-the-money. However, given that the maximum value for S/K is only 1.121, this explanation is not likely to hold. The results for the fourth hypothesis give the expected significantly positive coefficient for call options. However, the effect for put options is significantly negative, which is contrary to our hypothesis.[6] In this context it is interesting to notice that the empirical study of Jorion and Stoughton (1989) also finds a negative, albeit insignificant, relation between the early exercise premium for put options and volatility.[7]

6. A possible reason for this is that the data from American options are used in order to calculate implied volatilities. This may give volatility results that are not entirely reliable.
7. The t-statistics in their regression is -1.02.

Table 3. Regression Results.
This table includes the results for the following regression equations that explain the relative early exercise premium (REEP) of put and call options traded on the Philadelphia Stock Exchange between January 2, 1992, and September 24, 1997:

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(S / K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(S - K) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

$$REEP = \alpha + \beta_1(r - r_f) + \beta_2(T - t) + \beta_3(\log(S / K)) + \beta_4(\sigma_{call,t-1}) + \varepsilon$$

	Constant	*r-r_f*	*T-t*	*S/K*	*S-K*	*Log (S/K)*	$\sigma_{call,t-1}$	R^2
Group 1: Puts								
Coeff.	0.49	0.77	0.05	-0.43			-0.43	
t-stat.	2.43*	5.74*	3.49**	-2.13*			-4.20**	0.30
Coeff.	0.06	0.79	0.05		-0.03		-0.36	
t-stat.	6.34**	5.80**	3.46*		-0.20		3.76**	0.28
Coeff.	0.07	0.77	0.05			-0.41	-0.43	
t-stat.	6.67*	5.73*	3.49**			-2.14*	-4.20**	0.30
Group 2: Calls								
Coeff.	0.23	-0.90	0.10	-0.24			0.38	
t-stat.	1.11	-6.33**	4.92**	-1.16			2.80**	0.43
Coeff.	-0.02	-0.92	0.10		-0.31		0.40	
t-stat.	-1.16	-6.21**	5.01**		-1.49		2.98**	0.43
Coeff.	-0.02	-0.90	0.10			-0.25	0.38	
t-stat.	-1.10	-6.33**	4.92**			-1.10	2.79**	0.43

The premium REEP is calculated as the absolute value of (a) - (b) divided by the option price in which (a) is the difference between the American call and put price and (b) is the difference between the call and put price as implied by the put-call parity. In total 564 pairs of options are used with an identical exercise price and time to maturity. There are 295 observations for put options and 269 observations for call options. T-t is the remaining time to maturity of the option; r is the domestic (US) risk-free interest rate, rf is the foreign risk-free interest rate and $\sigma_{call,t-1}$ is the implied volatility of the call option with the same exercise price and time to maturity on the day before the estimation day. Puts are taken into account if the ratio of spot price (S) to exercise price (K), S/K < 0.995. Calls are taken into account if S/K>1.005 The equations are estimated using Newey-West standard errors. * = significant at the 5%-level; ** = significant at the 1%-level.

In addition, we include the results for regressions with alternative specifications for the relationship between S and K. In the first of these specifications, we replace S/K by S-K and in the second we replace S/K by log(S/K). The results for these alternative specifications are largely the same as for the original specification. The signs and the significance for the coefficients for time to maturity, interest rate differential, and volatility are the same. Only the level of significance changes between regressions (in all cases from 1% to 5% and vice versa). However, for put options the coefficient for the relationship between the spot price and the exercise price is no longer significant if we substitute (S/K) by (S-K). In the regression for log(S/K), the coefficient has again its hypothesized significant negative coefficient. All three different specifications of the regressions for call options show an insignificant negative coefficient for the relationship between S and K.

A final remark on the regression analysis concerns the R^2s of the regressions: These are high and vary between 0.28 and 0.30 for the put options. The regressions for the call options all show an R^2 of 0.43, indicating that the variables in these regressions explain a large part of the variation in REEP.

IV. CONCLUSIONS

Given the lack of a suitable formula for the valuation of American currency options, practitioners generally use a variant of the Black-Scholes option pricing model for the valuation of such options. For this reason it is important to acquire knowledge on the early exercise premium for American currency options. We use put-call parities to estimate these premiums and find that they are slightly higher for call options than for put options. We also find that these premiums are significantly influenced by time to maturity and the interest rate differential. This knowledge is important when valuing American currency options with a European model.

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8. In Table 1, the average price of the December 2006 futures contracts is €19.34 greater than €16.19 of the December 2007 futures. This is because the December 2006 futures cease trading at November 29, 2006, and there is also a downward trend for December 2007 futures after December 2006.

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