1 Appendix for 'Buyer Power in International Markets', Horst Raff and Nicolas Schmitt, August 2009

Proof of Proposition 1

The proof has two parts. First, we derive wholesale prices assuming an equilibrium exists. Second, we establish that a non-exclusive equilibrium exists for $b \leq 0.73205$.

The joint profit of retailer i and the manufacturer when the rival retailer, denoted by $-i$, offers contract (T_{-i}^N, w_{-i}^N) is equal to:

$$\begin{aligned}\Pi_i^N(w_i^N, w_{-i}^N) \equiv\ & (p_i(w_i^N, w_{-i}^N) - w_i^N)q_i(w_i^N, w_{-i}^N) \\ & + (w_i^N - c)q_i(w_i^N, w_{-i}^N) + (w_{-i}^N - c)q_{-i}(w_i^N, w_{-i}^N)) + T_{-i}^N.\end{aligned}$$

Using the linear demand specification, this can be rewritten as:

$$\begin{aligned}\max_{w_i}\{ & \frac{\left(2 - b - b^2 - (2-b^2)w_i^N + bw_{-i}^N\right)^2}{(4-b^2)^2(1-b^2)} \\ & + (w_i^N - c)\frac{\left(2 - b - b^2 - (2-b^2)w_i^N + bw_{-i}^N\right)}{(4-b^2)(1-b^2)} \\ & + (w_{-i}^N - c)\frac{\left(2 - b - b^2 - (2-b^2)w_{-i}^N + bw_i^N\right)}{(4-b^2)(1-b^2)} + T_{-i}^N\},\end{aligned}$$

where the first term is retailer i's profit, the second term the manufacturer's profit from selling to retailer i (both gross of retailer i's fixed transfer), and the third term is the manufacturer's profit from selling to the rival retailer $-i$. Retailer i's best-response function is

$$w_i^N = \frac{1}{4(2-b^2)}[(2-b-b^2)(b^2+(4-b^2)c) + 4bw_{-i}^N].$$

Setting $w_i^N = w_{-i}^N$ gives

$$\tilde{w}_i^N = c + \frac{b^2(1-c)}{4}.$$

Next, we show that the following contract offer of retailer $i = 1, 2$ constitutes an equilibrium strategy:

- $\widetilde{w}_i^N$,
- $\tilde{T}_i^N = \pi_i(\widetilde{w}_i^N, \widetilde{w}_{-i}^N) - \left[\tilde{\Pi}^N - \Pi_{-i}^m\right]$,
- $\tilde{w}_i^E = c$,
- $\tilde{T}_i^E = \Pi_1^m + \Pi_2^m - \tilde{\Pi}^N$.

Given these contracts, the manufacturer earns a profit of $\Pi_1^m + \Pi_2^m - \tilde{\Pi}^N$ either by accepting non-exclusive contracts from both retailers or by accepting an exclusive contract from one of them. Accepting a non-exclusive contract is hence a best response for the manufacturer, provided that the contract offers him at least this much profit. In the proposed non-exclusive-contract equilibrium retailer i earns

$$\tilde{\Pi}^N - \Pi_{-i}^m = \tilde{\Pi}^N - \frac{(1-c)^2}{4}.$$

This profit is non-negative for $b \leq 0.73205$. Since $\widetilde{w}_i^N$ constitutes a best response and the manufacturer does not accept a lower transfer, retailer i cannot gain by offering another non-exclusive contract. In addition, retailer i cannot benefit from offering a different exclusive contract, since any contract involving a smaller transfer to the manufacturer would not be accepted.

Proof of Lemma 1

Suppose an equilibrium exists in which both retailers are active. Denote the profits of retailer $i = 1, 2$ and the manufacturers by π_i^N, π_h^N and π_f^N, respectively. Let $\Pi^N(t) \equiv \pi_1^N + \pi_2^N + \pi_h^N + \pi_f^N$ denote the resulting total industry profit derived from sales in the home country given trade cost t. Then it must be the case that retailer i and manufacturer j together earn at least as much as they could if they foreclosed the rival retailer $-i$ while compensating the other manufacturer $-j$ for not selling to retailer $-i$:

$$\pi_i^N + \pi_j^N \geq \Pi_i^m - \hat{\pi}_{-j},$$

where $\hat{\pi}_{-j}$ is the compensation payment. Using the definition of $\Pi^N(t)$, this inequality can be rewritten as

$$\pi_{-i}^N \leq \Pi^N(t) - \Pi_i^m + (\hat{\pi}_{-j} - \pi_{-j}^N). \tag{1}$$

Note that $\hat{\pi}_{-j} \leq \pi_{-j}^N$ since there is no need to pay $-j$ strictly more than he would have earned in equilibrium. Since $\hat{\pi}_{-j} \leq \pi_{-j}^N$, (1) implies that a retailer's profit cannot exceed his contribution to total industry profit. Individual rationality implies $\pi_i^N \geq 0$ and hence a necessary condition for an equilibrium to exist is:

$$\Pi^N(t) \geq \Pi_i^m - (\hat{\pi}_{-j} - \pi_{-j}^N) \geq \Pi_i^m.$$

Next, given the definition of Π^N, it is the case that $\pi_h^N + \pi_f^N = \Pi^N - \pi_1^N - \pi_2^N$, so that, using (1),

$$\pi_h^N + \pi_f^N \geq \Pi^N(t) - (\Pi^N(t) - \Pi_2^m + (\hat{\pi}_h - \pi_h^N)) - (\Pi^N(t) - \Pi_1^m + (\hat{\pi}_f - \pi_f^N)).$$

Simplifying and re-arranging, $\hat{\pi}_h + \hat{\pi}_f \geq \Pi_1^m + \Pi_2^m - \Pi^N(t)$. Since $\Pi_1^m + \Pi_2^m - \Pi^m > 0$ for $b > 0$ and $\Pi^m \geq \Pi^N(t)$, it follows that $\Pi_1^m + \Pi_2^m - \Pi^N(t) > 0$ so that $\hat{\pi}_h + \hat{\pi}_f > 0$. Finally, since $\hat{\pi}_f \leq \pi_f^N$ and $\hat{\pi}_h \leq \pi_h^N$, we have $\pi_h^N + \pi_f^N > 0$.

Proof of Proposition 2

The proof has two parts. First, we establish that for $b \geq 0.61803$ there exists an equilibrium in which one of the retailers does not sell. Second, we show that there exists an equilibrium in which both retailers sell if $b \leq 0.67209$.

Suppose that retailer $-i$ offers an exclusive contract to both manufacturers, where $\hat{w}^E_{-i} = c$ and $\hat{T}^E_{-i} = \frac{(1-c)^2}{8}$ (so that each manufacturer receives half the monopoly profit). To break the exclusivity, retailer i has to make a better offer to a single manufacturer j. Given $\hat{w}^E_{-i} = c$ profit maximizing retail prices are:

$$p_i = \frac{(2 - b - b^2 + 2w_i + bc)}{4 - b^2} \quad \text{and} \quad p_{-i} = \frac{(2 - b - b^2 + 2c + bw_i)}{4 - b^2}.$$

The joint profit of retailer i and the single manufacturer j hence is

$$\Pi_{i,j}(w_i, c) = (p_i(w_i, c) - c)\frac{(2 - b - b^2 - (2 - b^2)w_i + bc)}{(4 - b^2)(1 - b^2)}.$$

Maximizing this joint profit over w_i yields

$$w_i = \frac{1}{4(2 - b^2)}[b^2(2 - b - b^2) + c(8 - 6b^2 + b^3 + b^4)],$$

and the resulting joint profit is equal to

$$\Pi_{i,j} = \frac{(1 - c)^2(1 - b)(2 + b)^2}{8(1 + b)(2 - b^2)}.$$

$\Pi_{i,j} > \frac{(1-c)^2}{8}$ if $b < 0.61803$. Hence only for $b \geq 0.61803$ does there exist an exclusive-contract equilibrium in which one of the retailers does not sell.

We have to show that the following contract offer of retailer i constitutes an equilibrium strategy for $b \leq 0.67209$:

- $\hat{w}^N_{i,j} = \hat{w}^N_i$, $\hat{w}^N_{i,-j} = 0$,
- $\hat{T}^N_{i,j} = \pi_i(\hat{w}^N_i, \hat{w}^N_{-i}) - \Pi^N(t = 0) + \frac{1}{2}\left(\Pi^m_1 + \Pi^m_2\right)$, $\hat{T}^N_{i,-j} = 0$,
- $\hat{w}^E_{i,j} = \hat{w}^E_{i,-j} = c$,
- $\hat{T}^E_{i,j} = \hat{T}^E_{i,-j} = \frac{1}{2}\left(\Pi^m_1 + \Pi^m_2 - \Pi^N(t = 0)\right)$.

Given these contract offers, each manufacturer earns a profit equal to $\frac{1}{2}\left(\Pi_1^m + \Pi_2^m - \Pi^N(t=0)\right)$ whether he accepts non-exclusive or exclusive contracts. Hence a manufacturer accepts a non-exclusive contract if he can earn at least this profit. Retailer i's profit in case of non-exclusive contracts is equal to $\Pi^N(t=0) - \Pi_{-i}^m = \Pi^N(t=0) - \frac{(1-c)^2}{4}$. This profit is greater or equal to zero for $b \leq 0.67209$. Retailer i cannot gain from a deviation to another non-exclusive contract since $\hat{w}_i^N$ is a best response, and since the manufacturers will not accept a contract that offers them a lower profit. By construction, i's profit is weakly greater than the profit he could earn by having both manufacturers sell exclusively to him, which cannot exceed $\Pi_i^m - \left(\Pi_1^m + \Pi_2^m - \Pi^N(t=0)\right) = \Pi^N(t=0) - \Pi_{-i}^m$.

Proof of Proposition 3

The nature of the contract and the number of active retailers in each case come directly from Propositions 1 and 2. Consider then the price comparisons. Given exclusive contracts in autarky, the price is $\widetilde{p}^E = c + \frac{1-c}{2}$. In free trade, either the price does not change, or we obtain non-exclusive contracts and the price is given by (8) in the paper. In this case, prices are lower in free trade because

$$\widetilde{p}^E - \widehat{p}_i^N = \frac{(1-c)(2-b)b}{2(4-b(2+b))} > 0.$$

With non-exclusive contracts in autarky, prices are given by (8) in the paper. If there are also non-exclusive contracts in free trade, then prices in free trade are lower because

$$\widetilde{p}_i^N - \widehat{p}_i^N = \frac{(1-c)b^3}{4[4-b(2+b)]} > 0.$$

Alternatively, contracts are exclusive in free trade and the price is $\widehat{p}^E = c + \frac{1-c}{2}$. In this case, the free-trade price is higher than in autarky since

$$\widetilde{p}_i^N - \widehat{p}^E = -\frac{b(1-c)}{4} < 0.$$

Proof of Proposition 4

Consider first the case of exclusive contracts. In this case, there is one active retailer so that consumer surplus is $CS = \frac{q_i^2}{2}$, where $i = 1, 2$ depending on which retailer is active. In the autarky equilibrium with exclusive contracts, $CS_{Aut}^E = \frac{(1-c)^2}{8}$, $\pi_i = \widetilde{\pi}_i^E = 0$ and $\pi_h = \widetilde{\pi}_h^E = \frac{(1-c)^2}{4}$. In free trade the equilibrium with exclusive contracts implies $CS_{FT}^E = CS_{Aut}^E$, $\pi_i = \widehat{\pi}_i^E = 0$. With exclusive contracts in both countries $\pi_h = \widehat{\pi}_h^E = \frac{(1-c)^2}{4}$ since the home manufacturer earns half the monopoly rents on domestic sales (the other half is earned by the foreign manufacturer) and the home manufacturer earns half the monopoly rent generated abroad. Thus, domestic social welfare with an exclusive contract equilibrium in both countries is the same in autarky and free trade, namely

$$W_{Aut}^E = W_{FT}^E = \frac{3(1-c)^2}{8}. \tag{2}$$

Consider next the case of non-exclusive contracts where both retailers are active. In this case, consumer surplus is $CS = q_1 + q_2 - \frac{1}{2}(q_1^2 + q_2^2) - bq_1q_2 - p_1q_1 - p_2q_2$. In autarky, $CS_{Aut}^N = \frac{(2+b)^2(1-c)^2}{16(1+b)}$ and $\sum_{i=1}^{2} \pi_i + \pi_h = \frac{(4-b^2)(1-c)^2}{8(1+b)}$. Hence

$$W_{Aut}^N = \frac{(1-c)^2(6-b)(2+b)}{16(1+b)}.$$

In free trade, $CS_{FT}^N = \frac{(2-b^2)^2(1-c)^2}{(1+b)(4-2b-b^2)^2}$. When both countries are in an equilibrium with non-exclusive contracts, then the rents accruing to the domestic manufacturer and the two retailers are equal to $\Pi^N = \frac{4(1-b)(2-b^2)(1-c)^2}{(1+b)(4-2b-b^2)^2}$ since the share of the rent earned by the foreign manufacturer in the home country is equal to the share of the rent earned by the home manufacturer in the foreign country. Hence when both countries are in an equilibrium with non-exclusive contracts we have

$$W_{FT}^{N}=\frac{(6-4b-b^{2})\,(2-b^{2})(1-c)^{2}}{(1+b)(4-2b-b^{2})^{2}}. \tag{3}$$

Two additional cases may arise in free trade when $0.61803 < b \leq 0.67209$. Specifically, one country may be in an equilibrium with exclusive contracts while the other is in an equilibrium with non-exclusive contracts. These cases are irrelevant for the proof since the welfare results for this range of b are ambiguous even when these cases do not arise.

Now examine how welfare changes with a move from autarky to free trade. (i) If $b \leq 0.61803$ and contracts remain non-exclusive, it can be verified that $W_{FT}^{N} - W_{Aut}^{N} > 0$. Alternatively, if there are exclusive contracts in autarky but non-exclusive contracts in free trade, we have $W_{FT}^{N} - W_{Aut}^{E} > 0$. (ii) If $0.61803 < b \leq 0.67209$, it is straightforward to verify that: 1) domestic welfare rises when, in both countries, contracts are non-exclusive in autarky and in free trade since $W_{FT}^{N} - W_{Aut}^{N} > 0$; 2) welfare does not change when, in both countries, there is exclusivity in autarky and in free trade, since $W_{FT}^{E} - W_{Aut}^{E} = 0$; 3) welfare decreases when, in both countries, there are non-exclusive contracts in autarky and exclusive contracts in free trade, since $W_{FT}^{E} - W_{Aut}^{N} < 0$. (iii) If $b > 0.67209$, either welfare does not change as $W_{FT}^{E} - W_{Aut}^{E} = 0$, or welfare decreases as $W_{FT}^{E} - W_{Aut}^{N} < 0$.

Proof of Proposition 5

When trade liberalization is unilateral, the only difference with respect to the proof of Proposition 4 concerns the free-trade level of welfare since the domestic manufacturer does not earn any rents abroad. With non-exclusive contracts in free trade, the rents accruing to the domestic manufacturer and the two retailers are now equal to $\Pi^{N} - \pi_{f}^{N}$, where $\pi_{f}^{N} = \frac{1}{2}\left(\Pi_{1}^{m} + \Pi_{2}^{m} - \Pi^{N}\right) = \frac{(1-c)^{2}}{4} - \frac{2(1-b)(2-b^{2})(1-c)^{2}}{(1+b)(4-2b-b^{2})^{2}}$. Hence π_{f}^{N} has to be subtracted from W_{FT}^{N}. We can show that $W_{FT}^{N} - \pi_{f}^{N} - W_{Aut}^{N} < 0$ for all feasible values of b and $W_{FT}^N - \pi_f^N - W_{Aut}^E > 0$ for $b < 0.67209$. With exclusive contracts in free trade, social welfare is equal to $\check{W}_{FT}^E = \frac{(1-c)^2}{4}$. As a result, $\check{W}_{FT}^E - W_{Aut}^E < 0$ and $\check{W}_{FT}^E - W_{Aut}^N < 0$.

Proof of Lemma 3

Suppose there exists an equilibrium in which both retailers accept an exclusive contract from manufacturer $i = h, f$, so that the other manufacturer, $-i$, does not sell. The highest total industry profit that can be generated is then given by $\bar{\pi}^m$ (see (9) in the paper) which is achieved when the active manufacturer sets the wholesale price $\bar{w}_i = c + b(1-c)/2$ (see (11) in the paper). Hence the highest payment that the active manufacturer can offer each retailer for not buying from the rival manufacturer is $\bar{\pi}^m/2$. Also note that an exclusive contract that offers both retailers strictly less than $\bar{\pi}^m/2$ cannot occur in equilibrium, since the inactive manufacturer would then have an incentive to offer the retailers an amount closer to $\bar{\pi}^m/2$, thereby realizing a positive profit.

Consider the joint profit that manufacturer $-i$ and a retailer could obtain by breaking the exclusive contract when manufacturer i chooses wholesale price $\bar{w}_i$. This profit is given by

$$\pi_{-i}(\bar{w}_i, w_{-i}) = (w_{-i} - c)\frac{(2 - b - b^2 - (2-b^2)w_{-i} + b\bar{w}_i)}{(4-b^2)(1-b^2)} + \frac{(2 - b - b^2 - (2-b^2)w_{-i} + b\bar{w}_i)^2}{(4-b^2)^2(1-b^2)}.$$

The wholesale price that maximizes this joint profit is given by

$$\bar{w}_{-i} = \frac{b^2(4 - 2b - b^2) + c(16 - 12b^2 + 2b^3 + b^4)}{8(2-b^2)},$$

which is strictly less than $\bar{w}_i$, and the resulting joint profit is equal to:

$$\pi_{-i}(\bar{w}_i, \bar{w}_{-i}) = \frac{(1-c)^2(4 - 2b - b^2)^2}{32(2-b^2)(1-b^2)}.$$

This profit is strictly greater than $\bar{\pi}^m/2$, since

$$\frac{(1-c)^2\left(4-2b-b^2\right)^2}{32\left(2-b^2\right)\left(1-b^2\right)}-\frac{(1-c)^2}{4(1+b)}=\frac{(1-c)^2\left(2-b\right)^2 b^2}{32\left(2-b^2\right)\left(1-b^2\right)}>0,$$

which implies that an exclusive contract cannot occur in equilibrium.