

Table 1

**Partial durations,  $\{n_t\}$  and extreme bounds  
for the 5 Year Zero Coupon Liability Using  
High and Low Surplus Examples\***

Date	High Surplus		Low Surplus	
	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>
0.5	0.687	0.0775	2.292	0.0236
1.0	0.075	0.0085	0.870	0.0089
1.5	0.107	0.0121	1.238	0.0128
2.0	0.136	0.0153	1.569	0.0162
2.5	0.161	0.0182	1.863	0.0192
3.0	0.183	0.0206	2.111	0.0217
3.5	0.201	0.0226	2.318	0.0239
4.0	0.214	0.0241	2.472	0.0254
4.5	0.226	0.0255	2.611	0.0269
5.0	-7.933	-0.89545	-86.115	-0.88658
5.5	0.244	0.0275	2.815	0.0290
6.0	0.246	0.0277	2.837	0.0292
6.5	0.247	0.0279	2.854	0.0294
7.0	0.246	0.0278	2.845	0.0293
7.5	0.243	0.0275	2.810	0.0289
8.0	0.242	0.0273	2.792	0.0287
8.5	0.239	0.0269	2.758	0.0284
9.0	0.231	0.0260	2.664	0.0274
9.5	0.221	0.0249	2.551	0.0263
10.0	3.786	0.42733	43.742	0.45033
Extreme Duration Bounds:	<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 8.860$		<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 97.131$	
Surplus:	33.8387		4.66735	

\* The High Surplus Portfolio is composed of (\$45.581) 1/2 year and (\$46.5963) 10 year bonds. The Low Surplus Portfolio is composed of (\$17.8382) 1/2 year and (\$74.343) 10 year bonds. The liability for the High Surplus Portfolio is a 5 year zero coupon bond with \$100 par value and market value of \$58.3427. The liability for the Low Surplus Portfolio is a 5 year zero coupon bond with \$150 par value and market value of \$87.514. The extreme Cauchy bounds are derived using  $\nabla N \nabla = 1$ .

Table 2

Partial durations,  $\{n_t\}$  and extreme bounds  
for the 5 Year Zero Coupon Liability Using  
the Maturity Bond and Split Maturity Examples\*

Date	Maturity Bond		Split Maturity	
	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>
0.5	0.476	0.0183	0.251	0.0063
1.0	0.454	0.0174	0.446	0.0112
1.5	0.646	0.0248	0.635	0.0159
2.0	0.818	0.0314	0.804	0.0201
2.5	0.971	0.0373	0.955	0.0239
3.0	1.101	0.0422	7.737	0.1939
3.5	1.208	0.0464	0.834	0.0209
4.0	1.288	0.0494	0.889	0.0223
4.5	1.361	0.0522	0.939	0.0235
5.0	-23.866	-0.91558	-36.998	-0.92716
5.5	0.636	0.0244	1.012	0.0254
6.0	0.641	0.0246	1.020	0.0256
6.5	0.645	0.0247	1.026	0.0257
7.0	0.643	0.0247	8.177	0.2049
7.5	0.635	0.0244	0.601	0.0151
8.0	0.631	0.0242	0.597	0.0150
8.5	0.623	0.0239	0.590	0.0148
9.0	0.602	0.0231	0.569	0.0143
9.5	0.577	0.0221	0.545	0.0137
10.0	9.884	0.37921	9.350	0.23430
Extreme Duration Bounds:	<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 26.07$		<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 39.905$	
Surplus:	10.32685		10.91804	

\* The Maturity Bond Portfolio is composed of (\$5.13) 1/2 year, (\$55.54) 5 year and (\$37.172) 10 year bonds. The Split Maturity Portfolio is composed of (\$0.4105) 1/2 year, (\$33.8435) 3 year, (27.0) 7 year and (\$37.172) 10 year bonds. The liability is a 5 year zero coupon bond with \$150 par value and market value of \$87.5121. The extreme Cauchy bounds are derived using  $\nabla N \nabla = 1$ .

Table 3

**Partial durations,  $\{n_t\}$  and extreme bounds  
for the 10 Year Annuity Liability Immunized with  
the Maturity Bond and Low Surplus Examples\***

<u>Date</u>	Maturity Bond		Low Surplus	
	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>	<u><math>D_t</math></u>	<u><math>n_t^*</math></u>
0.5	1.034	0.0841	2.856	0.0869
1.0	-0.275	-0.0224	-0.705	-0.0214
1.5	-0.392	-0.0319	-1.003	-0.0305
2.0	-0.497	-0.0404	-1.271	-0.0387
2.5	-0.590	-0.0480	-1.509	-0.0459
3.0	-0.668	-0.0543	-1.710	-0.0520
3.5	-0.734	-0.0597	-1.877	-0.0571
4.0	-0.782	-0.0636	-2.001	-0.0609
4.5	-0.826	-0.0672	-2.115	-0.0643
5.0	8.257	0.67159	-2.201	-0.0670
5.5	-1.401	-0.1139	-2.280	-0.0694
6.0	-1.412	-0.1148	-2.298	-0.0699
6.5	-1.420	-0.1155	-2.311	-0.0703
7.0	-1.415	-0.1151	-2.304	-0.0701
7.5	-1.398	-0.1137	-2.276	-0.0693
8.0	-1.389	-0.1130	-2.262	-0.0688
8.5	-1.372	-0.1116	-2.234	-0.0680
9.0	-1.325	-0.1078	-2.157	-0.0656
9.5	-1.269	-0.1032	-2.066	-0.0629
10.0	7.855	0.63888	31.647	0.9629
Extreme Duration Bounds:	<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 12.29$		<u>Cauchy = <math>\nabla D \nabla</math></u> $\pm 32.87$	
Surplus:	10.5979		4.68	

\* The Maturity Bond Portfolio is composed of (\$25.97) 1/2 year, (\$34.956) 5 year and (\$37.172) 10 year bonds. The Low Surplus Portfolio is composed of (\$31.388) 1/2 year and (\$60.791) 10 year bonds. The liability has market value of \$87.500 with annual coupon, paid semi-annually, of \$14.96. The extreme bounds are derived using  $\nabla N \nabla = 1$ .

Table 4

**Time Values, Convexity and Other Measures  
for the Immunizing Portfolios\***

<u>TABLE 1</u>	<u>High Surplus</u>	<u>Low Surplus</u>
Surplus	33.84	4.667
Time Value = $N_0'T$	0.03559	0.0139
$N_0' C_T N_0$	17.18	221.94
$N^* ' C_T N^*$	-25.34	-259.59
Max $CON_t$	37.23	430.09
Min $CON_t$	-41.34	-448.79
Cauchy Duration Bound	$\pm 8.86$	$\pm 97.13$
$N_0'D_T$	-0.000	-0.102
<u>TABLE 2</u>	<u>Maturity Bond</u>	<u>Split Maturity</u>
Surplus	10.327	10.918
Time Value = $N_0'T$	0.0332	0.0324
$N_0' C_T N_0$	42.37	44.21
$N^* ' C_T N^*$	-85.39	-142.06
Max $CON_t$	97.19	91.93
Min $CON_t$	-124.38	-192.81
Cauchy Duration Bound	$\pm 26.07$	$\pm 39.90$
$N_0'D_T$	-0.025	-0.022
<u>TABLE 3</u>	<u>Maturity Bond</u>	<u>Low Surplus</u>
Surplus	10.598	4.68
Time Value = $N_0'T$	0.0362	0.0255
$N_0' C_T N_0$	11.96	109.34
$N^* ' C_T N^*$	21.61	287.65
Max $CON_t$	77.23	311.17
Min $CON_t$	-11.89	-19.36
Cauchy Duration Bound	$\pm 12.29$	$\pm 32.86$
$N_0'D_T$	-0.019	-0.077

\* See Notes to Tables 1-3. Making appropriate adjustment for semiannual payments, the time value  $N_0'T$  is defined in (3). The sum of the partial convexities is  $N_0'C_T N_0$ . The quadratic form,  $N^* ' C_T N^*$ , is the sum of squares for the relevant  $N^*$  from Tables 1-3 multiplied term-by-term with the appropriate partial convexities. Max CON and Min CON are the maximum and minimum individual partial convexities.

Table 5

**Partial Durations and Convexities  
for the Immunizing Portfolios under  
Different Yield Curve Shift Assumptions\***

<u>TABLE 1</u>	<u>High Surplus</u>			<u>Low Surplus</u>		
Surplus		33.84			4.667	
	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>
$N_i ' D_T$	-0.606	-.879	-1.875	-8.205	-11.127	-23.75
$N_i ' C_T N_i$	-0.828	6.082	9.778	-9.398	71.151	112.23
$N_0 ' T_i$	.04572	.0439	.04461	.0218	.0204	.0277
Cauchy Bound ( $\nabla D_T$ )		±8.86			±97.13	
$N^* ' C_T N^*$		-25.34			-259.59	
$N_0 ' C_T N_0$		17.18			221.94	
Time Value = $N_0 ' T$		.03559			.0137	

  

<u>TABLE 2</u>	<u>Maturity Bond</u>			<u>Split Maturity</u>		
Surplus		10.327			10.918	
	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>
$N_i ' D_T$	-1.532	-2.328	-4.955	-1.451	-2.478	-5.289
$N_i ' C_T N_i$	-1.976	15.667	25.489	-2.039	16.281	25.51
$N_0 ' T_i$	.03637	.03432	.03699	.03554	.03337	.03591
Cauchy Bound ( $\nabla D_T$ )		±26.06			±39.90	
$N^* ' C_T N^*$		-85.39			-142.06	
$N_0 ' C_T N_0$		42.37			44.21	
Time Value = $N_0 ' T$		.0332			.0324	

  

<u>TABLE 3</u>	<u>Maturity Bond</u>			<u>Low Surplus</u>		
Surplus		10.598			4.68	
	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>	<u>YC1</u>	<u>YC2</u>	<u>YC3</u>
$N_i ' D_T$	-0.874	-0.638	-1.360	-5.086	-5.287	-11.298
$N_i ' C_T N_i$	-0.847	5.902	12.01	-5.039	36.520	62.55
$N_0 ' T_i$	.03868	.03704	.03796	.0308	.0300	.03578
Cauchy Bound ( $\nabla D_T$ )		±12.29			±32.86	
$N^* ' C_T N^*$		21.61			267.49	
$N_0 ' C_T N_0$		11.96			109.34	
Time Value = $N_0 ' T$		.0362			.0255	

\* See Notes to Tables 1-3. YC1 has the spot rate curve flattening up, with the T=10 rate constant; YC2 has the spot rate curve flattening down with the T=.5 (6 month) rate constant; and, YC3 has a flattening pivot with the T=5 year rate

constant.  $N_0 T_i$  evaluates (3) using the  $N_i$  shifted spot rates.