KRW conjecture

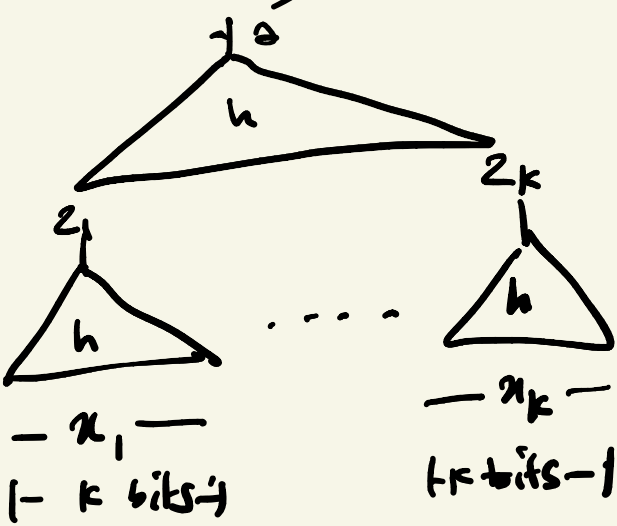
For any f, g $\quad d(f \circ g) \approx d(f) + d(g)$

Claim: KRW conjecture $\Rightarrow$ Superlog-Formula depth bounds

Say that there is some fn $h: \{0,1\}^k \to \{0,1\}$ which has good depth.

$d(h)$

$h \circ h : (\{0,1\}^k)^k \to \{0,1\}$



If KRW

$D(h \circ h) = D(h) + D(h)$

IF KRW

$$D(\overbrace{\underbrace{h\circ\cdots\circ h}_{n \text{ times}}}^{h^{(n)}}) = D(h\circ h^{(n-1)})$$

$$= D(h) + D(h^{(n-1)})$$

$$= D(h) + D(h) + D(h^{(n-2)})$$

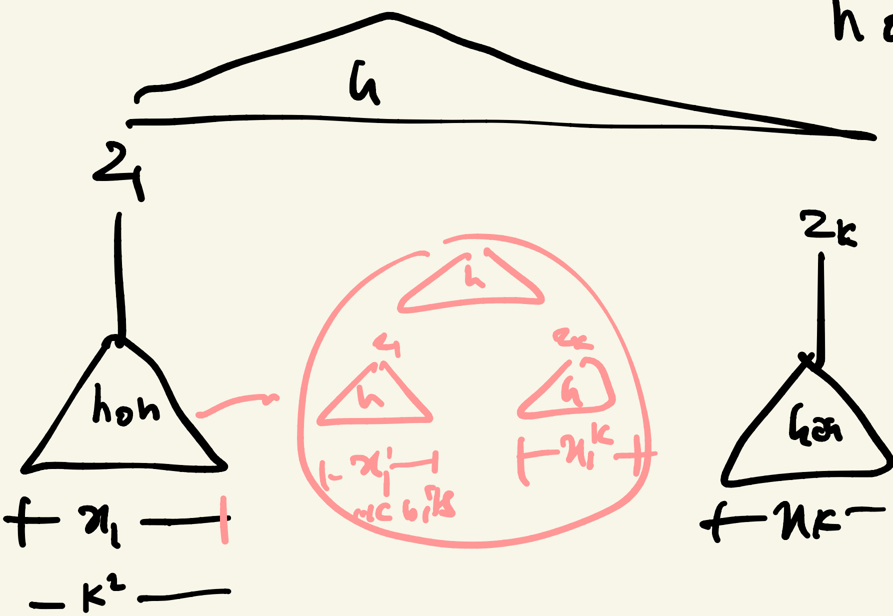
$$= nD(h)$$

$$D(\underline{h\circ h}\circ h) = D(h\circ g)$$

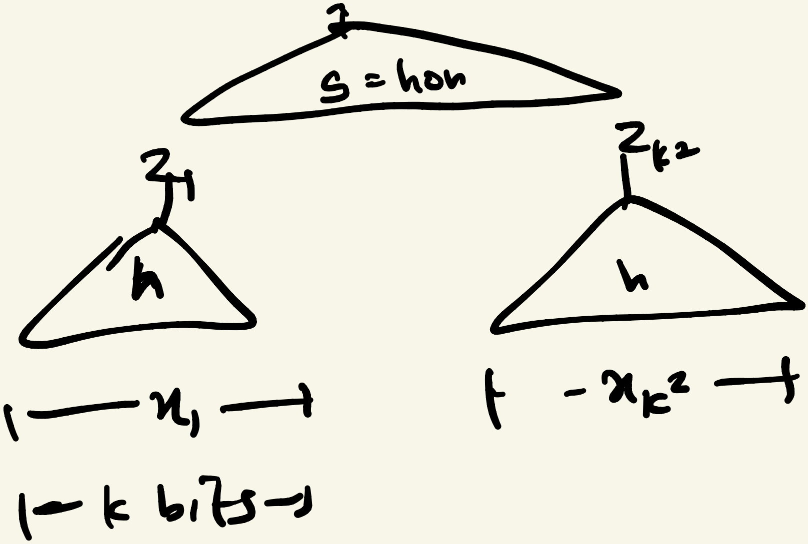
$$g = h\circ h : \{0,1\}^{k^2} \to \{0,1\}$$

$$h\circ h\circ h : ((\{0,1\}^k)^k)^k \to \{0,1\}$$

$h\circ g$



$g \circ h$



$$h^{(n)} = h \circ h^{(n-1)}$$

or

$$= h^{(n-1)} \circ h$$

If KRW conjecture

$$D(h^{(n)}) = n\, D(h)$$

T

(Recall $h: \{0,1\}^k \to \{0,1\}$)

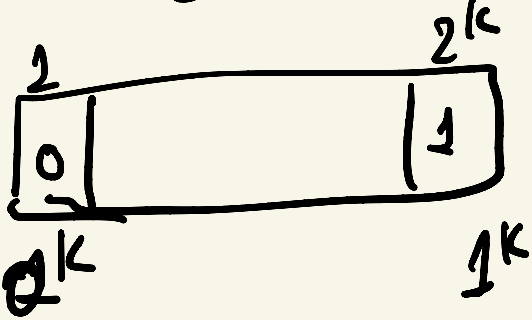
$h^{(n)}$ on k^n bits of input

Fact#1 For any i/p size k

$\exists\, h : \{0,1\}^k \to \{0,1\}$

$$D(h) \simeq \frac{k}{\log k}$$

Fact #2

T.T(h) = 

Explicit representation of any h takes $\simeq 2^k$ bits

$$D(h^{(n)}) \overset{\text{if KRW}}{=} D(h) \times n = \frac{nk}{\log k}$$

$\mathbin{\not{=}}$

Goal #1 : Turn into an explicit

fn.

Goal #2 Figure out best

n

$$D(h^{\otimes n}) = \underbrace{\frac{nk}{\log k} \qquad \text{inputsize}(h^{\otimes n}) = k^n}$$

Andreev

$$A\left(\boxed{\;x\;}, \boxed{\;y\;}\right)$$

$\leftarrow n$ bits $\rightarrow$ $\quad$ $|\!-$ n bits $-\!|$

$\downarrow$

TT(h) on $\log n$ bits

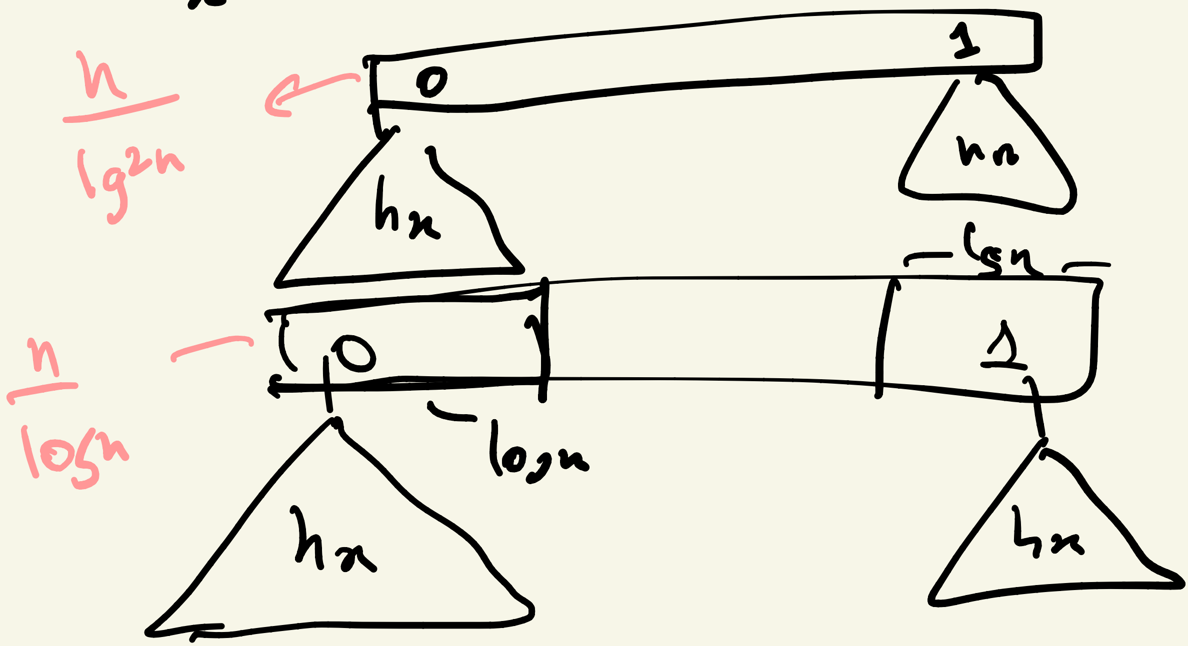
$$2^{\log n} = n$$

Fact #1 $\exists h \; (\exists x \in \{0,1\}^n)$

$$D(h) = \frac{k}{\log k}$$

$A(x\text{—}, y\text{—})$

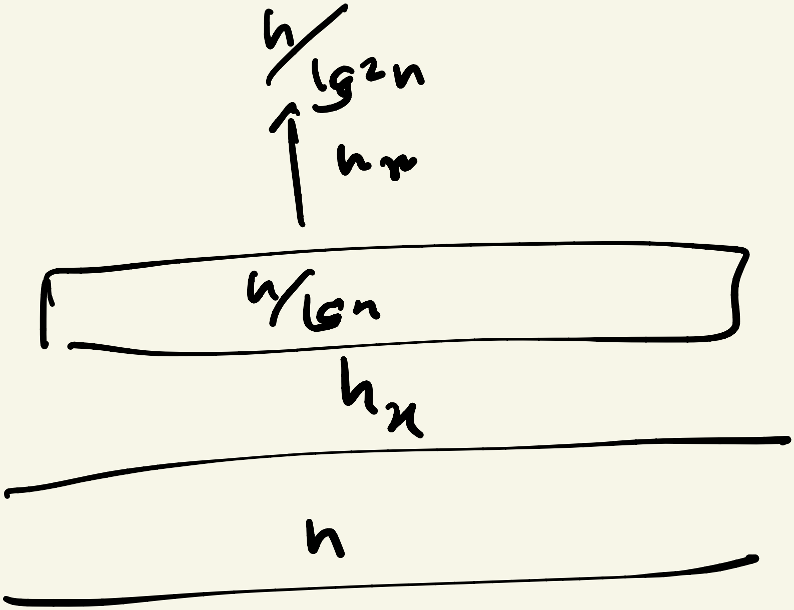
$h_x : \{0,1\}^{\log n} \to \{0,1\}$



$y_1\, y_2 \cdots y_{\log n} \quad \cdots \quad y_{n-\log n+1} \cdots y_n$

n bits $|\text{—} \log n \text{—}|$ $\qquad$ $|\text{—} \log n \text{—}|$

$$\frac{n}{(\log n)^j} = 1$$ at the j^{th} levels



Find j s.t.

$$\frac{n}{(\log n)^j} = 1$$

$$\Rightarrow (\log n)^j = n$$

$$j = \log n / \log\log n.$$

$$A(\text{---}x\text{---},\ \text{---}y\text{---})$$

$$= h_x^{\left(\frac{\log n}{\log\log n}\right)}(y)$$

We know $\exists\, h : \{0,1\}^{\log n} \to \{0,1\}$

$$\sim\ T\,T(h) = \underbrace{\boxed{0 \mid \ \cdots k}}_{x}$$

$\exists\, x,\ \underline{\text{st}}$ ..

$$D(h_x) = \frac{\log n}{\log\log n}$$

$$D(A()) \geq \underline{D(A(x, \cdot))}$$

$$A(x, y) = h_x^{(j)}(y)$$

$$= D(h_x^{(j)})$$

by KRW Conjecture

$$D(A) \geq j \times D(h_x)$$

$$= j \times \frac{\log n}{\log\log n}$$

$$= \frac{\log n}{\log\log n} \times \frac{\log n}{\log\log n}$$

$$D(A) \geq \frac{\log^2 n}{\log\log n}$$