Is math invented or discovered? A leading astrophysicist suggests that the answer to the millennia-old question is both

MOST OF US TAKE IT FOR GRANTED that math works--that scientists can devise formulas to describe subatomic events or that engineers can calculate paths for spacecraft. We accept the view, initially espoused by Galileo, that mathematics is the language of science and expect that its grammar explains experimental results and even predicts novel phenomena. The power of mathematics, though, is nothing short of astonishing. Consider, for example, Scottish physicist James Clerk Maxwell's famed equations: not only do these four expressions summarize all that was known of electromagnetism in the 1860s, they also anticipated the existence of radio waves two decades before German physicist Heinrich Hertz detected them. Very few languages are as effective, able to articulate volumes' worth of material so succinctly and with such precision. Albert Einstein pondered, "How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?"

As a working theoretical astrophysicist, I encounter the seemingly "unreasonable effectiveness of mathematics," as Nobel laureate physicist Eugene Wigner called it in 1960, in every step of my job. Whether I am struggling to understand which progenitor systems produce the stellar explosions known as type Ia supernovae or calculating the fate of Earth when our sun ultimately becomes a red giant, the tools I use and the models I develop are mathematical. The uncanny way that math captures the natural world has fascinated me throughout my career, and about 10 years ago I resolved to look into the issue more deeply.

At the core of this mystery lies an argument that mathematicians, physicists, philosophers and cognitive scientists have had for centuries: Is math an invented set of tools, as Einstein believed? Or does it actually exist in some abstract realm, with humans merely discovering its truths? Many great mathematicians--including David Hilbert, Georg Cantor and the group known as Nicolas Bourbaki--have shared Einstein's view, associated with a school of thought called Formalism. But other illustrious thinkers--among them Godfrey Harold Hardy, Roger Penrose and Kurt Gödel--have held the opposite view, Platonism.

This debate about the nature of mathematics rages on today and seems to elude an answer. I believe that by asking simply whether mathematics is invented or discovered, we ignore the possibility of a more intricate answer: both invention and discovery play a crucial role. I posit that together they account for why math works so well. Although eliminating the dichotomy between invention and discovery does not fully explain the unreasonable effectiveness of mathematics, the problem is so profound that even a partial step toward solving it is progress.

INVENTION AND DISCOVERY

MATHEMATICS is unreasonably effective in two distinct ways, one I think of as active and the other as passive. Sometimes scientists create methods specifically for quantifying real-world phenomena. For example, Isaac Newton formulated calculus for the purpose of capturing motion and change, breaking them up into infinitesimally small frame-by-frame sequences. Of course, such active inventions are effective; the tools are, after all, made to order. What is surprising, however, is their stupendous accuracy in some cases. Take, for instance, quantum electrodynamics, the mathematical theory developed to describe how light and matter interact. When scientists use it to calculate the magnetic moment of the electron, the theoretical value agrees with the most recent experimental value--measured at 1.00115965218073 in the appropriate units in 2008--to within a few parts per trillion!

Even more astonishing, perhaps, mathematicians sometimes develop entire fields of study with no application in mind, and yet decades, even centuries, later physicists discover that these very branches make sense of their observations. Examples of this kind of passive effectiveness abound. French mathematician Évariste Galois, for example, developed group theory in the early 1800s for the sole purpose of determining the solvability of polynomial equations. Very broadly, groups are algebraic structures made up of sets of objects (say, the integers) united under some operation (for instance, addition) that obey specific rules (among them the existence of an identity element such as 0, which, when added to any integer, gives back that same integer). In 20th-century physics, this rather abstract field turned out to be the most fruitful way of categorizing elementary particles--the building blocks of matter. In the 1960s physicists Murray Gell-Mann and Yuval Ne'eman independently showed that a specific group, referred to as SU(3), mirrored a behavior of subatomic particles called hadrons--a connection that ultimately laid the foundations for the modern theory of how atomic nuclei are held together.

The study of knots offers another beautiful example of passive effectiveness. Mathematical knots are similar to everyday knots, except that they have no loose ends. In the 1860s Lord Kelvin hoped to describe atoms as knotted tubes of ether. That misguided model failed to connect with reality, but mathematicians continued to analyze knots for many decades merely as an esoteric arm of pure mathematics. Amazingly, knot theory now provides important insights into string theory and loop quantum gravity--our current best attempts at articulating a theory of space-time that reconciles quantum mechanics with general relativity. Similarly, English mathematician Hardy's discoveries in number theory advanced the field of cryptography, despite Hardy's earlier proclamation that "no one has yet discovered any warlike purpose to be served by the theory of numbers." And in 1854 Bernhard Riemann described non-Euclidean geometries--curious spaces in which parallel lines converge or diverge. More than half a century later Einstein invoked those geometries to build his general theory of relativity.

A pattern emerges: humans invent mathematical concepts by way of abstracting elements from the world around them--shapes, lines, sets, groups, and so forth--either for some specific purpose or simply for fun. They then go on to discover the connections among those concepts. Because this process of inventing and discovering is man-made--unlike the kind of discovery to which the Platonists subscribe--our mathematics is ultimately based on our perceptions and the mental pictures we can conjure. For instance, we possess an innate talent, called subitizing, for instantly recognizing quantity, which undoubtedly led to the concept of number. We are very good at perceiving the edges of individual objects and at distinguishing between straight and curved lines and between different shapes, such as circles and ellipses-abilities that probably led to the development of arithmetic and geometry. So, too, the repeated human experience of cause and effect at least partially contributed to the creation of logic and, with it, the notion that certain statements imply the validity of others.

SELECTION AND EVOLUTION

MICHAEL ATIYAH, one of the greatest mathematicians of the 20th century, has presented an elegant thought experiment that reveals just how perception colors which mathematical concepts we embrace--even ones as seemingly fundamental as numbers. German mathematician Leopold Kronecker famously declared, "God created the natural numbers, all else is the work of man." But imagine if the intelligence in our world resided not with humankind but rather with a singular, isolated jellyfish, floating deep in the Pacific Ocean. Everything in its experience would be continuous, from the flow of the surrounding water to its fluctuating temperature and pressure. In such an environment, lacking individual objects or indeed anything discrete, would the concept of number arise? If there were nothing to count, would numbers exist?

Like the jellyfish, we adopt mathematical tools that apply to our world--a fact that has undoubtedly contributed to the perceived effectiveness of mathematics. Scientists do not choose analytical methods arbitrarily but rather on the basis of how well they predict the results of their experiments. When a tennis ball machine shoots out balls, you can use the natural numbers 1, 2, 3, and so on, to describe the flux of balls. When firefighters use a hose, however, they must invoke other concepts, such as volume or weight, to render a meaningful description of the stream. So, too, when distinct subatomic particles collide in a particle accelerator, physicists turn to measures such as energy and momentum and not to the end number of particles, which would reveal only partial information about how the original particles collided because additional particles can be created in the process.

Over time only the best models survive. Failed models--such as French philosopher René Descartes's attempt to describe the motion of the planets by vortices of cosmic matter--die in their infancy. In contrast, successful models evolve as new information becomes available. For instance, very accurate measurements of the precession of the planet Mercury necessitated an overhaul of Newton's theory of gravity in the form of Einstein's general relativity. All successful mathematical concepts have a long shelf life: the formula for the surface area of a sphere remains as correct today as it was when Archimedes proved it around 250 B.C. As a result, scientists of any era can search through a vast arsenal of formalisms to find the most appropriate methods.

Not only do scientists cherry-pick solutions, they also tend to select problems that are amenable to mathematical treatment. There exists, however, a whole host of phenomena for which no accurate mathematical predictions are possible, sometimes not even in principle. In economics, for example, many variables--the detailed psychology of the masses, to name one--do not easily lend themselves to quantitative analysis. The predictive value of any theory relies on the constancy of the underlying relations among variables. Our analyses also fail to fully capture systems that develop chaos, in which the tiniest change in the initial conditions may produce entirely different end results, prohibiting any long-term predictions. Mathematicians have developed statistics and probability to deal with such shortcomings, but mathematics itself is limited, as Austrian logician Gödel famously proved.

SYMMETRY OF NATURE

THIS CAREFUL SELECTION of problems and solutions only partially accounts for mathematics's success in describing the laws of nature. Such laws must exist in the first place! Luckily for mathematicians and physicists alike, universal laws appear to govern our cosmos: an atom 12 billion light-years away behaves just like an atom on Earth; light in the distant past and light today share the same traits; and the same gravitational forces that shaped the universe's initial structures hold sway over present-day galaxies. Mathematicians and physicists have invented the concept of symmetry to describe this kind of immunity to change.

The laws of physics seem to display symmetry with respect to space and time: They do not depend on where, from which angle, or when we examine them. They are also identical to all observers, irrespective of whether these observers are at rest, moving at constant speeds or accelerating. Consequently, the same laws explain our results, whether the experiments occur in China, Alabama or the Andromeda galaxy--and whether we conduct our experiment today or someone else does a billion years from now. If the universe did not possess these symmetries, any attempt to decipher nature's grand design--any mathematical model built on our observations--would be doomed because we would have to continuously repeat experiments at every point in space and time.

Even more subtle symmetries, called gauge symmetries, prevail within the laws that describe the subatomic world. For instance, because of the fuzziness of the quantum realm, a given particle can be a negatively charged electron or an electrically neutral neutrino, or a mixture of both--until we measure the electric charge that distinguishes between the two. As it turns out. the laws of nature take the same form when we interchange electrons for neutrinos or any mix of the two. The same holds true for interchanges of other fundamental particles. Without such gauge symmetries, it would have been very difficult to provide a theory of the fundamental workings of the cosmos. We would be similarly stuck without locality--the fact that objects in our universe are influenced directly only by their immediate surroundings rather than by distant phenomena. Thanks to locality, we can attempt to assemble a mathematical model of the universe much as we might put together a jigsaw puzzle, starting with a description of the most basic forces among elementary particles and then building on additional pieces of knowledge.

Our current best mathematical attempt at unifying all interactions calls for yet another symmetry, known as supersymmetry. In a universe based on supersymmetry, every known particle must have an as yet undiscovered partner. If such partners are discovered (for instance, once the Large Hadron Collider at CERN near Geneva reaches its full energy), it will be yet another triumph for the effectiveness of mathematics.

I started with two basic, interrelated questions: Is mathematics invented or discovered? And what gives mathematics its explanatory and predictive powers? I believe that we know the answer to the first question. Mathematics is an intricate fusion of inventions and discoveries. Concepts are generally invented, and even though all the correct relations among them existed before their discovery, humans still chose which ones to study. The second question turns out to be even more complex. There is no doubt that the selection of topics we address mathematically has played an important role in math's perceived effectiveness. But mathematics would not work at all were there no universal features to be discovered. You may now ask: Why are there universal laws of nature at all? Or equivalently: Why is our universe governed by certain symmetries and by locality? I truly do not know the answers, except to note that perhaps in a universe without these properties, complexity and life would have never emerged, and we would not be here to ask the question.

MORE TO EXPLORE

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SCIENTIFIC AMERICAN ONLINE Is mathematics invented, discovered, both or neither? See examples of remarkable mathematical structures that invite this question at ScientificAmerican.com/aug11/livio

PHOTO (BLACK & WHITE): Fractals, such as this stack of spheres created using 3-D modeling software, are one of the mathematical structures that were invented for abstract reasons yet manage to capture reality.

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IN BRIEF

The deepest mysteries are often the things we take for granted. Most people never think twice about the fact that scientists use mathematics to describe and explain the world. But why should that be the case?

Math concepts developed for purely abstract reasons turn out to explain real phenomena. Their utility, as physicist Eugene Wigner once wrote, "is a wonderful gift which we neither understand nor deserve.''

Part of the puzzle is the question of whether mathematics is an invention (a creation of the human mind) or a discovery (something that exists independently of us). The author suggests it is both.
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