# Imports, Pass-Through, and the Structure of Retail Markets Horst Raff and Nicolas Schmitt 

## 1 Comparative Statics for the Short Run

Totally differentiating

$$
\begin{gather*}
c_{I}=c_{D}+\frac{(w-t)}{2}-\frac{2 \beta F_{I}}{L(w-t)} ;  \tag{1}\\
N=\frac{\beta\left(\alpha-w-c_{D}\right)}{\gamma\left(w+c_{D}-\bar{p}\right)} ;  \tag{2}\\
\bar{p}=w+\frac{k c_{D}}{k+1}+\frac{c_{D}}{2(k+1)}-\frac{(w-t)}{2} \frac{c_{I}^{k}}{c_{D}^{k}}, \tag{3}
\end{gather*}
$$

and using $N=N_{E}\left(c_{D} / c_{M}\right)^{k}$, we have

$$
\left[\begin{array}{ccc}
a_{11} & a_{12} & 0  \tag{4}\\
0 & -1 & 1 \\
1 & a_{32} & a_{33}
\end{array}\right]\left[\begin{array}{c}
d \bar{p} \\
d c_{D} \\
d c_{I}
\end{array}\right]=\left[\begin{array}{c}
0 \\
b_{2} \\
b_{3}
\end{array}\right] d t
$$

where

$$
\begin{align*}
& a_{11}=\frac{\beta\left(\alpha-w-c_{D}\right)}{\gamma\left(w+c_{D}-\bar{p}\right)^{2}}>0,  \tag{5}\\
& a_{12}=-\left(\frac{\beta(\alpha-\bar{p})}{\gamma\left(w+c_{D}-\bar{p}\right)^{2}}+\frac{N_{E} k c_{D}^{k-1}}{c_{M}^{k}}\right)<0,  \tag{6}\\
& a_{32}=-\left(\frac{1+2 k}{2(1+k)}+\frac{k(w-t) c_{I}^{k}}{2 c_{D}^{k+1}}\right)<0,  \tag{7}\\
& a_{33}=\frac{k(w-t) c_{I}^{k-1}}{2 c_{D}^{k}}>0, \tag{8}
\end{align*}
$$

and

$$
\begin{align*}
b_{2} & =-\left(\frac{1}{2}+\frac{2 \beta F_{I}}{L(w-t)^{2}}\right)<0,  \tag{9}\\
b_{3} & =\frac{c_{I}^{k}}{2 c_{D}^{k}}>0 \tag{10}
\end{align*}
$$

The value of the determinant is given by $|D|=-a_{11}\left(a_{32}+a_{33}\right)+a_{12}$. After some algebraic manipulation we can show that

$$
\begin{aligned}
|D|=-\frac{\beta\left(\alpha-w-c_{D}\right)}{\gamma\left(w+c_{D}-\bar{p}\right)^{2}}\left[\frac{1}{2(1+k)}+\right. & \left.\frac{k(w-t) c_{I}^{k-1}\left(c_{D}-c_{I}\right)}{2 c_{D}^{k+1}}\right] \\
& -\frac{\beta}{\gamma\left(w+c_{D}-\bar{p}\right)}-\frac{N_{E} k c_{D}^{k-1}}{c_{M}^{k}}<0 .
\end{aligned}
$$

Using Cramer's Rule, we have

$$
\begin{aligned}
\frac{d \bar{p}}{d t} & =\frac{1}{|D|}\left|\begin{array}{ccc}
0 & a_{12} & 0 \\
b_{2} & -1 & 1 \\
b_{3} & a_{32} & a_{33}
\end{array}\right| \\
& =\frac{a_{12}\left(b_{3}-a_{33} b_{2}\right)}{|D|}>0
\end{aligned}
$$

since $|D|<0, a_{12}<0$ and $\left(b_{3}-a_{33} b_{2}\right)>0$;

$$
\begin{aligned}
\frac{d c_{D}}{d t} & =\frac{1}{|D|}\left|\begin{array}{ccc}
a_{11} & 0 & 0 \\
0 & b_{2} & 1 \\
1 & b_{3} & a_{33}
\end{array}\right| \\
& =\frac{-a_{11}\left(b_{3}-a_{33} b_{2}\right)}{|D|}>0
\end{aligned}
$$

since $|D|<0, a_{11}>0$ and $\left(b_{3}-a_{33} b_{2}\right)>0$;

$$
\begin{aligned}
\frac{d c_{I}}{d t} & =\frac{1}{|D|}\left|\begin{array}{ccc}
a_{11} & a_{12} & 0 \\
0 & -1 & b_{2} \\
1 & a_{32} & b_{3}
\end{array}\right| \\
& =\frac{a_{12} b_{2}-a_{11} b_{3}-a_{11} a_{32} b_{2}}{|D|}<0
\end{aligned}
$$

since $|D|<0$, and after some algebraic transformations

$$
\begin{aligned}
& a_{12} b_{2}-a_{11} b_{3}-a_{11} a_{32} b_{2} \\
& =\frac{\beta\left(\alpha-w-c_{D}\right)}{\gamma\left(w+c_{D}-\bar{p}\right)^{2}} \frac{1}{2}\left(\frac{c_{D}-c_{I}}{(w-t)}+1-\frac{c_{I}^{k}}{c_{D}^{k}}\right) \\
& \\
& +\left(\frac{1}{2}+\frac{2 \beta F_{I}}{L(w-t)^{2}}\right) \frac{\beta}{\gamma\left(w+c_{D}-\bar{p}\right)}>0 .
\end{aligned}
$$

In the short run, the pass-through rate with respect to the average retail price is also given by

$$
\begin{equation*}
\frac{d \bar{p}}{d t}=\frac{1}{2} \frac{c_{I}^{k}}{c_{D}^{k}}+\frac{k}{k+1} \frac{d c_{D}}{d t}+\frac{1}{2(k+1)} \frac{d c_{D}}{d t}+\frac{(w-t)}{2} \frac{c_{I}^{k}}{c_{D}^{k}}\left(\frac{k}{c_{D}} \frac{d c_{D}}{d t}-\frac{k}{c_{I}} \frac{d c_{I}}{d t}\right) . \tag{11}
\end{equation*}
$$

It is also easy to find examples for which the short-term pass-through rate is greater than one.

