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QUALITY COMPETITION AND THREAT OF ENTRY IN DUOPOLY *

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Threat of entry, sequential entry and cost of production are taken into account in a duopoly game in quality within the vertical product differentiation framework. A multi-stage perfect equilibrium solution is shown to exist and is fully characterized.

1. Introduction

In the framework of vertical differentiation, Shaked and Sutton (1982) showed that, in a market in which two firms at most can survive ('natural' duopoly), one of them selects the maximum product quality while the other chooses the minimum quality when no other firms threaten to enter the market. When this condition is relaxed, firms enter and jam the highest product quality. With zero cost, they earn zero profit, while with a fixed cost (however small), no perfect equilibrium exists if more than two firms enter.

These results raise some interesting issues. For instance, it is not known which firm selects the highest product quality or whether a large number of potential firms constantly threatening to enter the market affects the equilibrium.

In this note, we attempt to analyse these issues in a concise manner. Assuming sequential entry of firms and using the perfect equilibrium concept [Selten (1975)] in the vertical product differentiation approach, some new results are obtained. In particular, threat of entry reduces the equilibrium quality differentiation in the natural duopoly case and leads the first mover to choose indifferently its product quality within a specific range. As a consequence, the first mover does not necessarily select the maximum product quality. Moreover, the second mover is shown to select its quality such that the ratio of product quality is constant. This ratio depends solely on the cost parameters of the potential firms. Finally, sequential entry provides an advantage to the first mover. Based on its strategies, a condition is proposed under which the duopoly equilibrium is always the outcome. The model and the solution concept are outlined in the next section and the analysis is carried out in section 3.

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2. A model of quality competition with threat of entry

Following Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983), consumers have identical taste but different income which are uniformly distributed over the interval [a, b] with density equal unity. Their utility function takes the form

$$u(Y, j) = \theta_j Y. \tag{1}$$

Individual demands are thus inelastic at one unit. When no purchase is made, the utility achieved is $u(Y, 0) = \theta_r Y$, where θ_r stands for the reservation quality. In the 'natural' duopoly case, we can define Y_1 with

$$\theta_1(Y_1 - p_1) = \theta_2(Y_1 - p_2), \tag{2}$$

where Y_1 represents the income of the consumer who is indifferent between buying product 1 at price p_1 and product 2 at p_2. Assuming $\theta_1 > \theta_2$, consumers with $Y > Y_1$ prefer buying product 1 to 2. We can similarly define Y_2 as the income of the consumer who is indifferent between not making any purchase and buying low quality product 2 at price p_2 (i.e., $\theta_2(Y_2 - p_2) = \theta_r Y_2$), so that, for $Y < Y_2$, consumers do not buy anything. For $a \leqslant Y \leqslant b$, the revenue of firms 1 and 2 are thus the following:

$$\begin{aligned} R_1 &= p_1(b - Y_1), \\ R_2 &= p_2(Y_1 - Y_2) \quad \text{if} \quad Y_2 > a, \\ &= p_2(Y_2 - a) \quad \text{if} \quad Y_2 \leqslant a. \end{aligned} \tag{3}$$

We assume that each firm produces a single product and that entry is sequential with firm 1 being the incumbent firm. Let the feasible qualities lie in the interval $[\underline{\theta}, \bar{\theta}]$, where $\bar{\theta}$ ($\underline{\theta}$) represents the technological maximum (minimum) quality. For expository convenience, there is no variable cost of production. Moreover, we assume that firm 1 does not incur any entry cost, while firm 2, as well as any other potential firms, must pay an identical fixed cost F in order to enter. [1]

The game is cast in terms of a multi-stage Subgame Perfect Equilibrium. The three stages are the entry decision, the strategic choice of quality and the choice of price by the firms. These decisions are sequential and, after any stage, the firm strategies form a perfect equilibrium. In the first stage, only firm 2 must decide whether or not to enter, knowing that firm 1 has already decided to enter the market. This decision is conditional on the choice of quality by the two firms, and in particular on firm 1's strategic choice. Firm 2 enters the market only under a precise condition. Its choice of product quality in the second stage of the game, together with the choice made by firm 1, determine whether or not further entry is possible. Finally, in the third stage of the game, prices are set by the firms, conditional on their quality choice made in the previous stage. The interactions between entry, quality and price decisions, viewed as a three-stage non-cooperative game, are based on the belief that price decision is the easiest to change, whereas the decision of entry is the most difficult to reverse.

An important feature of our analysis is the existence of potential firms. Entry occurs whenever non-negative profit can be earned. In a market where duopoly is natural in Shaked and Sutton's sense, the entry of a third firm can only be deterred by strategic actions of the existing firms. This

[1] This assumption is not crucial in deriving the results reported in this note. Other cost structures have been investigated in Hung and Schmitt (1987) where a dynamic extension of the quality competition is analysed.

consideration leads to well defined firm strategies. The consequence of this game structure is now analysed in details.

3. Analysis

The different stages of the duopoly game are investigated in the reverse order of sequence of decisions, beginning with the price game, then the quality game, and finally the entry decision.

(a) The price game. The Nash equilibrium can be found by maximizing with respect to prices the revenue functions given by (5). We now state:

Lemma 1. (a) *For any Nash equilibrium in the price game (or Bertrand equilibrium):*

(i) when $2a < b < 4a$, *there are at most two firms having positive market share and covering the entire market with goods of distinct qualities.*

(ii) The Nash equilibrium prices are respectively

$$p_1^*(\theta_1, \theta_2) = K_1\left(1 - \frac{\theta_2}{\theta_1}\right) \quad \text{with} \quad K_1 = \frac{2b - a}{3}, \tag{4}$$

$$p_2^*(\theta_1, \theta_1) = K_2\left(\frac{\theta_1}{\theta_2} - 1\right) \quad \text{with} \quad K_2 = \frac{b - 2a}{3}. \tag{5}$$

The proof of the first part of this lemma can be found in Shaked and Sutton (1982). The second part of the lemma is obtained from the profit maximization conditions in prices. Since the two firms cover the market, $R_1 = p_1(b - Y_1)$ and $R_2 = p_2(Y_1 - a)$. Substituting Y_1 from (2) in these revenue functions and maximizing them with respect to prices yield (4) and (5).

Note that, from (2), we obtain $p_1^* > p_2^*$ when $\theta_1 > \theta_2$. Moreover, the price decision is independent of production cost since costs do not depend on quantity. It can also be seen that K_1 and K_2 represent respectively the market equilibrium production of goods 1 and 2. [2] We now have prices which depend upon quality choice. The decision about quality takes place in the second stage of the game.

(b) The quality game. Using (2), (3), (4), (5) and the assumption about costs, the following profit functions are obtained

$$\pi_1^*(\theta_1, \theta_2) = R_1^*(\theta_1, \theta_2) = K_1^2\left(1 - \frac{\theta_2}{\theta_1}\right), \tag{6}$$

$$\pi_2^*(\theta_1, \theta_2) = R_2^*(\theta_1, \theta_2) - F = K_2^2\left(\frac{\theta_1}{\theta_2} - 1\right) - F. \tag{7}$$

It can easily be checked that

$$R_1^*(\theta_1, \theta_2) > R_2^*(\theta_1, \theta_2). \tag{8}$$

[2] Substituting (4) and (5) in the definition of Y_1 yields $Y_1 = (b +)/3$. Thus, the quantity of the product of quality θ_1 is $(b - Y_1) = (2b - a)/3 = K_1$.

Moreover,

$$\frac{\partial R^{*}j(\theta_1, \theta_2)}{\partial \theta_1} > 0 \quad \text{and} \quad \frac{\partial R_j^{*}(\theta_1, \theta_2)}{\partial \theta_2} < 0 \quad \text{for} \quad j = 1,2. \tag{9}$$

In this stage of the game, firms maximize their profit with respect to product quality. In the absence of threat of entry, and when there are only two firms, it follows from (9) that $\theta_1 = \bar{\theta}$ and $\theta_2 = \underline{\theta}$. The differentiation between products depends only upon the technological constraints. This result no longer holds in the presence of threat of entry. Before showing this, we need the following remark:

Remark 1. Taking advantage of entering first, firm 1 always selects θ_1 which is higher than the product quality chosen by firm 2. This does not come from the fact that the first entrant has, by assumption, no cost of production, but rather because the firm setting a higher product quality enjoys a larger revenue [see (8)].

Remark 2. Since there are at most two firms which can earn non-negative profit, the threat of entry imposes an additional constraint on the strategic behaviour of the two established firms. It is now argued that it belongs to firm 2 to choose its product quality θ_2 such that $\pi_2^{*}(\theta_1, \theta_2) = 0$. Suppose $\pi_2^{*}(.) > 0$ and consider the behaviour of a potential firm, for instance firm 3. It can set $\theta_3 = \theta_2 + \epsilon$ (ϵ arbitrarily small), ignore firm 2 and charge a price according to the duopoly game with firm 1. Given θ_1, $p_3^{*}(\theta_1, \theta_2) < p_2^{*}(\theta_1, \theta_2)$ and thus firm 3 captures firm 2's entire market. It can safely adopt this price strategy because, with Lemma 1, firm 2 is forced to exit whatever its (rational) price strategy. Hence, in order to stay on the market, firm 2 must choose θ_2 from its zero-profit condition. This condition deters any further entry.

Remark 3. Given $\pi_2^{*} = 0$, it follows readily from (7) that the quality choice of firm 2 satisfies

$$\frac{\theta_1}{\theta_2} = \frac{F_2}{K_2^2} + 1. \tag{10}$$

Consider now firm 1's behaviour.

Remark 4. First, from (10), the ratio of product quality is now constant. Hence, from (6), firm 1's profit is also constant, whatever its product quality choice consistent with the duopoly equilibrium. The range of product quality of firm 1 consistent with the two-firm equilibrium depends upon the possible action of its rival.

Firm 2 can choose the lowest quality $\underline{\theta}$. Given this choice, there exists $\tilde{\theta}_1$ such that $\pi_2^{*}(\tilde{\theta}_1, \underline{\theta}) = 0$, that is

$$\tilde{\theta}_1 = \underline{\theta}\left(1 + \frac{F_2}{K_2^2}\right). \tag{11}$$

Note that $\pi_2^{*}(\theta_1, \underline{\theta}) \gtrless 0$ as $\theta_1 \gtrless \tilde{\theta}_1$. Thus $\tilde{\theta}_1$ represents the Lower bound of the product quality choice of firm 1 when firm 2 enters.

Assume now that $\theta_1 < \bar{\theta}$ is chosen. Firm 2 could select $\theta_2 > \theta_1$. This possibility of jumping over firm 1's product quality naturally exists as long as firm 2 can select $\theta_2 = \bar{\theta}$. Call $\tilde{\tilde{\theta}}_1$, firm 1's product quality such that $\pi_2^*(\tilde{\tilde{\theta}}_1, \bar{\theta}) = 0$. This quality is given by

$$\tilde{\tilde{\theta}}_1 = \bar{\theta}\left(1 - \frac{F_2}{K_1^2}\right). \tag{12}$$

If firm 1 chooses $\tilde{\tilde{\theta}}_1' = \tilde{\tilde{\theta}}_1 + \epsilon$, then, from (9), $\pi_2^*(\tilde{\tilde{\theta}}_1', \theta_2) < 0$ for any $\tilde{\tilde{\theta}}_1' \leqslant \theta_2 \leqslant \bar{\theta}$. Thus, by setting $\theta_1 \geqslant \tilde{\tilde{\theta}}_1'$, firm 1 prevents firm 2 from selecting a product quality superior to its own.

It follows that the range of product quality choice made by firm 1 is simply defined by the closed interval $[\max(\tilde{\theta}_1, \tilde{\tilde{\theta}}_1), \bar{\theta}]$. We can now sum up our discussion in

Proposition 1. Under threat of entry, the equilibrium product quality choice θ_1 of firm 1 lies in the interval $[\max \tilde{\theta}_1, \tilde{\tilde{\theta}}_1), \bar{\theta}]$ *while the product quality choice θ_2^* of firm 2 is such that*

$$\theta_2^* = \frac{K_2^2}{K_2^2 + F}\theta_1^*. \tag{13}$$

Proposition 1 calls for some comments. Firm 1 is indifferent between any product quality in the range $[\max(\tilde{\theta}_1, \tilde{\tilde{\theta}}_1), \bar{\theta}]$. This is so because the profit of firm 1 remains constant whatever its quality choice. Therefore, it is not necessarily true that the technological maximum product quality $\bar{\theta}$ is provided by the market. Also, the product quality ratio θ_1^*/θ_2^* is lower than $\bar{\theta}/\underline{\theta}$. For, even if firm 1 selects $\bar{\theta}$, firm 2 chooses θ_2^* according to (13), so that, unless F is sufficiently large, $\theta_2^* > \underline{\theta}$. Thus, in general, potential competition brings about a lower quality differentiation than it would be in its absence. When one assumes $F = 0$ for existing as well as potential firms, all potential firms enter and jam the highest product quality. We now turn to the first stage of the game.

(c) Entry decision. Even if the incumbent firm enjoys a cost advantage relative to its rival, it cannot always deter the entry of the later entrant. Recall the definition of $\tilde{\theta}_1$ and $\tilde{\tilde{\theta}}_1$ in (11) and (12). Now, we prove

Lemma 2. The entry of firm 2 is always accommodated by the incubent firm when $\tilde{\theta}_1 < \tilde{\tilde{\theta}}_1$.

Assume that there is a single firm on the market, producing a good of quality θ_m and selling it at a price p_m.[3] The profit function of this firm is $\pi_m = p_m(b - Y_m)$. Maximizing this function with respect to price, taking (2) into account, yields $p_m^* = b(\theta_m - \theta_r)/2\theta_m$, and henceforth, the profit function in terms of quality is $\pi_m^* = b^2(1 - \theta_r)/4\theta_m$.

If $\tilde{\theta}_1 > \tilde{\tilde{\theta}}_1$, any product quality θ_1 such that $\tilde{\tilde{\theta}}_1 < \theta_1 < \tilde{\theta}_1$ deters the entry of the second mover whatever its choice of product quality in the set $[\underline{\theta}, \bar{\theta}]$. In effect, $\theta_1 > \tilde{\tilde{\theta}}_1$ implies that firm 2 does not jump over firm 1 product quality and $\theta_1 < \tilde{\theta}_1$ implies that firm 2 cannot profitably enter the market even at $\underline{\theta}$. Since $\partial\pi_m^*/\partial\theta_m > 0$, firm 1, acting as a monopolist, chooses $\theta_m^* = \tilde{\theta}_1 - \epsilon$. By implication, if $\tilde{\theta}_1 < \tilde{\tilde{\theta}}_1$, any product quality θ_1 in $[\underline{\theta}, \bar{\theta}]$ leads to the entry of the second mover. Of course, firm 1 never lets firm 2 jump over, and therefore always chooses $\theta_1 > \tilde{\tilde{\theta}}_1$.

[3] At this price, we can define Y_m by

$$\theta_m(Y_m - p_m) = \theta_r Y_m.$$

Thus, consumers with $Y > Y_m$ prefer buying the product at price p_m rather than going without it; and conversely, for consumers with $Y < Y_m$.

Using (11) and (12), the inequality $\tilde{\theta}_1 < \tilde{\tilde{\theta}}_1$ is satisfied when

$$\frac{\bar{\theta}}{\underline{\theta}} > \frac{K_1^2}{K_2^2} \frac{\left(K_1^2 + F\right)}{K_1^2 - F}. \tag{14}$$

This is the condition under which the entry of firm 2 must be accommodated. We are therefore able to state:

Proposition 2. In the duopoly game with threat of entry, a three-stage perfect equilibrium exists provided (i) $2a < b < 4a$ *and* (ii) *that the condition given by (14) holds.*
(i) *follows directly from Lemma 1 and* (ii), *from Lemma 2.*

We have seen that the presence of threat of entry, sequential entry and cost of production alter in a significant way Shaked and Sutton's qualitative results. The threat of entry tends to reduce the quality differentiation between products and, with the presence of cost of production, can induce the incumbent firm to provide a lower product quality than the technological maximum quality. We also provide a condition for the existence of the two-firm perfect equilibrium in the presence of threat of entry and cost of production. This condition crucially depends on the sequential process of entry. These additional features bring some interesting results to the already rich set of implications of the vertical product differentiation approach.

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