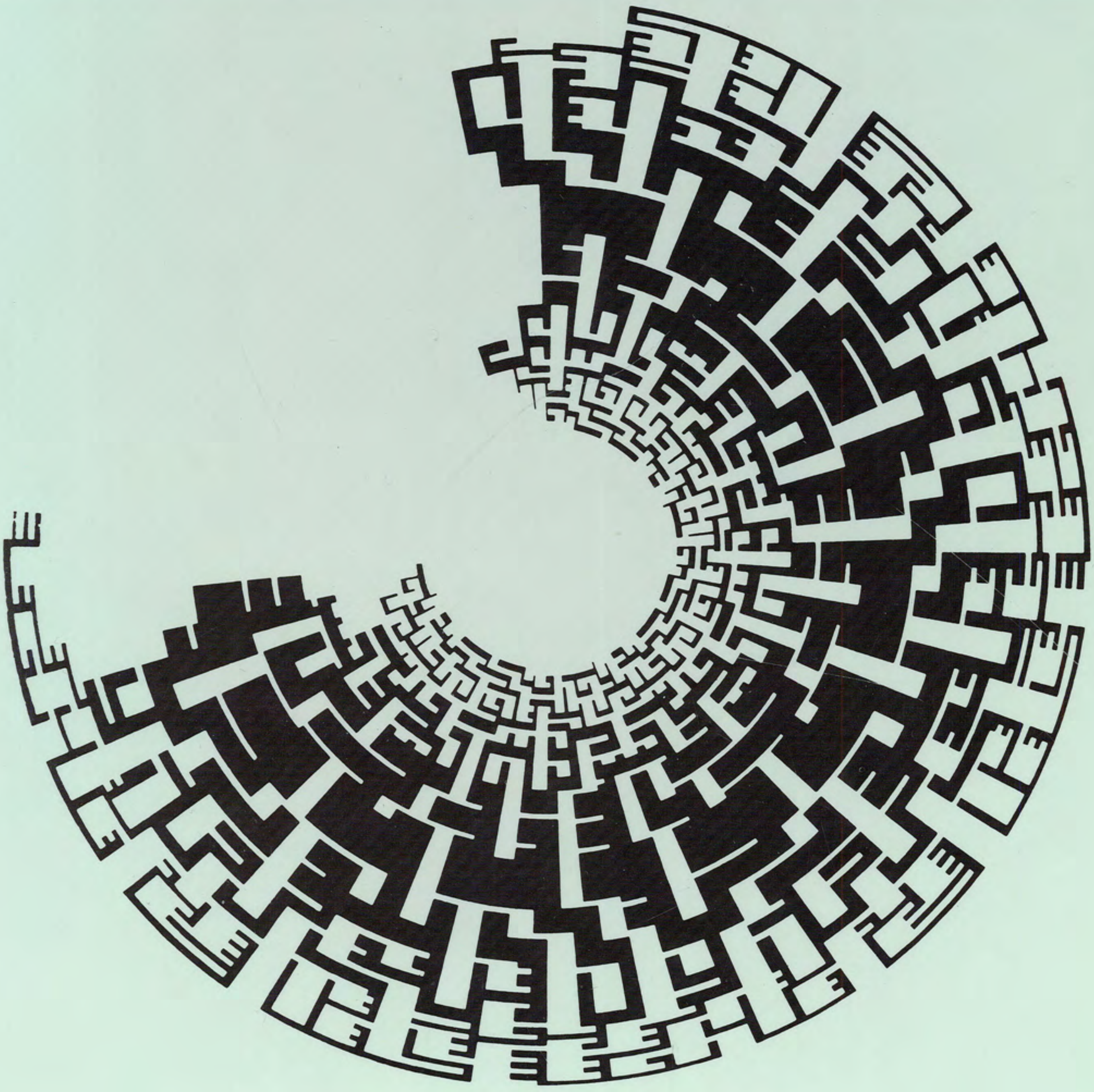
Possible Worlds an Introduction to Logic and Its Philosophy

Raymond Bradley / Norman Swartz



POSSIBLE WORLDS

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AN INTRODUCTION TO LOGIC AND ITS PHILOSOPHY

WORLDS

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To the members of our families, of whose company we were too often deprived during the years spent writing this book

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Preface

Talk of possible worlds is now a commonplace within philosophy. It began, nearly three hundred years ago, within philosophical theology. Leibniz thought it reassuring to say that although our world contains much that is evil, it is nonetheless the best of all possible worlds. Few philosophers today find this statement very plausible. Nevertheless, talk of the set of all possible worlds — when stripped of the suggestion that the actual world is better than any others — is nowadays frequently invoked as a means of illuminating other areas of philosophy. Ethics, epistemology, philosophy of psychology, philosophy of language, and — most notably of all — logic, are all benefiting from the insights of what is called "possible worlds semantics".

Unfortunately, most current talk of possible worlds is still regarded as the province of professionals; little of it has filtered down to those who are just beginning to learn their philosophy. Yet there is no good reason why this should be so. Although the higher reaches of possible-worlds semantics bristle with technical subtleties, its basic insights are really very simple. This book explains what those insights are and uses them to construct an integrated approach to both the philosophy of logic and the science of logic itself.

This approach, we believe, is especially suited to the needs of those who have difficulty with symbols. There are many persons who would like to learn something of philosophy and logic but who, because they are put off by the severely formal manner in which the logical part of philosophy is usually presented, are deterred from pursuing their intent. Their alienation is unfortunate and needless; we try to prevent it by taking more pains than usual to ensure that logical concepts are well understood before they are symbolized. Indeed, it is only in the last two chapters that the powers of symbolism are systematically exploited. Then, again, we would like to think that our approach will be helpful to those for whom symbolism holds no terrors but for whom the difficulty lies rather in seeing how there can be any real connection between the formal results of logic and the substantive inquiries undertaken in other parts of philosophy. Their intellectual schizophrenia reflects the fact that the rarefied results of formal logic resemble those of mathematics more closely than they do those of metaphysics, epistemology, and the rest. But it neglects the fact, which we here emphasize, that the basic concepts of formal logic are hammered out on the same anvil of analytical inquiry as are those of other parts of philosophy. Grounding logic in talk of possible worlds, we believe, is one way of making logic seem more at home with its philosophical kin.

Many of the arguments presented in this book are, and need to be, matters for philosophical debate. Yet seldom have we done more than hint at the parameters within which such debate arises. There are three main reasons for this. First, we believe that the kind of questioning which we hope this book will generate is likely to be deeper if it is provoked by sustained argument for a single coherent point of view rather than if it stems from exposure to an eclectic display of divergent doctrines. Secondly, we are confident that serious students will, sooner or later, be treated — by their reading or their teachers — to arguments which will put ours into a broader perspective. Thirdly, we could not hope to do justice to competing points of view without making this book even longer than it is.

Students in three countries — Australia, New Zealand, and Canada — have been the guinea pigs for the general approach and parts of the material in this book. We have benefited from the criticisms of many and also from the encouragement of the few who have gone on to become professional teachers of philosophy. We are indebted to scores of fine young minds.

Specific acknowledgements go to two institutions and to a number of individuals. The Canada Council and the President's Research Grant Committee of Simon Fraser University have provided generous financial assistance for some of the research and editorial work on this book. Three research assistants, Michael Beebe, Jeffrey Skosnik, and Moira Gutteridge, have assisted in the preparation of the manuscript: their help in compiling the references and index, in preparing some of the graphics, and in commenting on the manuscript, has been invaluable. Many professional colleagues have offered criticism and encouragement. We are especially indebted to Sidney Luckenbach, California State University at Northridge, Malcolm Rennie, Australian National University, and an anonymous reviewer for Basil Blackwell, our U.K. publisher. Each of these philosophers has offered extremely valuable comments on the manuscript. Even though we have not adopted all their suggestions, we hope that they will like the final version and will recognize their own contributions to it. Charles Hamblin, University of New South Wales, is the philosophical godfather of this book: his early unpublished classification of modal relations gave rise to the worlds-diagrams herein; his success in introducing students to logic through modal, rather than truth-functional concepts, has served as a model for our approach.

Responsibility for the final shape and substance of the book lies squarely with the authors. The book contains many imperfections. We are aware of some of them but have not wished to fall prey to the perils of perfectionism by further delaying publication. Besides, we trust that errors of which we are not yet aware will be communicated to us by those who think the possible worlds approach worth promoting and who would like to see it brought closer to that state of perfection which only a non-actual possible world is likely ever to contain.

NOTE ON THE SECOND PRINTING

All typographical errors known to us have been corrected. Substantive changes occur on pages 78, 143, 146, 286, 295 and 296.

To the Teacher

Approach

Three main features determine the complexion of this book: (1) the way we characterize the *subject matter* of logic; (2) the way we characterize the *methodology* of logic; and (3) the fact that we present the science of logic in its *philosophical* rather than in its formal guise.

(1) The *subject matter* of logic, as we present it, is explicated mainly in terms of metaphysical talk about the set of all possible worlds and the ways in which concepts apply and propositions are true or false within those worlds.

Philosophical reflection about the foundations of logic — and of mathematics, for that matter — tends to make *metaphysical realists* (Platonists, if you like) of us all. Both of us (the authors) would, if we could, happily live with the economies of nominalism. But, like so many others, from Plato to Gottlob Frege, Hilary Putnam, and David Lewis, we have felt compelled to posit a modestly rich ontology of abstract entities. Our own catalog extends not only to numbers and sets, but also to concepts and propositions and — above all — to possible worlds. Those of you who think it possible to be more parsimonious will probably relish the task of showing how it can be done. At the very least, you should find our arguments grist for your own philosophical mills.

One of the chief attractions of the Leibnizian metaphysic of *possible worlds* is that — as Kripke, Hintikka, and others demonstrated in the early 1960s — it enables us to give a semantical underpinning to much of the machinery of formal logic. And it enables us to do this in a way which flows naturally from the simple intuitions which most of us — laymen and philosophers alike — have about such concepts as consistency, inconsistency, implication, validity, and the like. We are all (if we have any logical insights at all) disposed to say such things as: that an argument is valid just when its premises imply its conclusion; that one proposition (or set of propositions) implies another just when it isn't possible for the former to be true in circumstances in which the latter is false; that one proposition is inconsistent with another just when it isn't possible for both to be true; and so on. It is only a short step from these modes of talking to those of Leibniz's possible worlds. And the step is worth taking. For once we take it, we have at our disposal an extremely powerful set-theoretic framework in terms of which to explain all these, and many other basic concepts of logic and philosophy.

The set-theoretic framework which does so much of this explanatory work is depicted in this book by means of what we call *worlds-diagrams*. Our worlds-diagrams are grounded, in chapter 1, in simple and pictureable intuitions about the ways in which the truth-values of propositions may be distributed across the members of the set of all possible worlds. They are elaborated, in chapters 5 and 6, in such a way as to yield a new and effective decision-procedure for evaluating sentences and sentence-forms within propositional logic, both truth-functional and modal.

The logic whose formalism most adequately reflects our Leibnizian intuitions about implication, necessity, and the like is, of course, *modal* logic. More particularly, we believe and argue that it is that system of modal logic which C. I. Lewis called S5. Like William and Martha Kneale, we are persuaded that S5 is the system whose theses and rules "suffice for the reconstruction of the whole of logic as that is commonly understood".[1] We give to modal logic in general, and to S5 in particular, both the philosophical and the pedagogical primacy which we believe is their due. This is why, for instance, we introduce the modal concept of logical implication (what Lewis called "strict implication") early in chapter 1 and postpone discussion of the truth-functional concept of material conditionality (often

1. *The Development of Logic*, Oxford, Clarendon Press, 1962, p. 583.

misleadingly called "material implication") until chapter 5. It is, you will undoubtedly agree, a purely empirical question as to whether this order of presentation is pedagogically viable and desirable. We believe that it is — on the basis of experience. But we await the reports of others.

(2) The *methodology* of logic, as we present it, is explicated in terms of epistemological talk about the a priori methods of analysis and inference whereby knowledge of noncontingent propositions is possible.

The epistemology of logic is an essential, though often neglected, part of the philosophy of logic. We try to treat it with more than usual care and thoroughness.

Chapter 3 serves as an introduction to *epistemology in general*, with as much emphasis on noncontingent propositions as on contingent ones. To the question, What is the *nature* of human knowledge? we answer with a version of defeasibility theory. It would have been nice, perhaps, to have espoused one of the currently fashionable causal theories of knowledge. But causal theories, though plausible for cases of experiential knowledge, do not seem able — as at present formulated — to account for the kinds of a priori knowledge which we have in mathematics and logic. To the question, What are the *limits* of human knowledge? we answer by arguing for the unknowability in principle of at least some propositions: of some contingent ones by virtue of the falsity of verificationism; of some noncontingent ones by virtue of the truth of Gödel's Proof. To the question, What are the standard *modes of knowledge-acquisition* for humans? we offer two answers. One has to do with the natural history, as it were, of human knowledge: with its sources in experience and its sources in reason. The other has to do with the Kantian dichotomy between empirical and a priori knowledge: with whether or not it is possible to know a proposition by means other than experience. The two distinctions, we argue, are by no means equivalent. Thus it is, for example, that we are able to accommodate Kripke's claim that some necessary propositions of logic are knowable both experientially and a priori without in any way compromising Kant's belief in the exclusiveness and exhaustiveness of the empirical/a priori distinction. The upshot of all this is that we offer ten different categories — rather than the usual four — under which to classify the epistemic status of various propositions; and further, that our knowledge of the truths of logic turns out to be possible under just two of them.

Although we acknowledge, with Kripke, that parts of the subject matter of logic may on occasion be known experientially, it by no means follows that appeal to experience forms part of the distinctive methodology whereby that subject matter may be systematically explored. On the contrary, we argue, the methodology of logic involves two wholly a priori operations: the ratiocinative operations of *analysis* and of *inference*. In chapter 4 we first show how analysis — "the greater part of the business of reason", as Kant put it — and inference can yield knowledge of noncontingent propositions; and then go on to illustrate, by surveying the three main branches of logic — Propositional Logic, Predicate Logic, and (what a growing number of philosophers call) Concept Logic — the sorts of knowledge which these methods can yield.

(3) The *philosophy* of logic which we present in this book is resolutely antilinguistic in several important respects.

With respect to the *bearers of truth-values,* we argue against those who try to identify them with any form of linguistic entity, such as sentences, and quasilinguistic entities such as sentences taken together with their meanings. Propositions may be expressed by linguistic entities and apprehended by language-using creatures; but their existence, we hold, is not dependent upon the existence of either of these.

With respect to the notion of *necessary truth*, we argue against those who suppose that necessary truth can somehow be explained in terms of rules of language, conventions, definitions, and the like. The truth of necessary propositions, we hold, does not require a different kind of explanation from the truth of contingent propositions. Rather it consists, as does the truth of contingent propositions, in "fitting the facts" — albeit the facts in all possible worlds, not just in some.

With respect to the notion of *a priori knowledge*, we argue against those who suppose that a priori knowledge is best explained in terms of our understanding of words and sentences and of the linguistic rules, conventions, or definitions which they obey. The linguistic theory of the a priori was never so persuasive as when it went tandem with the linguistic theory of necessary truth. But having rejected the latter we felt free, indeed obliged, to reject the former. Our alternative account of a priori knowledge relies, as already noted, on the twin notions of analysis and inference. It is, of course, propositions and their conceptual constituents which — on our view — are the proper objects of analysis. We try to show how what we call "constituent-analysis", when combined with possible-worlds analysis (roughly, truth-condition analysis), can yield knowledge both of the truth-value and of the modal status of logical propositions. And we try to show, further, how rules of inference may be justified by these same analytical methods.

In short, we replace linguistic theories about truth-bearers and necessity with an ontological theory about propositions, possible worlds, and relations between them. And we replace the linguistic theory of a priori knowledge with an epistemological theory which gives due recognition to what philosophers have long called "the powers of reason". This is not to say that we ignore matters having to do with language altogether. On the contrary, we devote a lot of attention to such questions as how sentential ambiguity may be resolved, how ordinary language maps onto the conceptual notation of symbolic logic, how sentence-forms can be evaluated in order to yield logical knowledge, and so on. But we resist the view that the theory of logic is best viewed as a fragment of the theory of language. Just as the subject matter of logic needs to be distinguished from its epistemology, so both — on our view — need to be distinguished from theory of language.

Many philosophical theses, other than those just canvassed, are developed in this book. We touch on, and in some cases discuss at length, the views of dozens of philosophers and a few mathematicians. This is seemly in a book whose primary emphasis is on the philosophy, rather than the formalism, of logic. And it serves to explain why we have found room in this book for little more than a brief sketch (in chapter 4) of the broad territory of logic, and why chapters 5 and 6, although lengthy, do no more than introduce the elements of propositional logic (truth-functional and modal, respectively). Which brings us to:

Place in a Curriculum

The book is written so as to be intelligible, and hopefully even intriguing, to the general reader. Yet it is expressly designed for use as a textbook in first- or second-year courses at colleges and universities. It is not designed to replace handbooks in 'speed reasoning' or informal reasoning. Nor is it designed only for those students who plan to go on to more advanced work in philosophy and logic.

As we see it, *Possible Worlds* could, either in part or as a whole, serve as an *introduction to philosophy in general* — as an introduction, that is, to the logical and analytical methods adopted by contemporary analytical philosophers.

It could, either in part or as a whole, serve as an introduction to formal logic, in particular — as a philosophical introduction to the basic concepts with which formal logicians operate. So viewed, it should be used as a *prolegomenon to the standard curricular offerings in logic programs:* to all those courses, that is, which develop natural deduction and axiomatic techniques for propositional logic, quantification theory, and the like. For we touch on these and such-like matters only for purposes of illustration (if at all).

Again, the book could, either in part or as a whole, serve as the text for a belated *course in the philosophy of logic* — as a way of opening up philosophical questions about logic to those whose introduction to logic has been of the more traditional formal kind. It has been our experience — and that of countless other teachers of logic — that the traditional approach, of plunging students straight into the formalism of logic, leaves so many lacunae in their understanding that additional instruction in the philosophy of logic is sooner or later seen as a necessity. Our own preference, of course, is to preempt these sorts of problems by introducing logic, from the outset, as an integral part of philosophy — to be more specific, as that part of philosophy which serves as a foundation for mathematics, but whose most intimate philosophical ties are with metaphysics on the one hand and with epistemology on the other. But those of you who do not share our pedagogical predilections on this matter may nevertheless find that *Possible Worlds* will help your students, later if not sooner, to understand why logic is as important to philosophers as it is to mathematicians.

Ideally, the material in this book should be covered in a single course of something like 40 - 50 lecture hours. Alternatively, the material might be spread over two courses each of 20 - 25 lecture hours. Barring that, one will have to design shorter courses around particular selections from the book's six chapters. Here are two possiblilities:

A short course on 'baby' formal logic, which keeps philosophical discussion at a minimum and maximizes formal techniques in propositional logic, could be structured around chapters 1, 5, and 6 (with chapter 4, perhaps, being treated in tutorial discussions).

A short course on philosophical logic, which maximizes philosophical discussion and minimizes formal techniques, could be structured around chapters 1, 2, 3, and 4.

Still again, the book may be used as an ancillary text for courses whose primary focus is importantly different from ours. For example, for courses in philosophy of language, teachers may wish their students to read chapter 2; for courses in epistemology, teachers may wish their students to read chapter 3; and for courses which — like so many general introductions to philosophy — include a brief survey of logic, teachers may wish their students to read chapter 1 and selections from chapter 4.

Some Practical Hints

Many of the exercises — especially in chapters 1, 5, and 6 — since they require non-prose answers — may be more readily corrected (perhaps by exchanging papers in class) if the instructor prepares *worksheets* for students to complete.

Exercises which require the completion (e.g., by addition of brackets) of *worlds-diagrams* are best handled by distributing photocopies of figure *(1.i)*. We hereby give our permission for the unlimited reproduction of this figure.

You may find it useful to make the progressive construction of a *glossary of terms* a formal requirement in the course.

Finally, much of the material in the book lends itself to testing by *multiple-choice examinations*. We would be happy to send sample copies to teachers whose requests are made on departmental letterheads.

To the Student

Most of you already understand that logic is theoretically important as a foundation for mathematics and practically useful as a set of principles for rational thinking about any topic whatever.

But what may not be clear to you is that logic also forms an integral part of philosophy.

Since this book aspires to introduce you to logic in its philosophical setting, a few words of explanation are in order.

What, for a start, is philosophy? More concretely: What is it to be a philosopher? Our answer — which will suffice for present purposes — is this:

> To be a philosopher is to reflect upon the implications of our experience, of the beliefs we hold, and of the things we say, and to try to render these all consistent — or, as it were, to try to get them all into perspective.

Four of the terms we have just used are particularly significant: "reflect", "implications", "consistent", and "perspective".

Philosophers are, first and foremost, *reflective* persons. We believe — as did Socrates — that an unexamined life is shallow, and that unexamined beliefs are often mischievous and sometimes dangerous. This is why we tend to be perplexed about, and to inspect more carefully, matters which less reflective persons either take for granted or brush aside as of no practical value. Any belief, dogma, or creed — social, political, moral, religious, or whatever — is subject to the philosopher's critical scrutiny. Nothing is sacrosanct; not even the beliefs of other philosophers. Do not be dismayed, then, to find in this book arguments which criticize other philosophical viewpoints; and do not be disturbed, either, if your teachers when dealing with this book find reason to criticize our own viewpoint. Out of such dialectic, philosophical insight may be born and progress towards truth may be made.

Of the notions of *implication* and *consistency* we shall have much to say before long. For the moment, it will suffice to say that one way a philosopher, or anyone else for that matter, has of testing the credentials of any belief or theory is to ask such questions as these: "Is it *implied* by something we already know to be true?" (if so, it must itself be true); "Does it *imply* something we know to be false?" (if so, it must itself be false); and "is it *consistent* with other beliefs that we hold?" (if not, we are logically obliged, whether we like it or not, to give up at least one of them). Plainly, if you wish to be able to answer questions such as these you will need to know a good deal about the concepts of implication and consistency themselves. But these concepts are *logical* concepts. Little wonder, then, that logic is — as we said at the outset — an "integral part of philosophy". The trouble is, however, that when — with others in coffeehouses, pubs, or university classrooms, or, alone, in moments of reflective solitude — we ponder such deep philosophical questions as those about existence, freedom, responsibility, etc., most of us do not know how to handle, in the requisite disciplined way, the logical concepts on which such questions turn. Learning a little logic can help us to get our thinking straight in philosophy as well as elsewhere.

As for trying to get everything into *perspective*: that, it is probably fair to say, is usually thought to be the most distinctive goal of philosophers. You will need to learn a good deal of logic, and a lot more philosophy, before achieving the kind of lofty vantage point reached by such great thinkers as Plato, Aristotle, Leibniz, Hume, Kant, Russell, and Wittgenstein. But one must begin somewhere. And perhaps one of the best places to start is by reflecting on the fact that the world of human experience is but one of many that could have been — that the actual world is, as we shall say, only one of many possible worlds. Thinking about other possible worlds is — as Leibniz recognized — a way of getting our own world into perspective. It is also — as we try to show — a way of providing both an introduction to, and secure theoretical foundations for, logic itself. Hence, the title of this book.

1

Possible Worlds

1. THIS AND OTHER POSSIBLE WORLDS

The Realm of Possibilities

The year is A.D. 4272. Lazarus Long is 2360 years old. Although he has been near death many times, he hasn't — unlike his biblical namesake — required the intervention of a miracle to recover. He simply checks himself (or is taken by force) into a Rejuvenation Clinic from time to time. When we last hear of him he is undergoing rejuvenation again. The year is now 4291 and Lazarus is being treated in his own portable clinic aboard the star-yacht "Dora" after traveling back in time to his birthplace in Kansas and being "mortally wounded" in the trenches "somewhere in France".

All this, and much more, happens to the Lazarus Long of Robert A. Heinlein's novel *Time Enough for Love*.[1] In his novel, Heinlein starts with a framework of persons who actually lived (e.g., Woodrow Wilson and Kaiser Wilhelm II), of places that actually existed (e.g., Kansas City and France), and of events that actually occurred (e.g., U-boat attacks and the entry of the U.S. into World War I), and builds up a world of fictional persons, places, and events. He carries us with him, in make-believe, to another world different from our actual one: to a merely possible world.

How much of this other possible world is believable? How much of it is *really* possible? Much of it is credible. For instance, there *could* have been a man named "Ira Howard" who died in 1873 and whose will instructed his trustees to set up a foundation devoted to the prolongation of human life. For all we know, Ira Howard may be just as historical a personage as Woodrow Wilson. To be sure, Ira Howard *may* be just as much a creature of Heinlein's imagination as is Minerva — a computer become flesh and blood and one of Lazarus' many mistresses. But this matters not. For, whatever the historical facts happen to be, we can always suppose — *counterfactually*, as we say — that they might have been otherwise. We constantly make such suppositions in the world of real life. The world of fiction needs no special indulgence. We easily can, and daily do, entertain all sorts of unactualized possibilities about past, present, and future. We think about things that *might* have happened, *might* be happening and *might* be about to happen. Not only do we ruefully ask "What if things *had been* thus and thus?"; we also wonder "What if things *are* so and so?" and "What if things *were to be* such and such?" Counterfactual supposition is not mere idle speculation. Neither is it just a fancy of the dreamer or a refuge for the escapist. Given that we are so often ignorant of what *is*, we need a rich

1. New York, G.P. Putnam's Sons, 1973.

sense of what *might be.* In matters of practice, we need to consider alternatives where knowledge is denied us. In matters of theory, we need to consider hypotheses where facts are unknown.

Actuality is, as it were, surrounded by an infinite realm of possibilities. Or, as we might otherwise put it, our actual world is surrounded by an infinity of other possible worlds. No wonder then that fiction writers like Heinlein have little difficulty in beguiling us with their stories of possibilities, most of which will never be actualized. Might there not come a time when incest will be socially and legally acceptable, when human life will be prolonged by periodic visits to Rejuvenation Clinics, when computers will be embodied in human flesh, or when travel from one galactic colony to another will be a commonplace? Maybe our everyday counterfactual suppositions are much more mundane. But who among us would rule the exciting ones entirely out of order? The fact is that we can, and do, conceive of social and legal, biological and technological, and perhaps even of physical, possibilities which the world of fact may never encompass.

What are the limits to the possible?

But are there not limits of some sort to what we can conceive or suppose to be possible? Does just anything go? How about time-travel, for instance? In Heinlein's novel, Lazarus recounts how he assumed a biological age of thirty-five so as to travel back in time twenty-three hundred years to observe how things were in his childhood and (as it turns out) to fall in love with his own mother.

SOME EVENTS IN THE LIFE OF LAZARUS LONG, SENIOR MEMBER OF THE HOWARD FAMILIES

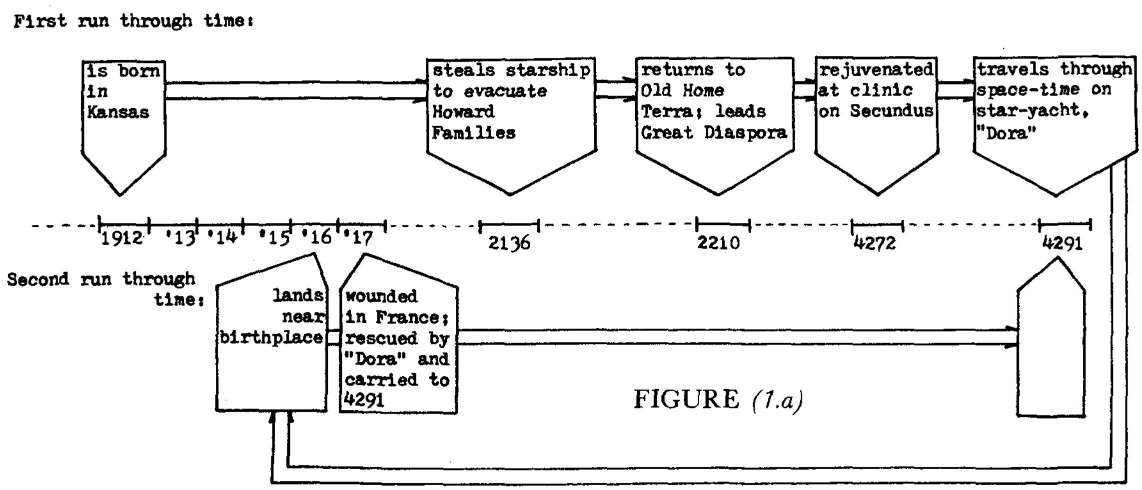

FIGURE *(1.a)*

Could time-travel occur? Can you be sure that there is not some other possible world in which it occurs even if it never occurs, or will occur, in our actual world? Is there any paradox in the idea of Lazarus observing himself as a four-year-old? See exercise 4 on p. 25.

Many of us may feel that the very concept of time-travel is paradoxical. If time-travel were possible then should we not be able to imagine a person traveling back in time and fathering himself? But is this really possible? Our minds boggle a bit. It is supposed by some that here we have reached the limits of conceivability, and that the world which Heinlein portrays through Lazarus is, in this respect at least, not in any sense a possible world. This, as it happens, is the view of another of the novel's characters: Carolyn Briggs, chief archivist of the Howard Foundation. As she put it in her Preface to the Revised Edition of Lazarus' memoirs: "An apocryphal and obviously impossible tale of the last events in his life has been included at the insistence of the editor of the original memoir, but it cannot be taken seriously." On the other hand, some of us may feel that time-travel, of the kind that Lazarus is supposed to have undertaken, is not wholly inconceivable; that paradox can be avoided; and that a world in which time-travel occurs may someday be seen to be possible if and when our concepts of space-time become more sophisticated. We may want to agree with Carolyn's predecessor — Justin Foote, chief archivist emeritus — when he appends the following note to her Preface: "My lovely and learned successor in office does not know what she is talking about. With the Senior [i.e., Lazarus], the most fantastic is always the most probable."[2]

But whatever we say about time-travel — whether we think it falls within or beyond the limits of possibility — it is certain that some things are *not* possible no matter how sophisticated our concepts become. The supposition that time-travel both *will* occur sometime in the future and will *not* occur anytime in the future, is a case of point. It takes us, in a sense, beyond the bounds of conceivability. It is not just paradoxical. It is, as we say, flatly self-contradictory. A supposed world in which something literally both is the case and is not the case is not, in any sense, a possible world. It is an impossible one.

Possibility is not the same as conceivability

So far we have spoken as if the boundary between the possible and the impossible were a function of human psychology: as if, that is, it coincided with the boundary between that which we find conceivable and that which we find inconceivable. Yet, while there is a great amount of overlap between what is conceivable and what is possible, the two are not the same.

In the first place, it should be clear on reflection that our inability to conceive of a certain state of affairs does not imply the impossibility of that state of affairs. Notoriously, there was a time when our ancestors thought it inconceivable that the earth should be round. Yet obviously the possibility of the earth's being round was in no way limited by their inability to conceive it.

Secondly, our seeming ability to conceive of a certain state of affairs does not imply the possibility of that state of affairs. For many centuries, mathematicians sought a means of squaring the circle (sought a procedure whereby to construct for any given circle a square of equal area). Plainly, they would not have done so unless they thought it conceivable that such a procedure existed. Yet we now know, and can prove, that the squaring of the circle is wholly beyond the bounds of possibility — that the concept itself is self-contradictory.

But if, as our first argument shows, conceivability is neither a necessary condition (conceivability is not *needed* for something to be possible), nor, as our second argument shows, a sufficient condition (conceivability doesn't *suffice* to establish possibility), what then *are* the conditions for something's being possible?

One might be tempted to answer that it is *conceivability without inconsistency* or *coherent conceivability* — not just conceivability itself — which is the measure of possibility. After all, although certain of our ancestors apparently did not have the psychological capacity to conceive of the earth being round, the concept of the earth being round is itself a perfectly coherent or self-consistent concept and hence is one of which they could — in the requisite sense — have conceived without inconsistency. And, again, although generations of mathematicians thought that the squaring of the circle was a goal which one could intelligibly pursue, we now know that they could not conceive of that goal without inconsistency.

Unfortunately, this answer will not do; or rather, it will not do from the standpoint of those who wish — as we do in this book — to explain the principal concepts of logic in terms of the notion of a possible world. For among those principal concepts are the concepts of consistency and inconsistency. And if we then invoke the concept of consistency in order to explain what a possible world is and how a possible world differs from an impossible one, we expose ourselves to a charge of circularity.

2. *Time Enough For Love*, p. xvii. [It will be our practice to give the complete bibliographical reference only on the first occasion in each chapter of our citing a work. Thereinafter we shall omit the details of publication.]

Our predicament is apparently this. On the one hand, we wish to avoid *psychologism,* viz., any form of theory which makes logic a function of human psychology. In its older form, psychologism held that the laws of logic were "the laws of thought". It treated the science of logic as if it were a branch of psychology concerned with the description of how human beings actually reason. Psychologism, in its traditional form, is now dead, largely as a result of the efforts, early in this century, of Frege, Russell, and Wittgenstein. We wish to avoid introducing it in a new form. Yet that is what we would be doing if we were to explain the principal concepts of logic in terms of possible worlds and then go on to explain possible worlds themselves in terms of the purely psychological concept of conceivability. On the other hand, we wish to avoid the *circularity* in which we would be involved if we tried to define possible worlds in terms of the logical notion of consistency, and then later defined consistency in terms of possible worlds.

Fortunately, there is a way out of this predicament. We can avoid circularity and still not be trapped by psychologism. Consider, for a moment, how the charge of circularity in definitions may in other cases be avoided. We look in a dictionary, for example, to find out what it is for something to be complex and find that the concept of complexity is opposed to that of simplicity; we try to find out what it is for something to be simple and find that the concept of simplicity is opposed to that of complexity. Yet such a circle of definitions can be, and standardly is, broken. It is broken by *citing examples:* sometimes by ostension (pointing); sometimes by naming; and sometimes by description. Thus, in the present case we can avoid the trap of circularity — and at the same time, the seduction of psychologism — by citing clear-cut examples (they are often called "paradigm examples") of possible worlds, and equally clear-cut examples of impossible worlds. Intuitively, we would want to include, among the possible worlds, worlds in which there are more objects than in the actual world (e.g., in which the earth has two moons); worlds in which there are fewer objects than in the actual world (e.g., in which the earth has no moon at all); worlds in which the same objects exist as in the actual world but have different properties (e.g., in which the long-supposed "canals" on Mars turn out, after all, to be relics of some past civilization); and so on. And intuitively, too, we would want to include, among the impossible worlds, worlds in which circles can be squared, worlds in which there is an even square root of nine, worlds in which time-travel both does and does not occur, and so on.

To be sure, descriptions can be given of worlds about whose possibility or impossibility we have no clear intuitions. But for nearly any distinction that we care to think of there will be problematic cases lying near the borderline of that distinction. The case of worlds in which time-travel occurs is just such a case; and so, until recently, was the case of worlds in which more than four colors are needed to demarcate between countries on a plane map. But the difficulty of deciding, for certain cases, on which side of the distinction they are to be located should not be allowed to discredit the distinction itself. The distinction between possible and impossible worlds is grounded in an appeal to paradigm cases. We do not have to be able to settle all boundary disputes in order to have a secure enough footing on which to proceed in our attempts to explain the workings of logic.

Possible worlds: actual and non-actual

Let us pause at this point and reflect, in philosophical fashion, on some of the things we have been saying. We have been working with a threefold distinction, which is implicit in much of our thinking, between the actual world, worlds which are non-actual but possible, and worlds which are neither. But what precisely do we mean by "possible world"? More basically still: What precisely do we mean by "the actual world"?

When we speak of "the actual world" we do not mean just the planet on which we live. Nor do we mean our solar system, or even our galaxy. We have spoken of the actual world in an all-encompassing way so as to embrace all that really exists — the universe as a whole.

Again, when we speak of "the actual world" we do not mean just the universe as it is now, in the present. When we identify it — as above — with all that really exists, we are using "exists" in a timeless sense, so as to encompass not only what exists now but also what once existed in the past and what will come to exist in the future. The actual world embraces all that was, is, or will be.

Now it is clear that the actual world is a possible world. If something actually exists then it is obviously possible that it exists. On the other hand, not everything that possibly exists does so actually. Not all possible worlds are actual. It follows, therefore, that the actual world is only one among many possible worlds: that there are possible worlds other than ours. Moreover, given that by "the actual world" we mean — as we agreed a moment ago — everything that was, is, or will be the case, it follows that by "another possible world" we do not mean some planet, star or whatnot that actually exists and which is located somewhere "out there" in physical space. Whatever actually exists, it must be remembered, belongs to the actual world even if it is light-years away. Other, non-actual, possible worlds, are not located anywhere in physical space. They are located, as it were, in conceptual space; or rather, as we may prefer to say, in logical space.

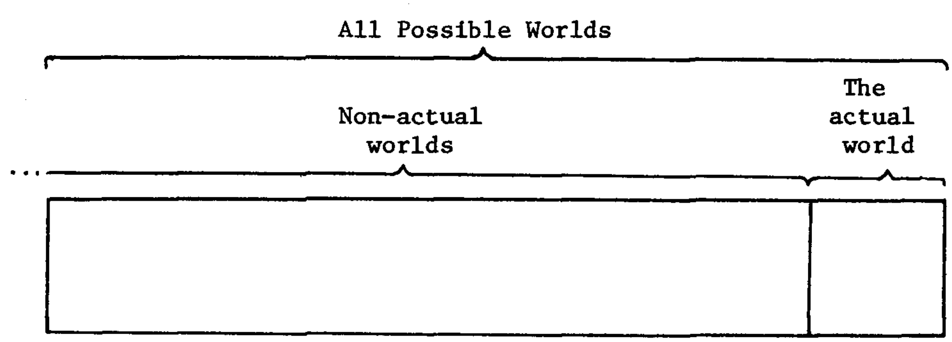

FIGURE *(1.b)*

Our world (everything that actually was, is, or will be) is only one of an infinite number of possible worlds. It is the actual world. The others are non-actual.

Note that in this and other similar diagrams we represent an infinite number of possible worlds by a rectangle of finite size. The rectangle may be thought of as containing an infinite number of points each of which represents a different possible world. It is only for the sake of diagrammatic convenience that we represent the actual world on this and a few subsequent figures by a segment of the rectangle rather than by a single point. From a logical point of view the actual world has little (if any) claim to privileged status. Indeed, in most of the worlds-diagrams featured later we shall have no need to make mention of the actual world, let alone to feature it prominently.

Note, further, that the bracket for non-actual worlds is left open at the left-hand side of this diagram and all diagrams on which the actual world is featured. This is to signify that there are more non-actual worlds than we have here depicted. The class of non-actual worlds contains all possible worlds other than the actual world and contains as well all impossible worlds. Every impossible world is a non-actual world.

Logical possibility distinguished from other kinds

By "a possible world", it should be emphasized, we do *not* mean only a *physically* possible world. Countless worlds which are physically *im*possible are numbered among the possible worlds we are talking about. Physically possible worlds form a proper subset of all possible worlds; or, to make the contrast somewhat sharper, we might say that the set of physically possible worlds forms a proper subset of all *logically* possible worlds.

A physically possible world is any possible world which has the same natural laws as does the actual world.[3] Thus the logically possible world depicted in Charles Dickens' novel *David Copperfield* is a physically possible one: no event in that novel violates any natural law. On the other hand, Washington Irving's short story *Rip Van Winkle* describes a physically impossible world: a world in which a person sleeps without nourishment for twenty years. The latter circumstance, a person's living for twenty years without nourishment (energy intake), violates certain laws of thermodynamics. Nonetheless, although such a situation is thus physically impossible, it is not logically impossible. In

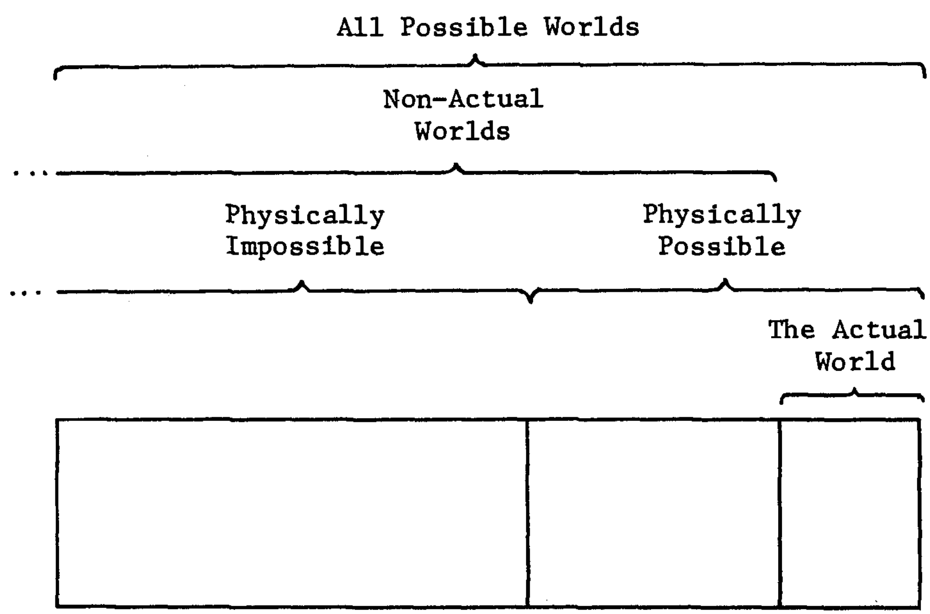

FIGURE *(1.c)*

3. Note that, on this account, the notion of a physically possible world is parasitic upon (needs to be defined in terms of) the broader notion of a logically possible world. Similarly, a state of affairs is physically possible, it is usually said, if its description is *consistent with* the natural laws (of the actual world). Yet the relation of consistency — as we show in section 4 — is itself to be defined in terms of possible worlds, i.e., of *logically* possible worlds.

some non-actual, physically impossible, but logically possible world in which the natural laws are different from those in the actual world, Rip Van Winkle does sleep (without nourishment) for twenty years. Of course, not every physically impossible world will be logically possible. The physically impossible world in which Rip Van Winkle sleeps for exactly twenty years without nourishment and does not sleep during those years without nourishment is a logically impossible one as well.

The class of *logically* possible worlds is the *most inclusive* class of possible worlds. It includes every other kind of possible world as well: all those that are physically possible and many, but not all, that are physically impossible; and it includes all worlds which are technologically possible, i.e., physically possible worlds having the same physical resources and industrial capacity as the actual world, and many, but not all, which are technologically impossible.

Very many different subsets of the set of all logically possible worlds may be distinguished. Many of these are of philosophical interest, e.g., physical, technological, moral, legal, etc. But we shall not much concern ourselves with them. For the most part, in this book our interest lies with the largest class of possible worlds, the class of logically possible worlds, and on a few occasions with the class of physically possible worlds. It is only in more advanced studies in logic that the special properties of various of these other subsets are examined.

Hereinafter, whenever we use the expression "all possible worlds" without any further qualification, we are to be understood to mean "all *logically* possible worlds".

The constituents of possible worlds

How are possible worlds constituted? It may help if we start with the possible world that we know best: the actual one. Following Wittgenstein, we shall say that the actual world is "the totality of [actually] existing states of affairs", where by "a state of affairs" we shall mean roughly what he meant, viz., an arrangement of objects, individuals, or things having various properties and standing in various relations to one another. It will help, however, if we adopt a slightly different terminology. Instead of the terms "objects", "things", and "individuals" we shall adopt the more neutral term "items".[4] And in addition to the terms "properties" and "relations" we shall adopt the more generic term "attributes".[5] An item (object or thing) is whatever exists in at least one possible world: physical objects like Model T Fords and starships; persons such as Woodrow Wilson and Lazarus Long; places such as Old Home Terra (the earth) and Secunda; events such as World War I and the Great Diaspora of the Human Race; abstract objects such as numbers and sets; and so on. An attribute (property or relation) is whatever is exemplified or instanced (has instances) by an item or by items in a world; properties such as being red, being old, being distant, or being frightening; and relations such as being faster than, being a lover of, being more distant than, or being earlier than. Typically, items are the sorts of things we would have to mention in giving a description of a possible world; they are things we can refer to. Attributes are the sorts of things that characterize items; they are the sorts of things which we ascribe to the objects of reference.

Now it is clear, from the examples just given of items and attributes, that items exist in possible worlds other than the actual one, and that attributes are instanced in possible worlds other than the

4. Other terms which play roughly the same role in philosophical literature are "particulars" and "logical subjects".

5. The term "attribute" with the meaning we have given it was introduced into recent philosophical literature by Carnap. As he put it: "In ordinary language there is no word which comprehends both properties and relations. Since such a word would serve a useful purpose, let us agree in what follows that the word 'attribute' shall have this sense. Thus a one-place attribute is a property, and a two-place (or a many-place) attribute is a relation." *Introduction to Symbolic Logic and Its Applications*, New York, Dover, 1958, p. 5.

actual one. How, then, do non-actual possible worlds differ from the actual one? They may differ in three basic ways. Other possible worlds may contain the very same items as the actual world but differ from the actual world in respect of the attributes which those items instance; e.g., differ in respect of the Eiffel Tower being purple, the Taj Mahal being green, and so on. Or they may contain at least some items which do not exist in the actual world (and ipso facto differ in respect of some attributes); e.g., they may contain Lazarus Long or Sherlock Holmes. Or still again, they may lack certain items which exist in the actual world (and ipso facto differ in respect of some attributes); e.g., they may lack Stalin or Shakespeare.

Some philosophers have thought that other possible worlds can differ from the actual one only in the first of these three ways. Other philosophers insist that they can differ from the actual one in the other two ways as well. Our own thinking in the matter is evident: in choosing the world of *Time Enough for Love* as our example, we are allowing the existence of non-actual possible worlds some of whose items do not exist in the actual world. In chapter 4, section 6, we will explore briefly some of the consequences of being less generous.

EXERCISES

In this and subsequent exercises, it is essential that one assume that all the terms being used have their standard meanings, e.g., that "square" refers to a plane, closed, four-sided figure having equal interior angles and equal sides. To be sure, for any word or sentence we care to think of, there will be possible worlds in which that word or sentence will mean something altogether different from what it means in the actual world. This fact, however, in no way tells against the fact that what *we* refer to by the word "square" has four sides in all possible worlds. The exercises posed ask whether the claims *we* are here considering are true in any possible world; and this is a wholly distinct matter from whether our words might be used by the inhabitants of other possible worlds to make different claims.[6]

Part A

For each of the following, say whether there is a logically possible world in which it is true.

1. *Frederick, of Gilbert and Sullivan's "Pirates of Penzance", has reached the age of 21 years after only 5 birthdays.*
2. *There is a square house all of whose walls face south.*
3. *Epimenides, the Cretan, spoke in truth when he said that everything that Cretans say is false.*
4. *Mt. Everest is lower than Mt. Cook.*
5. *Mt. Everest is lower than Mt. Cook, Mt. Cook is lower than Mt. Whistler, and Mt. Whistler is lower than Mt. Everest.*
6. $2 + 2 \neq 4$.
7. *The Pope believes that* $2 + 2 \neq 4$.
8. *The Pope knows that* $2 + 2 \neq 4$.

6. For more on this point, see our discussion in chapter 2, pp. 110ff, of the uni-linguo proviso.

9. *No objects are subject to the law of gravitation.*

10. *There is a mountain which is higher than* every *mountain.*

Part B

The following quotations are for reflection and discussion. For each of them try to decide whether the circumstance described could occur (a) in a logically possible world, and (b) in a physically possible world.

1. *" 'All right,' said the Cat; and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone." (Lewis Carroll,* Alice's Adventures in Wonderland & Through the Looking Glass, *New York, Signet Classics, 1960, p. 65.)*

2. *"As Gregor Samsa awoke one morning from uneasy dreams he found himself transformed in his bed into a gigantic insect. He was lying on his hard, as it were armor-plated, back and when he lifted his head a little he could see his dome-like brown belly divided into stiff arched segments on top of which the bed quilt could hardly keep in position and was about to slide off completely. His numerous legs, which were pitifully thin compared to the rest of his bulk, waved helplessly before his eyes." (Franz Kafka,* The Metamorphosis, *1912, translated by Willa and Edwin Muir, New York, Schocken Books, 1948, p. 7.)*

3. *"NAT BARTLETT is very tall, gaunt, and loose-framed. His right arm has been amputated at the shoulder, and the sleeve on that side of the heavy mackinaw he wears hangs flabbily or flaps against his body as he moves. . . . He closes the door and tiptoes carefully to the companionway. He ascends it a few steps and remains for a moment listening for some sound from above. Then he goes over to the table, turning the lantern very low, and sits down, resting his elbows, his chin on his hands, staring somberly before him." (Eugene O'Neill,* Where the Cross is Made, *copyright 1919 by Boni and Liveright. Reprinted in* Twelve One-Act Plays for Study and Production, *ed. S. Marion Tucker, Boston, Ginn and Co., 1929, pp. 202, 208.)*

2. PROPOSITIONS, TRUTH, AND FALSITY

Truth and falsity defined

Items (i.e., objects, things), we have said, may exist in possible worlds other than the actual one. Likewise, attributes (i.e., properties and relations) may be instanced in possible worlds other than the actual one. Consider, now, any arbitrarily selected item and any arbitrarily selected attribute; and let us name them, respectively, a and **F**. Then we can *define* "truth" and "falsity" as follows:

(a) it is *true* that a has **F** if, and only if, a has **F**;

and

(b) it is *false* that a has **F** if, and only if, it is not the case that a has **F**.

These definitions tell us, for instance, that where "a" stands for Krakatoa Island and "**F**" stands for the property of being annihilated in a volcanic eruption, then

> It is true that Krakatoa Island was annihilated by a volcanic eruption if and only if Krakatoa Island was annihilated by a volcanic eruption

and again, that

> It is false that Krakatoa Island was annihilated by a volcanic eruption if and only if it is not the case that Krakatoa Island was annihilated by a volcanic eruption.

These definitions accord with the insight which Aristotle, more than two thousand years ago, expressed thus:

> To say of what is that it is not or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true.[7]

Several points about these definitions are worth noting:

(i) Although Aristotle's account suggests that it is persons' sayings which are true or false (i.e., which are, as we shall put it, the bearers of truth-values), our account in (a) and (b) leaves open the question as to what it is which is true or false. There is no doubt that sayings, along with believings, supposings, etc., are *among* the things which can be true or false. But many contemporary philosophers prefer to say that it is primarily *propositions* which have the properties of truth or falsity (i.e., that it is primarily propositions which are bearers of *truth-values*), and would hold that the things persons say, believe, suppose, etc., are true or false just when the propositions which they utter, believe, or entertain, etc., are true or false. For the reasons given at length in chapter 2, we adopt the latter way of talking.[8] If we let the letter "P" stand for the proposition that *a* has **F**, we can restate (a) and (b) as

(a)* The proposition P (that *a* has **F**) is true if and only if *a* has **F**;

and

(b)* The proposition P (that *a* has **F**) is false if and only if it is not the case that *a* has **F**.

(ii) Our account, in (b) and (b)*, of the conditions in which the proposition that *a* has **F** is false, allows for two such conditions: the possible state of affairs in which the item *a* exists but fails to have the attribute **F**; and the possible state of affairs in which the item *a* does *not* exist. Since an attribute can be instanced by an item in a possible world only if that item exists in that possible world, the failure of an item to exist in a given possible world precludes it from having any attributes whatever in that world.

(iii) Strictly speaking, our account — and Aristotle's — of what it is for a proposition to be true or false applies only to propositions which ascribe *properties* to items. But it is easily enough extended to deal with those propositions which ascribe *relations* to two or more items. We can deal with so-called "two-place" attributes (i.e., relations holding between just two items) as follows: where P is a proposition, *a* and *b* are items, and **R** is the two-place attribute (relation) which P asserts to hold between *a* and *b*, then P is true if and only if *a* and *b* stand to each other in the relation **R**, while P is

7. *Metaphysics,* Γ, 7(1011^b 26 - 27), *The Basic Works of Aristotle,* ed. R. McKeon, New York, Random House, 1941.

8. A proposition, we shall argue, is to be distinguished from the various sentences which language-speakers may use to express it, in much the same way as a number is to be distinguished from the various numerals which may be used to express it.

false if and only if it is not the case that *a* and *b* stand in the relation **R**. And accounts of truth-values for other relational propositions involving three or more items may be constructed along similar lines. Alternatively, we can deal with relational propositions by pointing out that whenever an item *a* stands in a relation **R** to one or more other items, *b, c*, etc., *a* can be said to have the *relational property* of standing in that relation to *b, c*, etc. And since relational properties are properties, the straightforward account given in (a) and (b) suffices.

(iv) The account of truth which we are here espousing has been described variously as "the Correspondence Theory", "the Realist Theory", or even "the Simple Theory" of truth. In effect, it says that a proposition, P, is true if and only if the (possible) state of affairs, e.g., of *a*'s having **F**, is as P asserts it to be. It defines "truth" as a property which propositions have just when they "correspond" to the (possible) states of affairs whose existence they assert. It is a "realist" theory of truth insofar as it makes truth a real or objective property of propositions, i.e., not something subjective but a function of what states of affairs really exist in this or that possible world. And it is a "simple" theory of truth insofar as it accords with the simple intuitions which most of us — before we try to get too sophisticated about such matters — have about the conditions for saying that something is true or false.

(v) If it seems to some that this theory is overly simple and not profound enough, this is probably because it is deceptively easy to confuse the question "What are the conditions of truth and falsity?" with the question "What are the conditions for our *knowledge* of truth and falsity?" It is not surprising, as a consequence, that the most commonly favored alleged rivals to the Correspondence Theory are the so-called Coherence and Pragmatist Theories of Truth. Proponents of these theories tend either to deny or to ignore the distinction between a proposition's *being* true and a proposition's *being known* to be true. We may accept the coherence theorist's claim that one way of getting to know what propositions are true is to determine which of them cohere with the rest of the beliefs that we hold to be true. And we may also accept the pragmatist's claim that one way of getting to know what propositions are true is to determine which of them it proves useful or practical to believe. But at the same time we would wish to point out that neither claim warrants *identifying* truth with coherence or truth with practical usefulness. And further we would point out that both theories are parasitic upon the "simple" theory of truth. The pragmatist states that certain hypotheses *are* useful; the coherence theorist states that certain sets of beliefs *are* coherent. Yet each, in so doing, is implicitly making a claim to the *simple* truth of *these* propositions. The concept of simple truth, it seems, is not easily dispensed with. Even those who would like most to avoid it cannot do so.

Truth in a possible world

It should be evident from our definitions (i) that a proposition, P, may assert that an item *a* has attribute **F** even if *a* exists only in a non-actual possible world, and (ii) that P will be true in that non-actual world provided that in that world *a* has **F**. Let us use the letter "W" (with or without numerical indices) to refer to any possible world of our choosing (e.g., the world of Heinlein's *Time Enough for Love*). Then our definitions allow us, for instance, to assert the truth in some specified possible world, W_1, of the proposition that Lazarus Long has the relational property of being a lover of Minerva even though in some other specified possible world, W_2 (e.g., the actual world), he does not exist and a fortiori does not have that property. Likewise, they allow us to assert the falsity in some specified possible world, W_2, of the proposition that Lazarus had Minerva as a lover even though in some other specified possible world, e.g., W_1, that proposition is true. Needless to say, they also allow us to assert that a proposition is true (or false) in some *un*specified world, W; or, as we shall later put it, to assert that a proposition is possibly true (or false, as the case may be).

Truth in the actual world

It follows from what we have just said that — contrary to what is often supposed — the expressions "is true" and "is false" do *not* mean the same as "is actually true" and "is actually false". To say that a proposition is actually true (or false) is to say that among the possible worlds in which it is true (or false) there is the actual world. Thus, for instance, the proposition that Canada is north of Mexico is actually true because in the *actual* world Canada stands in the relation of being north-of to Mexico, i.e., has the relational property of being north of Mexico. And equally, the proposition that Lazarus has Minerva as a lover is actually false because in the *actual* world Lazarus fails to exist.

The fact that "true" does not mean "actually true" should be evident from two considerations. First, if it did, then the occurrence of the qualifier "actually" would be wholly redundant (pleonastic). Yet, as we have already seen, it is not. Secondly, if it did mean the same, then persons in other possible worlds would not be able to invoke the concept of truth without thereby making claims about *this* world, the actual one in which you and we find ourselves. Consider, for illustrative purposes, Lazarus' claim (on the first page of the narrative of *Time Enough for Love*): "It's true I'm not handy with the jabber they speak here." The year is A.D. 4272 and the place is the fictional Howard Rejuvenation Clinic on the fictional planet of Secundus. Lazarus asserts that a certain proposition, viz., that he doesn't have facility with the local language, is true. We understand him to be saying something which is true of the fictional world in which he exists. Yet if "true" *meant* "actually true" we should be obliged to understand him as saying something very different — as saying something about his lack of facility with a language that exists in the actual world which you and we inhabit.

Although "true" does not *mean* "actually true", it can be — and often is — used to *refer* to actual truth; what matters is *who* uses it. When persons in the actual world claim that a proposition is true they are claiming that it is true in their own world; and since their world is the actual world, it turns out that they are attributing actual truth to the proposition. However, when a person in a non-actual world attributes truth to a proposition, he is attributing to that proposition truth in his *own* world; and since his world is *not* the actual world, it turns out that he is attributing non-actual (but possible) truth to that proposition.

The myth of degrees of truth

It is worth pointing out that it is implicit in both Aristotle's account and the one in (a)* and (b)* that truth and falsity do not admit of degrees. Although there is a common manner of speech which fosters this belief — for example, "Jones' report was more true than Roberts'" — strictly speaking this manner of speech is logically untenable. A proposition is either wholly true or wholly false. There is no such thing as partial truth. Consider the simplest sort of case, that in which a proposition P ascribes an attribute **F** to an item *a*: P is true if *a* has attribute **F**, otherwise P is false. There is no provision, no room, in this explication of truth, for P to be anything other than wholly true or wholly false; for either *a* has the attribute **F**, or it is not the case that *a* has the attribute **F**.

There is, however, a way to reconcile the common manner of speech just reported with the stringencies of our definition of truth; and that is to regard such utterances as "Jones' report was more true than Roberts'" as elliptical for something of this sort: "A greater number of the details reported by Jones were true than those reported by Roberts." Similarly, an article in *Time* magazine[9] which bore the title "How True is the Bible?" could instead have been better presented under the title "How Much of the Bible is True?"

9. Dec. 30, 1974.

3. PROPERTIES OF PROPOSITIONS

Among the items which exist in various possible worlds must be included propositions; and among the attributes which propositions instance in various possible worlds are the properties of truth and falsity. In every possible world each proposition has one or other of these properties of truth and falsity. But truth and falsity are not the only properties which propositions can have. By virtue of the fact that in any given possible world, a proposition has either the property of truth or the property of falsity, we can distinguish other properties which a proposition can have, viz., possible truth, possible falsity, contingency, noncontingency, necessary truth, and necessary falsity. These are properties — *modal properties* we shall call them — which propositions have according to the way in which their truth-values are distributed across the set of *all* possible worlds; according, that is, to whether they are true (or false) in just some, in all, or in none of the totality of possible worlds.[10] In distinguishing them we begin to approach the heart of logic itself.

Possibly true propositions

Consider, for a start, those propositions which are true in at least one possible world. The proposition that a particular item, Lazarus, had a particular attribute, that of having lots of time for love in his twenty-three hundred odd years, is a case in point. It is a proposition which is true in that possible though (so far as we know) non-actual world of Heinlein's novel. We shall say that it is *possibly true*, or again that it is a *possible truth*. Another example of a proposition which is true in at least one possible world is the proposition that Woodrow Wilson was president of the U.S. in 1917. In this case, of course, one of the possible worlds in which the proposition is true is also the actual one. Nevertheless it makes sense to say that this proposition is a possible truth even if, in saying so, we are not saying all that we are entitled to say, viz., that it is also an actual truth. For actual truths form a subclass of possible truths. A proposition is a possible truth, then, if it is true in at least one possible world — actual or non-actual. A proposition is *actually true* if among the possible worlds in which it is true there occurs the actual world.

In saying of a proposition that it is true in at least one possible world it should not be thought that it is also being claimed that that proposition is false in some other possible world. When we say that a proposition is true in some possible worlds we leave it an open question as to whether that proposition is true in all other possible worlds as well, or is false in some of those other possible worlds.

Possibly false propositions

How about propositions which are false in at least one possible world? Such propositions, we shall want to say, are *possibly false*, or are *possible falsities*. Now some (but, as we shall see later, not all) propositions which are possibly true are also possibly false. The proposition that Lazarus had lots of time for love is not only possibly true, because there is a possible world in which it is true; it is also possibly false, because there are other possible worlds — our own actual one among them — in which (so far as we know) it is false, i.e., actually false. Similarily, the proposition that Woodrow Wilson was president of the United States in 1917 is not only possibly (as well as actually) true, because there is a possible (as it happens the actual) world in which it is true; it is also possibly false, because there are other possible worlds — our own actual one *not* among them — in which it is false.

10. Note that although attributions of noncontingent truth and noncontingent falsity *will* count as attributions of modal status, the corresponding attributions of contingent truth and contingent falsity will *not*. The reason for allowing an ascription of contingency to count as an ascription of modal status but not allowing ascriptions of contingent truth or contingent falsehood is given in chapter 6, section 5.

Contingent propositions

Any proposition which not only is true in some possible worlds but also is false in some possible worlds is said to be a *contingent proposition.* A contingent proposition, that is, is both possibly true and possibly false. The proposition about Woodrow Wilson is contingent and happens to be true in the actual world, while the proposition about Lazarus is contingent and happens to be false in the actual world. The former is a possible truth which *could* have been false, even though as a matter of actual fact it is not; the latter is a possible falsity which *could* have been true, even though as a matter of actual fact it is not.

Contradictories of propositions

Suppose we have a proposition which is contingent and true in the actual world, such as the proposition that the U.S. entered World War I in 1917: then it is true in some possible worlds, including the actual one, but false in all those possible worlds in which it is not true. What might such a possible world be in which it is false that the U.S. entered World War I in 1917? It could be, for example, a possible world in which the U.S. entered World War I, not in 1917, but in 1918; or, it might be a possible world in which World War I never took place at all, perhaps because universal peace had been established in that world some years earlier, or because mankind had managed to destroy itself quite accidently in 1916; or again it might be a possible world in which the North American continent, and hence the U.S., simply did not exist. In each and every one of these latter possible worlds, and in countless others besides, it is false that the U.S. entered World War I in 1917, or in other words, it is true that it is not the case that the U.S. entered World War I in 1917.

Let us call any proposition which is true in all those possible worlds, if any, in which a given

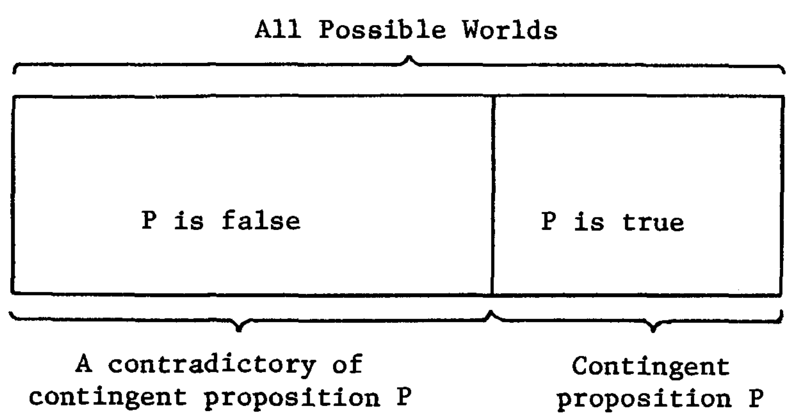

FIGURE *(1.d)*

A contingent proposition, P, is represented as being true in some possible worlds and false in all the others. Any contradictory of proposition P will then be false in all those possible worlds in which P is true and true in all those possible worlds in which P is false.

proposition is false, and which is false in all those possible worlds, if any, in which a given proposition is true, a *contradictory* of that proposition.[11] Then the proposition that it is not the case that the U.S. entered World War I in 1917 is a contradictory of the proposition that the U.S. did enter World War I at that time, and vice versa. The proposition that it is not the case that it did, since it is a contradictory of a true contingent proposition, will be contingent and false. It will be false, that is, in all those possible worlds, including the actual one, in which the U.S. entered World War I in 1917. Again, suppose we have a proposition which is contingent and false, such as the proposition that no fighting occurred in France during World War I. Then, although this contingent proposition will be false in some possible worlds including the actual one, there will be other possible worlds in which it is true. These other possible worlds will be precisely those in which a contradictory of that proposition, e.g., that fighting did occur in France at that time, will be false.

Noncontingent propositions

We have just been talking about those propositions which are both possibly true *and* possibly false. They are, we said, the contingent propositions. Are there any propositions which are *not* both possibly true and possibly false? Any propositions which are just the one and not the other? Any propositions, for instance, which could not possibly be false, i.e., which must be true? Or again, any propositions

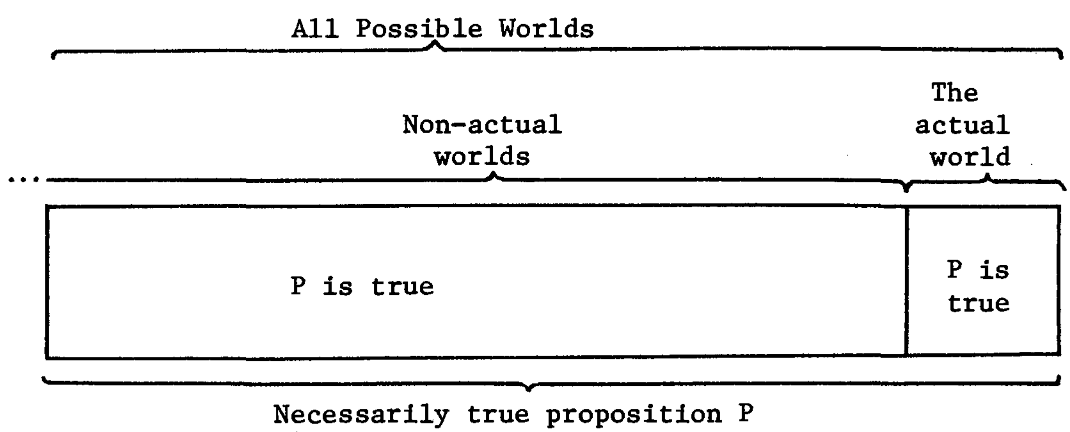

FIGURE *(1.e)*

If P is a necessarily true (noncontingently true) proposition, then P is true in all possible worlds — the non-actual ones as well as the actual one.

11. It is commonly said that a proposition has one and only one contradictory. If this claim were true, we ought, accordingly, to speak of *the* contradictory of a proposition rather than, as we have here, of *a* contradictory. However, subsequently in this chapter — after we have made a distinction between propositional-identity and propositional-equivalence — we will show that every proposition has an infinite number of non-identical equivalents and hence that every proposition has an infinite number of non-identical, but equivalent, contradictories.

which could not possibly be true, i.e., which must be false? Such propositions, if there are any, would not be contingent. They would be what we should want to call *noncontingent* propositions.

Consider, first, what it would be like for there to be propositions which *must be true.* Such propositions we should call *necessarily true* propositions. A necessarily true proposition would, of course, be a possibly true proposition. Indeed, since it would not only be true in at least one possible world but true in *all* possible worlds, it would also be true in the actual world. See figure *(1.e)*.

A necessarily true proposition, in short, would be both possibly true and actually true. But it would not be contingent and true. For a true contingent proposition, remember, is false in some possible world, whereas a necessarily true proposition — since it would be true in every possible world — could not be false in any. A necessarily true proposition, then, as well as being both actually and possibly true, would be *noncontingently true.*

Can we give an example of such a proposition? Examples abound, and we shall examine many in the course of this book. But for present purposes we shall satisfy ourselves with a particularly straightforward example.

It is a fairly obvious fact that not only can we ascribe truth and falsity (and other attributes) to individual propositions, but also we can ascribe various attributes to *pairs* of propositions. For example, we can assert of a pair of propositions (1) that neither is true, (2) that only one is true, (3) that at least one is true, or (4) that both are true, etc. In asserting something of a *pair* of propositions one is, of course, expressing a proposition which is itself either true or false. For example, if one were to assert of the two false contingent propositions

(1.1) Benjamin Franklin was a president of Spain,

(1.2) Canada is south of Mexico,

that one or the other of them is true, then the proposition which one has expressed is itself, obviously enough, contingent and false.

Our ability to ascribe truth to neither of, or to one of, or to both of, etc., a pair of propositions, provides a means to construct an example of a necessarily true proposition.

Necessarily true propositions

Consider the case in which we ascribe truth to one or the other of a pair of propositions. We can, of course, do this for any arbitrarily selected pair of propositions whatever. But let's see what happens when we do so in the case of ascribing truth to one or the other of a pair of contradictory propositions. Take, for instance, the contradictory pair

(1.3) The U.S. entered World War I in 1917;

(1.4) It is not the case that the U.S. entered World War I in 1917.

As we saw, *(1.3)* is true in all those possible worlds in which *(1.4)* is false, and *(1.4)* is true in all those possible worlds in which *(1.3)* is false. So in every possible world one or the other is true. If we then assert of such a pair that one or the other is true, the proposition we express will be true in all possible worlds. This proposition will be

(1.5) Either the U.S. entered World War I in 1917 or it is not the case that the U.S. entered World War I in 1917.

The same holds for any pair of contradictories whatsoever. Think of any proposition you like: for example, the contingent proposition that you (the reader) are now (at the moment of reading this sentence) wearing a pair of blue jeans. It doesn't matter whether this proposition is actually true or false. Being contingent, it is true in at least some possible worlds — even if not the actual one — and false in all those in which it is not true. Now think of a contradictory of that proposition, e.g., that it is not the case that you are wearing a pair of blue jeans. Whether this latter proposition is actually true or false, it will be true in all those possible worlds in which the proposition that you are wearing blue jeans is false, and false in all those possible worlds in which this proposition is true. Finally, then, think of the proposition that *either* you are wearing a pair of blue jeans *or* it is not the case that you are wearing a pair of blue jeans. This latter proposition, just like the proposition *(1.5)*, is true in all possible worlds, and hence, like *(1.5)*, is necessarily true.

Any proposition which asserts of two simpler ones that either one or the other is true, i.e., that at least one is true, is called a *disjunctive* proposition: it *disjoins* two simpler propositions each one of which is a *disjunct*. The disjuncts in a disjunctive proposition need not be contradictories of one another, as they are in the case of *(1.5)*. For instance, the proposition that either you are wearing a pair of blue jeans or you are dressed for a formal occasion is a disjunction whose disjuncts are *not* contradictories of one another. But in the case when disjuncts are contradictories, it follows, as we have seen, that the disjunction itself is necessarily true.

It is important not to think that the only necessarily true propositions are those which assert of a pair of contradictories that one or the other is true. Later we shall cite examples of necessarily true propositions which are not of this sort.

Necessarily false propositions

Consider, now, what it would be like to have a noncontingent proposition which *must be false.* Such a proposition we should call a *necessarily false* proposition. A necessarily false proposition would, of course, be a possibly false proposition. Indeed, since it would not only be false in at least some possible worlds but false in *all* possible worlds, it would also be false in the actual world. A necessarily false proposition, in short, would be both possibly false and actually false. But it would not be contingent and false. For a false contingent proposition, remember, is true in some possible world whereas a necessarily false proposition — since it is false in every possible world — is not true in any. A necessarily false proposition, then, as well as being both actually and possibly false would be *noncontingently false.*

What would be an example of such a proposition? Again, our ability to say something about the way truth and falsity are distributed between the members of a *pair* of propositions gives us the means of constructing an example of a necessarily false proposition.

Just as we can say of any pair of propositions that one or the other of them is true, we can also assert of any pair that *both* are true. But consider what happens when we say of a pair of propositions that they both are true in the case in which the two propositions happen to be contradictories. Consider, again, the contradictory pair of propositions *(1.3)* and *(1.4)*. Suppose we were to assert of them that they both are true. The proposition to this effect would be

> *(1.6)* The U.S. entered World War I in 1917 and it is not the case that the U.S. entered World War I in 1917.

It is easy to see that there is no possible world in which this latter proposition is true. For, as we saw earlier when discussing the limits of possibility and conceivability, a supposed world in which something literally both is the case and is not the case is not, in any sense, a possible world; it is an

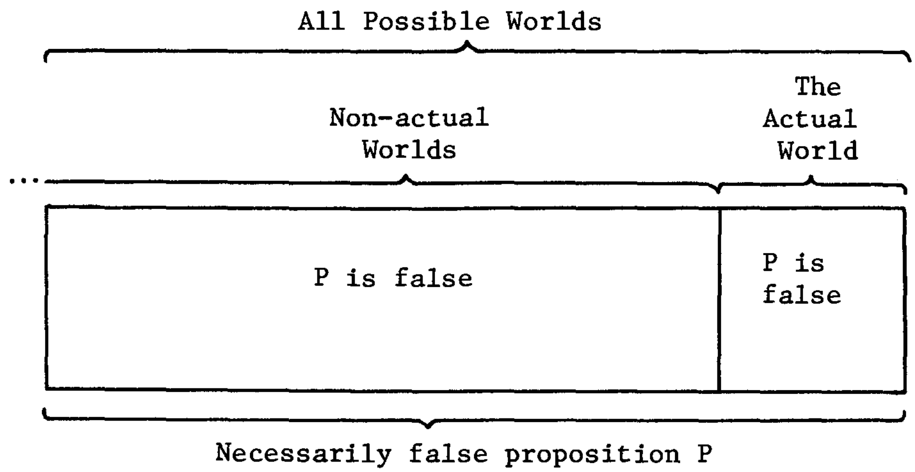

FIGURE *(1.f)*

If P is a necessarily false (i.e. noncontingently false) proposition, then P is false in all possible worlds — the non-actual ones as well as the actual one. Such propositions are also said to be *self-contradictory, self-inconsistent,* or *logically impossible.*

impossible one. The case we considered earlier was that of the supposition that time travel both *will* occur sometime in the future and *will not* occur at any time in the future. We can see that it, like the case we are presently considering, involves ascribing truth to two propositions one of which is a contradictory of the other. And perhaps we can also now see why we earlier said that that proposition was *self-contradictory,* and why we would now want to say that the present case is also self-contradictory. For if any proposition ascribes truth to both members of a pair of contradictories, then that proposition is one which has a contradiction within itself.

Any proposition which asserts of two simpler ones that both of them are true, is called a *conjunctive* proposition: it *conjoins* two simpler propositions each of which is called a *conjunct.* The conjuncts in a conjunctive proposition need not be contradictories of one another, as they are in the case of *(1.6)*. For instance, the proposition that you are wearing a pair of blue jeans and your friend is wearing a pair of tweeds is a conjunction whose conjuncts are *not* contradictories. But in the case where conjuncts are contradictories, it follows, as we have seen, that the conjunction is necessarily false.

More about contradictory propositions

A simple way of describing the relation which holds between two propositions which are contradictories of one another is to say that in each possible world one or other of those propositions is true and the other is false. This description has two immediate consequences which we would do well to note.

First, note that any contradictory of a contingent proposition is itself a contingent proposition. For if a proposition is contingent, i.e., true in some possible worlds and false in all the others, then since any proposition which is its contradictory must be false in all possible worlds in which the former is true, and true in all possible worlds in which the former is false, it follows immediately that this second proposition must be false in some possible worlds and true in some others. But this is just to say that the latter proposition is contingent.

Second, note that any contradictory of a noncontingent proposition is itself a noncontingent proposition. For if a proposition is noncontingently true, i.e., true in all possible worlds and false in none, then any proposition which is its contradictory must be false in all possible worlds and true in none, which is just to say that it itself must be noncontingent, and more especially, noncontingently false. Similarly, if a proposition is noncontingently false, i.e., false in all possible worlds and true in none, then any proposition which is its contradictory must be true in all possible worlds and false in none, which is just to say that it itself must be noncontingent, and more especially, noncontingently true. In short, if one member of a contradictory pair of propositions is necessarily true or necessarily false, then the other member of that pair must be necessarily false or necessarily true respectively. To cite an example of such a pair we need only return to propositions *(1.5)* and *(1.6)*: *(1.5)* is true in all possible worlds and *(1.6)* is false in all possible worlds; they are, in short, contradictories of one another.

Between them, then, a proposition and any contradictory of that proposition divide the set of all possible worlds into two subsets which are *mutually exclusive* and *jointly exhaustive*. One of these subsets will comprise all of the possible worlds, if any, in which the former proposition is true, and the other subset will comprise all of the possible worlds, if any, in which the latter proposition is true. Each possible world will belong to one or the other, but not to both, of these subsets.

Some main kinds of noncontingent propositions

So far, the only example cited of a necessarily true proposition was the disjunction of a pair of contradictories, viz., *(1.5)*; and the only example of a necessarily false proposition was the conjunction of a pair of contradictories, viz., *(1.6)*. Yet many noncontingent propositions are of kinds very different from these. In what follows, we cite some other kinds of examples (in no special order), all of necessarily true propositions. It should be evident, from what we said in the previous subsection, that the contradictories of each of the necessarily true propositions cited will be necessarily false.

1. One main kind of necessarily true proposition is exemplified by

(1.7) If something is red then it is colored.

Note that this proposition is true even in those possible worlds in which there are no red things. To assert *(1.7)* is simply to assert that *if* anything is red then it is colored; and this proposition is true both in worlds in which there are red things and in worlds in which there are not.[12] Philosophers have spent time analyzing the *reasons* for the necessary truth of propositions like *(1.7)*. Properties, in general, tend to come in *ranges:* ranging from the more or less specific to the more general. Thus the property of being located in Ray Bradley's drawer in his desk at Simon Fraser University is a highly specific property. Or, as we shall prefer to say, it is a highly *determinate* property. The property of being located in British Columbia is less determinate; that of being located in Canada even less so; that of being located in the northern hemisphere even less so again; and so on. As we proceed along the scale of increasing generality, i.e., decreasing determinateness, we come finally to the least determinate property in the range. This is the property which, intuitively, falls just short of the most general of all properties — that of being a thing — viz., in the present instance the property of being

12. It has become standard practice, in the past hundred years or so, to construe sentences of the form "All . . . are. . . ." as if they said the same as sentences of the form "For anything whatever, if it is . . . then it is. . . ." On this account, the sentence "All red things are colored" expresses the necessary truth *(1.7)*.

located somewhere or other. Such properties we shall call *determinable* properties. Determinable properties are those under which all the more or less determinate properties of the range are subsumed. They are those which we would cite if we wished to give the least determinate possible answer to such general questions about a thing as: "Where is it?" (location); "How many are there?" (number); "How heavy is it?" (weight); "What color is it?" (color); and so on. Plainly, on this analysis, the property of being colored — of having some color or other — is a determin*able* property under which the relatively determin*ate* property of being red is subsumed. To be red is, therefore, ipso facto to be colored; and nothing could possibly be red without being colored.

2. In the light of this understanding of determinable properties, we are able to give a characterization of a second main kind of necessary truth; that of which

(1.8) Any event which occurs, occurs at some time or other

is an example. This kind of proposition is sometimes called a *category proposition* (or even a categorial[13] proposition): it is that kind of proposition in which a determinable property is truly ascribed to an item of a certain sort, or (as we shall say) of a certain *category*. Just as determinate properties are subsumable under highly general determinable properties, so are particular items and classes of items subsumable under highly general categories of items, e.g., the categories of material objects, of events, of persons, of mental processes, of sounds, and so on. Now what distinguishes items belonging to one category from those belonging to another is just the set of determin*able* (as distinct from determinate) properties which are essential to such items.[14] Material objects and shadows, for instance, share certain determin*able* properties, e.g., having some number, some shape, and some location. And these determinable properties are *essential* to them in the sense that nothing could possibly be a material object or a shadow if it lacked any of these determinable properties. By way of contrast, the determin*ate* properties, which particular items in a given category happen to have, are *not* all essential. The determinate property of being in Ray Bradley's drawer, for instance, is not essential to his copy of *Anna Karenina* although the determinable property of being located somewhere or other is. What makes material objects and shadows comprise different categories is the fact that, though sharing certain determinable properties, they do not share others. They do not share, for instance, the determinable property of having some mass or other. And neither of them has the determinable property which seems distinctive of propositions, viz., being true or false. The question as to what the essential determinable properties of items in a given category turn out to be, is one which we cannot pursue here. Suffice it to say that any true proposition which, like *(1.8)*, ascribes to items in a certain category a determinable property which is essential to those items, will be necessarily true. And so, for that matter, will be any true propositions which, like

(1.9) Propositions have no color

deny of items in a certain category that they have a certain determinable property.

13. Note the spelling of this term. Do not confuse it with "categorical". The two are not synonyms.

14. A related kind of necessary truth, that which truly ascribes essential properties to so-called *natural kinds* — as, for example, the proposition that gold is a metal — has recently been the subject of much philosophical discussion. See, in particular, Saul Kripke, "Naming and Necessity" in *Semantics of Natural Language,* ed. Harman and Davidson, Dodrecht, D. Reidel, 1972. Kripke, somewhat controversially, maintains that having atomic number 79 is an essential property of gold.

3. Still another main kind of necessary truth is exemplified by the *relational proposition*

> *(1.10)* If Molly is taller than Judi then it is not the case that Judi is taller than Molly.

The necessary truth of this proposition, it should be noted, is not in any way a function of which particular items it asserts as standing in the relation of being taller than. Rather, it is a function of the essential nature of the relation of being taller than; of the fact, that is, that no matter what possible worlds or what possible items are involved, if the relation of being taller than holds between two items, x and y, in that order, then it does not also hold between x and y in the reverse order. This fact about the essential nature of the relation of being taller than is sometimes referred to by saying that the relation of being taller than is an "asymmetrical" relation. It is a fact about the relation of being taller than which serves to explain why not only *(1.10)* but countless other propositions, viz., *any* propositions of the form

> *(1.11)* If x is taller than y then it is not the case that y is taller than x

are necessarily true. More generally, the property of being asymmetrical is essential to countless other relations as well, e.g., being the mother of, being older than, being the successor of, and so on. And for each of these relations there will be countless necessarily true propositions analogous to *(1.10)*. In chapter 6, section 6, we will investigate other of the essential properties which relations may have: (a) symmetry or nonsymmetry (as alternatives to asymmetry); (b) transitivity, intransitivity, or nontransitivity; and (c) reflexivity, irreflexivity, or nonreflexivity. We will see that every relation has at least one essential property drawn from each of (a), (b), and (c), i.e., has at least three essential properties *in toto*. That this is so provides a rich source of necessary truths. For all those propositions whose truth can be ascertained by an appeal to a fact about an essential property of a relation will themselves be necessary truths.

4. In saying that propositions of the several main kinds discussed so far are logically necessary truths we are, it should be noted, using the term "logically necessary" in a fairly broad sense. We are not saying that such propositions are currently recognized as truths within any of the systems of formal logic which so far have been developed. Rather, we are saying that they are true in all logically possible worlds. Now, in this broad sense of the term, the *truths of mathematics* must also count as another main kind of logically necessary truth. Bertrand Russell and A. N. Whitehead, it is worth mentioning, tried soon after the turn of this century to show that the truths of mathematics were logically necessary in the rather stricter sense of being derivable in accordance with the rules of and from the axioms of then-known systems of formal logic. But whether or not they were successful — and this is still a topic for debate — there can be little doubt that mathematical truths are logically necessary in the broader sense. Indeed, it seems that early Greek philosophers recognized them as such well before Aristotle laid the foundation of formal logic in the third century B.C. The true propositions of mathematics seemed to them to be necessary in a sense in which those of history, geography, and the like are not. Admittedly, they doubtless would not then have offered a "true in all (logically) possible worlds" analysis of the sense in which mathematical truths seemed to them to be necessary. But such an analysis seems to fit well with their judgments in the matter.

It may seem to some that there are two important classes of exceptions to the general account we are giving of mathematical truths. The first has to do with propositions of what we often call *applied* mathematics. If by "propositions of applied mathematics" is meant certain true propositions of physics, engineering, and the like which are inferred by mathematical reasoning from other propositions of physics, engineering, etc., then of course such propositions will not count as necessary truths but only as contingent ones. But then it is misleading even to say that they are, in any sense, propositions of mathematics. If, on the other hand, we mean by "propositions of applied mathematics" certain propositions which, like

(1.12) If there are 20 apples in the basket then there are at least 19 apples in the basket

are *instances* (i.e., applications) of propositions of pure mathematics, then our account is sustained. For there are no possible worlds — even worlds in which apples, baskets, and whatnot fail to exist — in which a proposition like *(1.12)* is false. This sort of mathematical proposition should be distinguished from propositions about the physical results of putting things together, e.g.,

(1.13) One liter of alcohol added to one liter of water makes 2 liters of liquid

which not only is not necessarily true but is contingent and false. (See chapter 3, p. 170).

A second supposed exception has to do with the propositions of geometry. Thus, it may be said that although there was a time when propositions like Euclid's

(1.14) The sum of the interior angles of a triangle is equal to two right angles

were universally thought to be necessarily true, we now know much better: the development, in the nineteenth century, of non-Euclidean (e.g., Riemannian and Lobachevskian) geometries in which the sum of the interior angles of triangles are asserted to be more than or less than (respectively) two right angles, shows that *(1.14)* is not necessarily true. In effect, the objection is that with the advent of non-Euclidean geometries in the nineteenth century, we have come to see that the answer to the question as to what is the sum of the interior angles of a triangle, is a contingent one and will vary from possible world to possible world according to the natural laws in those worlds.

But this objection fails to make an important distinction. It is correct so long as by "triangles" it refers to physical objects, e.g., triangles which surveyors might lay out on a field, or paper triangles which one might cut out with scissors. But there are other triangles, those of pure geometry, whose properties are *not* subject to the physical peculiarities of any world. These idealized, *abstract* entities have invariant properties. What the advent of non-Euclidean geometries has shown us about *these* triangles is that we must distinguish various kinds: a Euclidean triangle will have interior angles whose sum is equal to two right angles; a Riemannian triangle, angles whose sum is greater than two right angles; and a Lobachevskian triangle, angles whose sum is less than two right angles. Specify which kind of abstract triangle you are refering to, and it is a necessary truth (or falsehood) that its angles sum to two right angles, less than two, or greater than two.

5. Among the most important kinds of necessarily true propositions are those true propositions which ascribe *modal* properties — necessary truth, necessary falsity, contingency, etc. — to other propositions. Consider, for example, the proposition

(1.15) It is necessarily true that two plus two equals four.

(1.15) asserts of the simpler proposition — viz., that two plus two equals four — that it is necessarily true. Now the proposition that two plus two equals four, since it is a true proposition of mathematics, is necessarily true. Thus in ascribing to this proposition a property which it does have, the proposition *(1.15)* is *true*. But is *(1.15)* necessarily true, i.e., true in all possible worlds? The answer we shall

want to give — and for which we shall argue in chapter 4, pp. 185-88, and again in chapter 6, p. 335 — is that true propositions which ascribe necessary truth to other propositions are themselves necessarily true, i.e., logically necessary. As Hintikka has put it:

> It seems to me obvious that whatever is logically necessary here and now must also be logically necessary in all logically possible states of affairs that could have been realized instead of the actual one.[15]

Similar considerations lead us to say further that any true proposition which ascribes necessary falsity or contingency to another proposition is itself true in all possible worlds, i.e., is necessarily true. And the same holds for true propositions which, like

(1.16) The proposition that there are 20 apples in the basket is *consistent* with the proposition that the basket is green

assert of two simpler propositions that they are logically related by one of the *modal* relations, implication, equivalence, consistency, or inconsistency (discussed in the next section).

6. The preceding five kinds of necessary truth are not wholly exclusive of one another and are certainly not exhaustive of the whole class of necessary truths. There are other propositions which are true in all possible worlds but for which it is difficult to find any single apt description except to say that they are propositions whose truth can be ascertained by *conceptual analysis.* When Plato, for instance, in his dialogue, *Theatetus,* analyzed the concept of knowledge and came to the conclusion that knowing implies (among other things) believing, he provided grounds for us to ascertain, as being necessary truths, a host of propositions, such as

(1.17) If the Pope knows that there are nine planets then the Pope believes that there are nine planets.

That hosts of such necessary truths exist should be evident from the fact that the analytic truth which Plato discovered is quite general, admitting countless instances ranging from the Pope's knowledge (and consequent belief) about the number of the planets to the knowledge (and consequent belief) that most of us have about the earth's being round, and so on. Likewise, when a moral philosopher analyzes the concept of moral obligation and concludes that being morally obliged to do something implies being able to do it, that philosopher establishes that a host of propositions, of which

(1.18) If the foreign minister is morally obliged to resign then he is able to do so

is only one instance, are necessary truths. Characteristically, philosophers — in their analyses of these and other concepts which figure centrally in our thinking — are not trying to compete with scientists in discovering contingent truths about how the actual world happens to be constituted. Rather, they are trying to discover what is *implied* by many of the concepts which scientists and the rest of us take for granted. And the propositions in which they report their analytical discoveries, if true, are necessarily true.

15. Jaakko Hintikka, "The Modes of Modality" in *Acta Philosophica Fennica,* vol. 16 (1963), pp. 65–81.

Summary

Much of what we have said so far concerning worlds — actual and non-actual — and concerning propositions — true and false, contingent and noncontingent — about items in these worlds can be summarized in the following diagram:

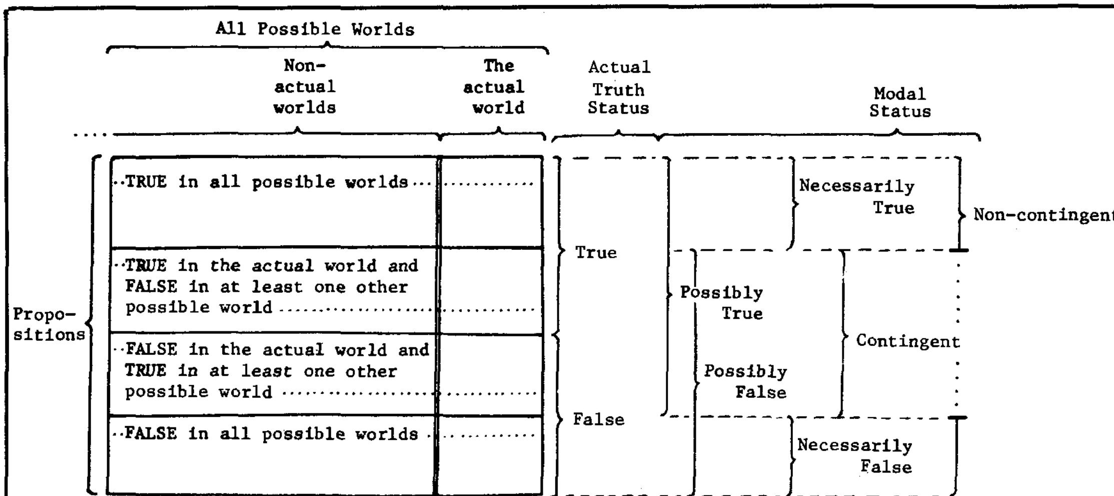

FIGURE *(1.g)*

Note that only those propositions are contingent which are *both* possibly true *and* possibly false, whereas those propositions are noncontingent which are *either* not possibly false (the necessarily true ones) *or* not possibly true (the necessarily false ones).

Other important relationships between classes of propositions may be read off the diagram in accordance with the following rule: if the bracket representing one class of propositions (e.g., the necessarily true ones) is contained within the bracket representing another class of propositions (e.g., the actually true ones), then any proposition belonging to the former class is a proposition belonging to the latter class. For instance, all necessarily true propositions are actually true, although not all actually true propositions are necessarily true.

EXERCISES

1. *For each of the following propositions, say (1) whether it is contingent or noncontingent, and (2) if noncontingent, then whether it is true or false.*

   a. *All aunts are females.*

   b. *On April 13, 1945, all females are aunts.*

   c. *If a bird is entirely black, then it is not also white.*

   d. *All black birds are black.*

e. All blackbirds are black.

f. Some squares have six sides.

g. No mushrooms are poisonous.

h. It is illegal to jaywalk in Moscow.

i. Whenever it is summer in New Zealand, it is winter in Canada.

j. What will be, will be.

2. *Briefly explain why each of the following propositions is* false.

k. If a proposition is actually true, then that proposition is also contingent.

l. If it is possible for a proposition to be true, then it is possible for that same proposition to be false.

m. Even though a proposition is actually false, it need not be.

n. If a proposition is noncontingent, then it is actually true.

o. If a proposition is possibly true, then it is contingent.

3. *Write a short story such that a person reading that story could tell from subtle clues that there is no possible world in which all the events related occur.*

4. *Does the concept of time-travel really generate paradoxes? Here is what Lazarus says about the matter when discussing it with two of his descendants before making his own time-trip: "That old cliché about shooting your grandfather before he sires your father, then going* fuff! *like a soap bubble — and all descendants, too, meaning both of you among others — is nonsense. The fact that I'm here and you're here means that I* didn't *do it — or won't do it; the tenses of grammar aren't built for time-travel — but it does* not *mean that I never went back and poked around. I haven't any yen to look at* myself *when I was a snot-nose; it's the era that interests me. If I ran across myself as a young kid, he — I — wouldn't recognize me; I would be a stranger to that brat. He wouldn't give me a passing glance. I know, I* was *he." Discuss. (Robert A. Heinlein,* Time Enough For Love, p. 358.)

* * * * *

Symbolization

Much of the success and promise of modern logic, like that of mathematics, arises from the powerful and suggestive symbolization of its concepts. From time to time, as we introduce and discuss various of the most fundamental concepts of logic, we will also introduce symbols which are widely accepted as standing for those concepts.

Already we have casually introduced some of these symbols as when, for example, we used the capital letter "P" of the English alphabet to represent any arbitrarily chosen proposition; when we used "*a*" to represent any arbitrarily chosen item; and when we used "**F**" to represent an arbitrarily chosen attribute.

Let us extend our catalog of symbols even more. We have just introduced several of the most important concepts of modern logic. They are standardly represented in symbols as follows:

(1) The concept of falsity: "$\sim$" [called *tilde*[16]]

(2) The concept of possibility: "$\Diamond$" [called *diamond*]

(3) The concept of necessary truth: "$\Box$" [called *box*]

(4) The concept of contingency: "∇" [called *nabla*]

(5) The concept of noncontingency: "Δ" [called *delta*]

Each of these symbols may be prefixed to a symbol (such as "P", "Q", "R") which stands for a proposition, to yield a further propositional symbol (e.g., "$\sim$P", "$\Diamond$R"). Any sequence of one or more propositional symbols is called a "formula".[17]

Each of these symbols may be defined contextually as follows:

"$\sim$P" $=_{df}$ "The proposition P is false"[18] [or "It is false that P"]

"$\Diamond$P" $=_{df}$ "The proposition P is possible" [or "It is possible that P"]

"$\Box$P" $=_{df}$ "The proposition P is necessarily true" [or "It is necessarily true that P"]

"∇P" $=_{df}$ "The proposition P is contingent" [or "It is contingent that P"]

"ΔP" $=_{df}$ "The proposition P is noncontingent" [or "It is noncontingent that P"]

These symbols may be concatenated, that is, linked together in a series, as for example, "$\sim\sim$P". This gives us the means to express such propositions as that P is possibly false ("$\Diamond\sim$P"), necessarily false ("$\Box\sim$P"), impossible ("$\sim\Diamond$P"), etc.

Certain of these combinations of symbols are sometimes unwittingly translated into prose in incorrect ways, and one should beware of the pitfalls. In particular, combinations beginning with "∇" and "Δ" prove troublesome. Consider, for example, "∇P". There is a temptation to translate this as "it is contingently true that P," or equivalently as "P is contingently true". Neither of these proposed translations will do. The correct translation is "It is contingent that P is true." Why? What exactly is the difference between saying on the one hand that P is contingently true, and on the other, that it is contingent that P is true? Just this: to say that P is contingently true is to say that P is both contingent and true; to say that it is contingent that P is true, is to say only that P's being true is contingent, i.e., that P is true in some possible worlds and false in some others. Clearly, then,

16. Rhymes with "Hilda".

17. In later chapters we will distinguish between formulae which are constructed so as to make sense (i.e., which are well-formed) and those which are not. The exact rules for constructing well-formed formulae within certain systems of symbols are given in chapters 5 and 6.

18. The "$=_{df}$"-symbol may be read as "has the same meaning as" or alternatively as "equals by definition". The expression on the left side of the "$=_{df}$"-symbol is the expression being introduced; it is known by the technical name *definiendum*. The expression on the right hand side of the "$=_{df}$"-symbol is the one whose meaning is presumed already understood and is being assigned to the definiendum. The right hand expression is called the *definiens*.

to say that a proposition is contingently true is to say *more* than merely that its truth is contingent. In other words, the expression "P is contingently true" is to be rendered in our symbolic notation as "P and ∇P." It is thus incorrect to translate "∇P" alone as "P is contingently true." Similarly, "$\nabla \sim$ P" is to be translated as "it is contingent that it is false that P"; and not as "P is contingently false."

An easy rule to bear in mind so as to avoid mistakes in translations is this: Never translate "∇" or "Δ" adverbially, i.e., with an "ly" ending; always translate them as "it is contingent that" and "it is noncontingent that" respectively. Adverbial translations in the other cases, i.e., for "$\Box$" and "$\Diamond$" are freely permitted: "it is necessarily true that" and "it is possibly true that" respectively.

EXERCISES

1. *Let "A" stand for the proposition that Canada is north of Mexico. Translate each of the following expressions into English prose.*

   (a) A (f) $\sim\Diamond\sim A$ (k) ∇A (p) $\Delta\sim A$

   (b) $\sim A$ (g) $\Box A$ (l) $\nabla\sim A$ (q) $\sim\Delta A$

   (c) $\Diamond A$ (h) $\Box\sim A$ (m) $\sim\nabla A$ (r) $\sim\Delta\sim A$

   (d) $\Diamond\sim A$ (i) $\sim\Box A$ (n) $\sim\nabla\sim A$

   (e) $\sim\Diamond A$ (j) $\sim\Box\sim A$ (o) ΔA

2. *For each of the cases* (c) *through* (r) *above say whether the proposition expressed is true or false.*
3. *Letting "A" now stand for the proposition that all squares have four sides, say for each of the expressions* (a) – (r) *in question 1, whether the proposition is true or false.*
4. *Explain why "Δ P" is* not *to be translated as "P is noncontingently true" but as "it is noncontingent that P is true." Find a proposition of which it is true that it is noncontingent that it is true, but of which it is false that it is noncontingently true.*
5. *Say of each of the following which is true and which is false. (Note: it is actually true that some cows are infertile.)*

   (s) *It is contingent that some cows are infertile.*

   (t) *It is contingent that it is not the case that some cows are infertile.*

   (u) *It is contingently true that some cows are infertile.*

   (v) *It is contingently false that some cows are infertile.*
6. *Say for each of the following whether it is true or false.*

   (w) *It is noncontingent that* $2 + 2 = 4$.

   (x) *It is noncontingently true that* $2 + 2 = 4$.

   (y) *It is noncontingent that it is false that* $2 + 2 = 4$.

(z) *It is noncontingently false that* $2 + 2 = 4$.

7. *Explain the difference in meaning in the two phrases "noncontingently true" and "not contingently true".*

4. RELATIONS BETWEEN PROPOSITIONS

Just as the modal properties of propositions are a function of the ways in which the truth-values of those propositions singly are distributed across the set of all possible worlds, so the *modal relations* between propositions are a function of the ways in which the truth-values of the members of *pairs* of propositions are distributed across the set of all possible worlds. We single out four modal relations for immediate attention, viz.,

inconsistency; consistency; implication; and *equivalence.*

In terms of these we will find it possible to explain most of the central logical concepts discussed in this book. Yet these modal relations, we shall now see, can themselves be explained — in much the same way as the above discussed modal properties — in terms of possible worlds.

Inconsistency

Two propositions are *inconsistent* with one another, we ordinarily say, just when it is necessary that if one is true the other is false, i.e., just when they cannot both be true. Translating this ordinary talk into talk of possible worlds we may say that two propositions are inconsistent just when in any possible world, if any, in which one is true the other is false, i.e., just when there is no possible world in which both are true.

Inconsistency is a generic modal relation. It has two, and only two species: contradiction and contrariety.

Contradiction is that species of inconsistency which holds between two propositions when they not only cannot both be true but also cannot both be false. As we saw in section 2, it is a relation which holds between, e.g., the contingent propositions

(1.3) The U.S. entered World War I in 1917

and

(1.4) It is not the case that the U.S. entered World War I in 1917;

and also holds between, e.g., the noncontingent propositions

(1.5) Either the U.S. entered World War I in 1917 or it is not the case that the U.S. entered World War I in 1917

and

(1.6) The U.S. entered World War I in 1917 and it is not the case that the U.S. entered World War I in 1917.

In any possible world, one or other of the propositions in a contradictory pair is true and the remaining proposition is false. Where two propositions are contradictories of one another there is no possible world in which both propositions are true and no possible world in which both propositions are false. To repeat: contradictories divide the set of all possible worlds into two mutually exclusive and jointly exhaustive subsets.

Contrariety is that species of inconsistency which holds between two propositions when although they cannot both be true, they nevertheless can both be false. Consider, for instance, the relation between the contingent propositions

(1.3) The U.S. entered World War I in 1917

and

(1.19) The U.S. entered World War I in 1914.

Plainly, if one member from this contrary pair is true, the other is false. The truth of one excludes the truth of the other. So if two propositions are contraries it must be that at least one is false. Moreover both may be false. After all, we can conceive of its having been the case that the U.S. entered World War I neither in 1914 nor in 1917 but rather, let us suppose, in 1916. Thus while propositions *(1.3)* and *(1.19)* cannot both be true, they *can* both be false. There is some possible world in which *(1.3)* and *(1.19)* are false. Between them, then, the propositions of a contrary pair do not exhaust all the possibilities. In short, contraries divide the set of all possible worlds into two mutually exclusive subsets which are *not* jointly exhaustive.

Both members of the contrary pair just considered are contingent propositions. Can noncontingent propositions also be contraries? Can a noncontingent proposition be a contrary of a contingent proposition? As we have here defined "contrariety", the answer is "Yes" to both questions.[19] Consider, first, two propositions which are necessarily false. Since there are no possible worlds in which either is true, there is no possible world in which both are true. That is to say, since both are necessarily false, they cannot both be true. But equally, since both are necessarily false, they are both false in all possible worlds, and hence there is a possible world in which both are false. Hence, since two necessarily false propositions cannot both be true, but can both be false, they are contraries. Consider, secondly, a pair of propositions one of which is necessarily false and the other of which is contingent. Since there are no possible worlds in which the necessarily false proposition is true, there can be no possible worlds in which both it and the contingent proposition are true. To be sure, there will be some possible worlds in which the contingent proposition is true. But in all those possible worlds the necessarily false proposition will be false. Hence, even in those possible worlds it will not be the case that *both* are true. Moreover, both propositions may be false. They will both be false in all those possible worlds in which the contingent proposition is false. Hence, since two propositions one of which is necessarily false and the other of which is contingent cannot both be true, but can both be false, they are contraries.

Necessarily false propositions, it is clear, are profligate sources of inconsistency. Every necessarily false proposition is a contradictory of, and hence inconsistent with, every necessarily true proposition. Every necessarily false proposition is a contrary of any and every contingent proposition. And every necessarily false proposition is a contrary of every other necessarily false proposition. Indeed, we need only add that the term "self-inconsistent" is a synonym for the term "necessarily false", in order to conclude that a necessarily false proposition is inconsistent with every proposition whatever, including itself, i.e., that a necessarily false proposition is *self*-inconsistent.

From the fact that two propositions are inconsistent it follows that at least one is *actually* false. For since, by the definition of "inconsistency", there is no possible world in which both members of an inconsistent pair of propositions are true, every possible world — including the actual world — is a world in which at least one of them is false. Inconsistency, we may say, provides a guarantee of falsity. But the converse does not hold. From the fact that one or both of a pair of propositions is

19. Historically some logicians have used the terms "contradiction" and "contrariety" as if they applied only in cases in which both propositions are contingent. For more on this, see the subsection entitled *"A Note on History and Nomenclature"*, pp. 53–54.

actually false, it does not follow that they are inconsistent, i.e., that in every possible world one or both of them is false. Thus *(1.4)* and *(1.19)* are false in the actual world. Yet they are not inconsistent. This is fairly easy to show. Consider the fact that *(1.19)* is contingent and hence is true in some possible worlds, in particular, in all those possible worlds in which the U.S. entered World War I in 1914. But in each of these possible worlds, it turns out that *(1.4)* is also true: any possible world in which the U.S. entered World War I in 1914 is also a world in which it is not the case that the U.S. entered World War I in 1917. And this is just to say that in all those possible worlds in which *(1.19)* is true, *(1.4)* is also true. Hence there is a possible world in which these actually false propositions are true together. In short, from the fact that they are false in fact it does not follow that they are inconsistent. Being false in the actual world, it turns out, provides no guarantee of inconsistency.

EXERCISE

Can two propositions be contraries as well as contradictories of one another? Explain your answer.

* * * * *

Consistency

What does it mean to say that two propositions are *consistent* with one another? Given that we already know what it means to say that two propositions are *in*consistent with one another, the answer comes easily: two propositions are consistent with one another if and only if it is not the case that they are inconsistent. It follows that two propositions are consistent if and only if it is not the case that there is no possible world in which both are true. But this means that they are consistent if and only if there *is* a possible world in which both are true.

As an example of the modal relation of consistency, consider the relation between the contingent propositions

(1.3) The U.S. entered World War I in 1917

and

(1.20) Lazarus Long was born in Kansas in 1912.

Whatever other relation may hold between them, plainly the relation of consistency does: it need not be the case that if one is true the other is false; both can be true. No matter what the facts happen to be about the actual world (no matter, that is, whether either *(1.3)* or *(1.20)* is actually true), it is possible that both of them should be true — which is just to say that there is at least one possible world in which both are true.

If two contingent propositions happen both to be true in the actual world, then since the actual world is also a possible world, there is a possible world in which both are true, and hence they are consistent. Actual truth, that is to say, provides a guarantee of consistency. But the converse does not hold. From the fact that two propositions are consistent it does not follow that they are both true in the actual world. What does follow is that there is *some* possible world in which both are true; yet that possible world may be non-actual. Thus *(1.3)* and *(1.20)* are consistent. Yet they are not both true in the actual world. *(1.3)* is true in the actual world and false in some non-actual worlds, while *(1.20)* is false in the actual world and true in some non-actual worlds. Hence propositions can be consistent even if one or both is false. In short, consistency does not provide a guarantee of truth.

Since a necessarily true proposition is true in all possible worlds, a necessarily true proposition will be consistent with any proposition which is true in at least one possible world. It will be consistent, that is, with any contingent proposition and with any necessarily true proposition. It will be inconsistent only with those propositions which are not true in any possible worlds, i.e., with necessarily false ones.

Implication

Of the four modal relations we are currently considering, it is probably implication which is most closely identified, in most persons' minds, with the concerns of philosophy in general and of logic in particular. Philosophy, above all, is concerned with the pursuit of truth; and logic — its handmaiden — with discovering new truths once established ones are within our grasp. To be sure, philosophers and logicians alike are concerned to *avoid inconsistency* (since the inconsistency of two propositions is a sufficient condition of the *falsity* of at least one of them) and thus to *preserve consistency* (since the consistency of two propositions is a necessary — but not sufficient — condition of the *truth* of both). But it is in *tracing implications* that they most obviously advance their common concern with the discovery of new truths on the basis of ones already established. For implication is the relation which holds between an ordered pair of propositions when the first cannot be true without the second also being true, i.e., when the truth of the first is a sufficient condition of the truth of the second.

Like the relations of inconsistency and consistency, the relation of implication can be defined in terms of our talk of possible worlds. Here are three equivalent ways of so defining it:

(a) a proposition P implies a proposition Q if and only if Q is true in all those possible worlds, if any, in which P is true;

(b) a proposition P implies a proposition Q if and only if there is no possible world in which P is true and Q false;[20]

(c) a proposition P implies a proposition Q if and only if in each of all possible worlds if P is true then Q is also true.

As an example of the relation of implication consider the relation which the proposition

(1.3) The U.S. entered World War I in 1917

has to the proposition

(1.21) The U.S. entered World War I before 1920.

Whatever other relations may hold between *(1.3)* and *(1.21)*, plainly the relation of implication does. All three of the above definitions are satisfied in this case. Thus: [definition (a)] *(1.21)* is true in all those possible worlds in which *(1.3)* is true; [definition (b)] there is no possible world in which *(1.3)* is true and *(1.21)* is false; and [definition (c)] in each of all possible worlds, if *(1.3)* is true then *(1.21)* is true.

To say that a proposition Q *follows from* a proposition P is just to say that P *implies* Q. Hence the relation of following from, like its converse, can be explained in terms of possible worlds. Indeed, the explanation can be given by the simple expedient of substituting the words "a proposition Q

20. Definition (b), it should be noted, amounts to saying that P implies Q if and only if the truth of P is *inconsistent* with the falsity of Q. Implication, in short, is definable in terms of inconsistency.

follows from a proposition P" for the words "a proposition P implies a proposition Q" in each of the definitions (a), (b), and (c) above.

In terms of the relation of implication (and hence in terms of our talk of possible worlds) we can also throw light on another important logical concept: that of the *deductive validity* of an inference or argument. True, we have not hitherto had occasion to use the words "deductively valid". Yet it will be evident to anyone who has even a superficial understanding of the meanings of these words that much of our discussion has consisted in marshalling deductively valid arguments and drawing deductively valid inferences.[21] Time and again we have signaled the presence of arguments and inferences by means of such words as "hence", "consequently", "therefore", and "it follows that"; and implicitly we have been claiming that these arguments and inferences are deductively valid. But what does it mean to say that an argument or inference is deductively valid? As a preliminary it may help if we remind ourselves of some familiar facts: that it is propositions from which and to which inferences are drawn. Consider, then, the simplest sort of argument (or corresponding inference) which features just one proposition as its premise and just one proposition as its conclusion; and let us designate the premise "P" and the conclusion "Q". Then we can reformulate our question by asking: What does it mean to say that an argument or inference from P to Q is deductively valid? To say that an argument or inference from a proposition P to a proposition Q is deductively valid is just to say that P implies Q, or (conversely) that Q follows from P.[22] Deductive validity, then, which is a property of arguments or inferences, can be explained in terms of the modal relations of implication and following from. And, like them, it can be explained in terms of possible worlds. In this case, we need only adopt the expedient of substituting the words "an argument or inference from P to Q is deductively valid" for the words "a proposition P implies a proposition Q" in each of the definitions (a), (b), and (c) above.

A casual reading of our three definitions of "implication" may suggest to some that only true propositions can have implications. After all, we defined "implication", in (c) for instance, as the relation which holds between a proposition P and a proposition Q when in all possible worlds if P is *true* then Q is also true. And we illustrated the relation of implication by citing a case where a true proposition, viz., *(1.3)*, stood in that relation to another true proposition, viz., *(1.21)*. Does this mean that false propositions cannot have implications? Does it mean, to put the question in other words, that nothing follows from false propositions, or that deductively valid arguments cannot have false premises?

Not at all. On a more careful reading of these definitions it will be seen that they say nothing whatever about the actual truth-values of P or Q; i.e., that they say nothing at all about whether P or Q are true in the actual world. Hence they do not rule out the possibility of a proposition P implying a proposition Q when P is not true but false. In (c), for instance, we merely said that where P implies Q, in all possible worlds Q will be true *if* P is true. We have not *asserted* that P is true in the actual world but merely entertained the *supposition* that P is true in some world or other; and that is something we can do even in the case where P is false in the actual world or even where P is false in all possible worlds.

21. When persons draw a conclusion out of a proposition or a set of propositions they can be correctly said to be "inferring a proposition". Inferring is something persons *do;* it is *not* a logical relation between propositions. It is not only grammatically incorrect to speak of one proposition inferring another, it is logically confused as well. See H.W. Fowler, *A Dictionary of Modern English Usage, 2nd Edition,* revised by Sir Ernest Gowers, Oxford, Clarendon Press, 1965, p. 282.

22. Note that here we are defining "deductive validity", not "validity" *per se.* Later, in chapter 4, we shall define the wider concept of validity.

Moreover, we might just as easily have chosen to illustrate the relation of implication by citing a case where a false proposition implies another proposition. Consider such a case. The proposition

(1.19) The U.S. entered World War I in 1914

is contingent and happens to be false; it is false in the actual world even though it is true in at least some non-actual worlds. Does this (actually) false proposition have any implications? (Equivalently: Do any other propositions follow from *(1.19)*? Can any other proposition be inferred with deductive validity from *(1.19)*?) Obviously enough, the answer is: Yes. A false proposition, like a true one, will have countless implications. For instance, the false proposition *(1.19)* implies all the countless propositions that we could express by uttering a sentence of the form "The U.S. entered World War I before . . . " and filling in the blank with the specification of any date whatever later than 1914, e.g., 1915, 1916, 1917, etc., etc. The crucial difference between the implications of a false proposition and the implications of a true proposition lies in the fact that on the one hand, a false proposition has implications some of which are false — as is the proposition that the U.S. entered World War I before 1915 — and some of which are true — as is the proposition *(1.21)* that the U.S. entered World War I before 1920 — while, on the other hand, a true proposition has implications all of which are true.

Here, then, are two important logical facts about the relation of implication. (i) All the implications of a true proposition have the same truth-value as that proposition, i.e., they 'preserve' its truth. For this reason implication is said to be a *truth-preserving* relation. In tracing out the implications of a true proposition we can be led only to further true propositions, never to false ones. Or, in other words, the only propositions that follow from or can be inferred with deductive validity from propositions which are true are propositions which are also true. (ii) The implications of a false proposition need not have the same truth-value as that proposition. Some of the implications of a false proposition are themselves false; but others are true. Implication, we may say, is *not* falsity-preserving. Among the propositions which follow from or can be inferred with deductive validity from propositions which are false, there are some true propositions as well as some false ones.

These simple logical facts have important practical and methodological applications when it comes to the pursuit of truth. By virtue of (i), it follows that one of the ways in which we can advance the frontiers of human knowledge is simply to reflect upon, or reason out, the implications of propositions we already know to be true. This is, paradigmatically, the way in which advances are made in mathematics and logic. But it is also the way in which unrecognized truths can be discovered in other areas as well. Many of the advances made in technology and the applied sciences, for instance, occur because someone has reasoned out for particular circumstances the implications of universal propositions already accepted as true in the pure sciences. By virtue of (ii), it follows that we can also advance the frontiers of human knowledge, negatively as it were, by testing the implications of hypotheses whose truth-values are as yet unknown, weeding out the false hypotheses, and thus narrowing down the range of alternatives within which truth may yet be found. An exploratory hypothesis is put forward and then tested by seeing whether its implications 'hold up' (as we say) in the light of experience. Of course, if a hypothesis has implications which experience shows to be true, this does not entitle us to conclude that the hypothesis itself is true. For as we have seen, there always are some implications of a proposition which are true even when the proposition itself is false. But if, on the other hand, a hypothesis has any implications which experience shows to be false, this does entitle us to conclude that the hypothesis itself is false. For as we have seen, there can be no false implications of a proposition in the case where that proposition itself is true. Hence if any of the implications of a hypothesis turn out to be false, we may validly infer that that hypothesis is false.

These two facts about the relation of implication are reflected in the standard methodology (or 'logic' as it is often called) of scientific enquiry. The cost of ignoring them, when one is conducting scientific research or when one is pursuing knowledge in any field whatever, is that the discovery of truth then becomes a completely haphazard matter.

Two further important logical facts about the relation of implication deserve notice and discussion. It follows from the definitions given of implication that: (iii) a necessarily false proposition implies any and every proposition; and (iv) a necessarily true proposition is implied *by* any and every proposition whatever. Conclusion (iii) follows from the fact that, if a proposition P is necessarily false then there is no possible world in which P is true and *a fortiori* no possible world such that in it both P is true and some other proposition Q is false; so that [by definition (b)] P must be said to imply Q. Conclusion (iv) follows from the fact that if a proposition Q is necessarily true then there is no possible world in which Q is false and *a fortiori* no possible world such that in it both Q is false and some proposition P is true; so that [again by definition (b)] Q must be said to be implied by P.

These conclusions, however, strike many persons as counterintuitive. Surely, it would be said, the necessarily false proposition

(1.6) The U.S. entered World War I in 1917 and it is not the case that the U.S. entered World War I in 1917

does not imply the proposition

(1.2) Canada is south of Mexico.

And surely, it again would be said, the necessarily true proposition

(1.7) If some thing is red then it is colored

is not implied by the proposition

(1.20) Lazarus Long was born in Kansas in 1912.

For the propositions in the first pair have 'nothing to do with' each other; they are not in any sense about the same things; one is 'irrelevant' to the other. And the same would be said for the propositions in the second pair.

These admittedly counterintuitive results are ones to which we devote a good deal of discussion in chapter 4, section 6, pp. 224–30. For the present, just three brief observations must suffice.

In the first place, (iii) and (iv) ought not to be viewed solely as consequences of some recently developed artificial definitions of implication. They are consequences, rather, of definitions which philosophers have long been disposed to give; indeed, comparable definitions were adopted by, and the consequences recognized by, many logicians in medieval times. Moreover, they are immediate (even if not immediately obvious) consequences of the definitions which most of us would *naturally* be inclined to give: as when we say that one proposition implies another if the latter can't *possibly* be false if the former is true; or again, as when we say that one proposition implies another just when if the former is true then *necessarily* the latter is true.[23] Once we recognize this we may be more ready

23. For discussion of an ambiguity, and a possible philosophical confusion, lurking in these natural ways of speaking, see chapter 6, section 3.

to educate our intuitions to the point of recognizing (iii) and (iv) as the important logical truths which they are.

Secondly, it is not hard to understand why our uneducated intuitions tend to rebel at accepting (iii) and (iv). For the plain fact of the matter is that most of the instances of implication which we are likely to think of in connection with the inferences we perform in daily life, or in scientific enquiry, are instances in which the relation of implication holds between *contingent* propositions; and one contingent proposition, as it happens, implies another only if there *is* a certain measure of 'relevance' to be found between them — only if they are, in some sense, 'about' the same things. Not surprisingly, then, we are inclined to indulge our all-too-common disposition to *generalize* — to suppose, that is, that all cases of implication must be like the ones with which we are most familiar. Had we, from the beginning, attended both to the consequences of our definitions and to the fact that they allow of application to noncontingent propositions as well as contingent ones, we might never have come to expect that all cases of implication would satisfy the alleged relevance requirement when, in the nature of the case, only some do.

Thirdly, in chapter 4, section 6, pp. 224–30, we press the case further for acceptance of (iii) and (iv) by showing, among other things, that those who are disposed to reject them are likely to have other *competing* and even more compelling intuitions on the basis of which they will be strongly disposed, as well as logically obliged, to accept (iii) and (iv). But the detailed argument on that can wait.

EXERCISES

1. *Explain the difference between asserting (1) that Q is a false implication of P, and asserting (2) that it is false that P implies Q.*
2. *Give an example of two propositions such that the latter is a false implication of the former.*
3. *Give an example of two propositions such that it is false that the former implies the latter.*

* * * * *

Equivalence

Once we have the concept of implication in hand it is easy to give an account of the modal relation of equivalence. To say that a proposition P is equivalent to a proposition Q is just to say that they imply one another, i.e., that not only does P imply Q but also Q implies P, i.e., that the relation of *mutual* implication holds between P and Q.

Now the relation of implication, as we have already seen, can itself be defined in terms of possible worlds. It follows that the relation of equivalence is likewise definable.

Consider once more how we defined "implication". Any of the definitions, (a), (b), or (c), will do. Let us choose (a). There we said that a proposition P implies a proposition Q if and only if Q is true in all those possible worlds, if any, in which P is true. Suppose, now, that P and Q are equivalent, i.e., that not only does P imply Q but also Q implies P. Then not only will Q be true in all those possible worlds, if any, in which P is true, but also the converse will hold, i.e., P will be true in all those possible worlds, if any, in which Q is true. It follows that where two propositions are equivalent, if there are any possible worlds in which one of them is true, then in exactly the same worlds the other is also true. More briefly, two propositions are equivalent if and only if they have the same truth-value in precisely the same sets of possible worlds, i.e., there are no possible worlds in which they differ in truth-value.

As an example of the relation of equivalence consider the relation which the contingent proposition

(1.2) Canada is south of Mexico

has to the contingent proposition

(1.22) Mexico is north of Canada.

Even if we were merely to rely on our untutored intuitions most of us would find it natural to say that these two propositions are equivalent. But now we can explain why. We can point out that not only does *(1.2)* imply *(1.22)*, but also *(1.22)* implies *(1.2)*. Or, getting a little more sophisticated, we can point out that in any possible world in which one is true the other is also true and that in any possible world in which one is false the other is false. It matters not at all that both propositions happen to be false in the actual world. As we have already seen, false propositions as well as true ones can (and do) have implications. And as we can now see, false propositions as well as true ones can be equivalent to one another.

Noncontingent propositions as well as contingent ones can stand in relations of equivalence to one another. Indeed, if we attend carefully to the definition we have given for equivalence it is easy to see: (i) that all noncontingently true propositions form what is called an equivalence-class, i.e., a class all of whose members are equivalent to one another;[24] and (ii) that all noncontingently false propositions likewise form an equivalence-class. Conclusion (i) follows from the fact that if a proposition is necessarily true it is true in all possible worlds and hence is true in precisely the same set of possible worlds as any other necessarily true proposition. Conclusion (ii) follows from the fact that if a proposition is necessarily false it is false in all possible worlds and hence is false in precisely the same set of possible worlds as any other necessarily false proposition.

These two conclusions strike many persons as counterintuitive. Surely, it would be said, there is a *difference* between the necessarily true proposition

(1.5) Either the U.S. entered World War I in 1917 or it is not the case that the U.S. entered World War I in 1917

and the necessarily true proposition

(1.23) Either Canada is south of Mexico or it is not the case that Canada is south of Mexico.

After all, the concepts involved are not the same. One of these propositions makes reference to an item called "the U.S." and to an event that occurred at a specific moment in time. The other makes reference to two very different items called "Canada" and "Mexico" and to the geographical location of one with respect to the other. How, then, can the two propositions be equivalent? Likewise, it would be said, there is a difference — a conceptual difference, one might say — between the necessarily false proposition

24. In this book we are using the term "equivalence-class" as a synonym for "a class of equivalent propositions". On this reading, it is possible for a proposition to belong to several equivalence-classes. More standardly, however, the term "equivalence-class" is used in such a way that a proposition can be a member of only one equivalence-class. There should be little cause for confusion. The more usual conception of equivalence-class can simply be regarded as the logical union of all the equivalence-classes (as here defined) of a proposition.

> *(1.6)* The U.S. entered World War I in 1917 and it is not the case that the U.S. entered World War I in 1917

and the necessarily false proposition

> *(1.24)* Canada is south of Mexico and it is not the case that Canada is south of Mexico

which prevents us, in any ordinary sense of the word, from saying that they are "equivalent".

The same problem arises in connection with contingent propositions. Let us see how.

Consider, for a start, the fact that any proposition which asserts of two other propositions that both are true will be true in all and only those possible worlds in which *both* are true. After all, in any possible world, if any, in which one were true and the other false, the claim that both of them are true would be false. Suppose, now, that we want to assert of a contingent proposition that both it and a noncontingently true proposition are true. Then the proposition in which we assert their *joint* truth will be true in all and only those possible worlds in which both are true together. But they will be true together only in those possible worlds in which the contingent proposition is true. Hence the proposition which asserts the joint truth of two propositions, one of which is contingent and the other of which is necessarily true, will be true only in those possible worlds in which the contingent proposition is true. But this means that any proposition which asserts the joint truth of two propositions one of which is contingent and the other of which is necessarily true will itself be contingent and equivalent to the contingent proposition. For example, suppose that we have a proposition which asserts both that a contingent proposition, let us say

> *(1.2)* Canada is south of Mexico

and that a necessarily true proposition, let us say

> *(1.5)* Either the U.S. entered World War I in 1917 or it is not the case that the U.S. entered World War I in 1917

are true. This will be the proposition

> *(1.25)* Canada is south of Mexico and either the U.S. entered World War I in 1917 or it is not the case that the U.S. entered World War I in 1917.

Then it follows from what we have said that *(1.25)* is true in all and only those possible worlds in which it is true that Canada is south of Mexico, i.e., in which *(1.2)* is true. But this means not only that *(1.25)* is contingent but also that it is true in precisely the same set of possible worlds as *(1.2)*, and hence that *(1.2)* and *(1.25)* form an equivalence-class, i.e., are equivalent.

But are *(1.2)* and *(1.25)* *identical?* Our intuitions are likely to rebel at the very suggestion. And this is for the very same sorts of reasons which would lead them to rebel at the suggestion that the necessarily true propositions *(1.5)* and *(1.23)* are identical.

Perhaps the first point that needs to be made in reply to these objections is that the sense in which we are saying that two contingent propositions *may* be equivalent, and that any two necessarily true propositions *are* equivalent, and that any two necessarily false propositions *are* equivalent — is simply that which is conveyed in our definition, viz., that members of each of these sets of propositions have the same truth-value in the same set of possible worlds. We are *not* claiming that equivalent propositions are identical with one another. To be sure, there are uses of the term "equivalent" in ordinary discourse which foster the idea that "equivalent" is a precise synonym for "identical". For instance, someone who says that a temperature of zero degrees Celsius is equivalent to a temperature of thirty-two degrees Fahrenheit might just as well claim that the temperature as measured on one scale is the same as, or is identical with, the temperature as measured on the other scale. But the claim that two propositions are equivalent is not to be construed in this way. Two propositions can have identical truth-values in identical sets of possible worlds without themselves being identical. They can be identical in *these* respects without being identical in *all* respects. That is to say, they can be equivalent without being one and the same proposition. Draw a distinction between equivalence and identity, and conclusions (i) and (ii) no longer will seem counterintuitive. Similarly, we shall then be able, with consistency, to say that the two equivalent contingent propositions *(1.2)* and *(1.25)* are likewise non-identical. What *would* be counterintuitive would be the claims that all necessarily true propositions are identical with one another, that all necessarily false propositions are identical with one another, and that all equivalent contingent propositions are identical with one another. For then we should have to conclude that there are only two noncontingent propositions — a single necessarily true one and a single necessarily false one; and that all equivalent contingent propositions are identical.

But precisely how is the distinction between propositional equivalence and propositional identity to be drawn? More particularly, since we have already said what it is for two propositions to be equivalent, can we give an account of propositional identity which will enable us to say that propositions may be equivalent but non-identical?

It is worth noting, for a start, that discussions of identity — whether of the identity of propositions or of people, of ships or of sealing wax — are all too often bedeviled by difficulties even in posing the problem coherently. We can say, without any sense of strain, that two propositions are equivalent. But what would it mean to say that two propositions are identical? If they are identical how can they be *two?* Indeed, how can we sensibly even use the plural pronoun "they" to refer to that which we want to say is *one?* One's head spins, and we seem to be hedged in between inconsistency and futility. One of the greatest philosophers of the twentieth century, Ludwig Wittgenstein, put it aphoristically:

> Roughly speaking, to say of *two* things that they are identical is nonsense, while to say of *one* thing that it is identical with itself is to say nothing at all.[25]

One way out of this incipient dilemma lies in the recognition that on most, if not all, of the occasions when we are tempted to say that two things are identical, we could equally well — and a lot more perspicuously — say that two linguistic items symbolize (refer to, mean, or express) one and the same thing. Instead of saying — with all its attendant awkwardness — that two people, let us say Tully and Cicero, are identical, we can say that the names "Tully" and "Cicero" refer to one and the same person. Instead of saying that two propositions, let us say that Vancouver is north of Seattle and that Seattle is south of Vancouver, are identical, we can say that the sentences "Vancouver is north of Seattle" and "Seattle is south of Vancouver" express one and the same proposition.[26] This essentially, is the solution once offered, but subsequently rejected, by the great German philosopher and mathematician, Gottlob Frege. As he put it:

25. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trans. D.F. Pears and B.F. McGuinness, London, Routledge & Kegan Paul, 1961 (original edition published in German under the title *Logisch-Philosophiche Abhandlung*, 1921), proposition 5.5303.

26. Further reasons for adopting the distinction between sentences and propositions will be developed at length in chapter 2.

> What is intended to be said by $a = b$ seems to be that the signs or names 'a' and 'b' designate the same thing.[27]

Although Frege's suggestion works well enough when we want to make specific identity-claims, it does not enable us to avoid the dilemma when we try to formulate the conditions of identity for things quite generally, things for which there are no linguistic symbols as well as for things for which there are. As it stands, Frege's claim suggests that we should be able to say something along these lines: "Two signs or names 'a' and 'b' designate the same thing if and only if. . . . " (where the blank is to be filled in by the specification of appropriate conditions of identity). But it is just plain false that to make an identity-claim is, in general, to assert that two expressions have the same reference. Frege's reformulation works well enough in the case of items which happen to have been named or referred to by someone or other in some language or other. But are there not at least some unnamed items, for which linguistic symbols have not yet and perhaps never will be, devised? Surely there must be identity-conditions for these items as well. Yet if this is so, how can we even begin? As we have already seen, we can hardly start off by saying "Two things are identical if and only if . . . "

In order to give quite general identity-conditions for any items whatever, we would do well to, as it were, 'turn the problem around' and ask for the conditions of *non*-identity. We would do well, that is, to ask, "Under what conditions should we feel compelled to say that there are *two* items rather than just one?" Not only is this way of putting the question paradox-free, but it also avoids the limitations implicit in Frege's formulation.

The answer which commends itself to most thinking people, philosophers and nonphilosophers alike, is essentially that which has come to be known as Leibniz's Principle. Leibniz put it this way:

> There are never in nature two beings which are exactly alike in which it is not possible to find an internal difference. . . . [28]

In effect, Leibniz claimed that it is impossible for *two* items to have *all* their attributes — including relational ones — in common. Putting the point in still another way: there are two items rather than one if and only if one item has at least one attribute which the other does not.

Armed with this account of identity, let us return to the task of distinguishing between propositional equivalence and propositional identity. Can we give an account of the conditions of propositional identity which will enable us to preserve our intuitions that propositions may be equivalent and yet non-identical?

It seems clear that what guides our intuitions when we insist that propositions, contingent and noncontingent alike, need not be identical even when they are members of the same equivalence-classes is something like Leibniz's Principle. We note that in each of the problematic cases preceding, one proposition has at least one attribute which the other lacks, and so conclude — in accordance with Leibniz's Principle — that the two, though equivalent, are not identical.

Let us call any attribute which serves to sort out and differentiate between two or more items a *differentiating attribute.* Then we may say that what guides our intuitions as to the non-identity of

27. Gottlob Frege, "On Sense and Reference", in *Translations from the Philosophical Writings of Gottlob Frege,* ed. P. Geach and M. Black, Oxford, Basil Blackwell, 1952, p. 56.

28. G.W.F. Leibniz, *Monadology,* trans. R. Latta, London, Oxford University Press, 1965, Section 9, p. 222. This principle is widely known as the Principle of Identity of Indiscernibles. More aptly, it might be called the Principle of Non-Identity of Discernibles.

the equivalent contingent propositions *(1.2)* and *(1.25)* is the fact that there is at least one differentiating attribute which makes them non-identical. Indeed there are several. The attribute of making reference to the U.S. is one of them: it is an attribute which *(1.25)* has but *(1.2)* does not. And there are still further differentiating attributes which, in the case of these two propositions, serve to differentiate one from the other: *(1.25)* makes reference to an event which *(1.2)* does not; *(1.25)* makes reference to a date which *(1.2)* does not; and so on. In short, the items and attributes to which one proposition makes reference are not entirely the same as the items and attributes to which the other makes reference. Hence the propositions themselves are not the same but different.

Similarly, by invoking Leibniz's Principle we may distinguish the two equivalent noncontingent propositions *(1.5)* and *(1.23)*: *(1.5)* refers to the U.S., *(1.23)* does not; *(1.23)* refers to Canada, *(1.5)* does not; etc. Once again, we may conclude that two equivalent propositions are not identical.

To sum up, equivalent propositions cannot differ from one another in respect of the attribute of having the same truth-value in the same sets of possible worlds. But they can differ from one another in respect of other attributes. Identical propositions, by way of comparison, cannot differ from one another in respect of this or any other attribute. They have all of their attributes in common.

In chapter 2, section 2, we return to the problem of drawing a line between propositional equivalence and propositional identity and come up with a more precise statement (in section 3) of the conditions for propositional identity.

EXERCISES

1. *Which propositions (a. – e.) are* inconsistent *with which propositions (i. – v.)? Which propositions (a. – e.) are* consistent *with which propositions (i. – v.)? Which propositions (a. – e.)* imply *which propositions (i. – v.)? And which propositions (a. – e.) are* equivalent *to which propositions (i. – v.)?*

   a. *There are 8,098,789,243 stars.*

   b. *All squares have four sides.*

   c. *Some squares have six sides.*

   d. ***There are 8,098,789,243 stars or it is not the case that there are 8,098,789,243 stars.***

   e. *The U.S. entered World War I in 1917.*

   i. *All triangles have three sides.*

   ii. *There are fewer than 17,561,224,389 stars.*

   iii. *There are more than 8,098,789,242 stars and fewer than 8,098,789,244 stars.*

   iv. *There are 124,759,332,511 stars.*

   v. *The U.S. entered World War I after 1912.*

   (Partial answer: *a.* *is inconsistent with iv.*
   c. *is inconsistent with i., ii., iii., iv., and v.*
   a. *is consistent with i., ii., iii., and v.)*

2. *a.* *Is proposition A, defined below, consistent or inconsistent with proposition B?*

   "A" = Bill is exactly 6′ tall.
   "B" = Bill is exactly 6′ 2″ tall.

b. *Is proposition C, defined below, consistent or inconsistent with proposition D?*

"C" = Someone is exactly 6′ tall.
"D" = Someone is exactly 6′ 2″ tall.

c. *Is proposition E, defined below, self-consistent or self-inconsistent?*

"E" = Someone is exactly 6′ tall and 6′ 2″ tall.

3. *Explain why it is* misleading *to say such things as:*

"In the actual world, Canada's being north of Mexico is inconsistent with Mexico's being north of Canada";

or

"In the world of Time Enough for Love, *the proposition that Lazarus falls in love with two of his daughters implies the proposition that Lazarus is a father."*

* * * * *

Symbolization

Our repertoire of symbols can now be extended to encompass not only the modal properties represented by "$\Box$", "$\Diamond$", "∇", and "Δ", but also the modal relations of consistency, inconsistency, implication, and equivalence. They are standardly represented in symbols as follows:

(1) The concept of consistency: "$\circ$"

(2) The concept of inconsistency: "ϕ"

(3) The concept of implication: "$\rightarrow$" [called *arrow*]

(4) The concept of equivalence: "$\leftrightarrow$" [called *double-arrow*][29]

Each of these symbols may be written between symbols which stand for propositions to yield further propositional symbols (e.g., "$P \circ Q$", "$P \rightarrow Q$").

Each of these symbols may be defined contextually as follows:

"$P \circ Q$" $=_{df}$ "P is consistent with Q"

"$P \phi Q$" $=_{df}$ "P is inconsistent with Q"

"$P \rightarrow Q$" $=_{df}$ "P implies Q"

"$P \leftrightarrow Q$" $=_{df}$ "P is equivalent to Q"

29. These latter two symbols are not to be confused with the two symbols "$\supset$" and "$\equiv$" with which some readers may already be familiar. The two symbols "$\supset$" (called *hook* or *horseshoe*) and "$\equiv$" (called *triple-bar*) will be introduced later in this book and will there be used to stand for the relations of material conditionality and material biconditionality respectively. At that time we shall take some pains to argue that the relations of material conditionality and material biconditionality are distinctly different from any relations which have been introduced in this first chapter; in particular, that they are distinct from implication and equivalence.

EXERCISE

Refer to question 1 in the preceding exercise. Let the letters "A" – "E" stand for propositions a. – e. and the letters "J" – "N" for the propositions i. – v. Re-do question 1 expressing all your answers in the symbolism just introduced.

(Partial answer: $A \Phi M$

$C \Phi J$, $C \Phi K$, $C \Phi L$, $C \Phi M$ *and* $C \Phi N$

$A \circ J$, $A \circ K$, $A \circ L$, $A \circ N$*)*

5. SETS OF PROPOSITIONS

The truth-values, modal properties, and the modal relations which may be ascribed to individual propositions and to pairs of propositions, may, with equal propriety, be ascribed to *sets* of propositions and to pairs of sets of propositions.

Truth-values of proposition-sets

A set of propositions will be said to be *true* if every member of that set is true. And a set of propositions will be said to be *false* if not every member of that set is true, i.e., if at least one member of that set is false. Note carefully: a set of propositions may be false even though not every member of that set is false. A single false member in a set of propositions is sufficient to render the set false. And of course it follows from this that if a set is false, we are not entitled to infer of any particular member of that set that it is false; we are entitled to infer only that at least one member is false.

Example 1: A true set of propositions

(1.26) {Snow is white, The U.S. entered World War I in 1917}.[30]

Example 2: A false set of propositions

(1.27) {Snow is white, The U.S. entered World War I in 1914}.

It should be clear that the two expressions (1) "a set of false propositions" and (2) "a false set of propositions", do *not* mean the same thing. A set of false propositions is a set *all* of whose members are false; a false set of propositions is a set *at least one of whose* members is false.

Modal properties of proposition-sets

A set of propositions will be said to be *possibly true* or *self-consistent* if and only if there exists a possible world in which every member of that set is true.

Self-consistency is a 'fragile' property. It is easily and often unwittingly lost (see chapter 6, section 7). Consider the following example:

30. Here and subsequently in this book we use a pair of braces (i.e., "{" and "}") as a means to designate a set.

{April is taller than Betty,
Betty is taller than Carol,
Carol is taller than Dawn,
Dawn is taller than Edith,
Edith is taller than Frances,
Frances is taller than April}.

Notice how every subset consisting of *any* five of these propositions is self-consistent. Remove the first, *or* the second, *or* the third, etc., and the *remaining* set is self-consistent (which of course is *not* to say that it is true). But in reintroducing the removed proposition and consequently enlarging the set to what it was, self-consistency is lost. And once self-consistency is lost, in this set as in any other, it can never be regained by *adding* more propositions. Some persons think that self-consistency can be restored by inserting a proposition of the following kind into a self-inconsistent set: The immediately preceding proposition is false. But this device can *never* restore self-consistency. (See the exercises on p. 44.)

A set of propositions is *possibly false* if and only if there exists a possible world in which at least one member of that set is false.

A set of propositions is *necessarily true* if and only if every member of that set is necessarily true, i.e., if in every possible world every member of that set is true.

A set of propositions is *necessarily false* or *self-inconsistent* if and only if there does not exist any possible world in which that set is true, i.e., if in every possible world at least one proposition or another in that set is false.

And finally, a set of propositions is *contingent* if and only if that set is neither necessarily true nor necessarily false, i.e., if and only if there exists some possible world in which every member of that set is true and there exists some possible world in which at least one member of that set is false.

EXERCISES

Part A

For each set below tell whether that set is (1) possibly true and/or possibly false; and (2) necessarily true, necessarily false or contingent.

i. {*Canada is north of Mexico, Hawaii is in the Pacific Ocean, Copper conducts electricity*}

ii. {*Snow is white, Pine is a softwood, Coal is red*}

iii. {*There were exactly twelve tribes of Israel, There were exactly fourteen tribes of Israel*}

iv. {*All sisters are female, All triangles have three sides, All squares have four sides*}

v. {*Some coffee cups are blue, Some coffee cups are green, Some coffee cups are yellow*}

vi. {*Some triangular hats are blue, All triangles have three sides, Some squares have five sides*}

vii. {*All triangles have three sides, Some triangular hats are blue*}

viii. {*Someone believes that today is Monday, Someone believes that today is Wednesday*}

ix. {*Grass is green, Someone believes that grass is green*}

x. {*All sisters are females, All females are sisters*}

Part B

1. *Explain why self-consistency can never be restored to a self-inconsistent set of propositions by the device of inserting into that set a proposition of the sort: The immediately preceding proposition is false.*
2. *The example used above which reports the relative heights of April and Betty, etc., can be made self-consistent by the removal of the last proposition. What argument is to be used against the claim that it is the last proposition in the above set which 'induces' the self-inconsistency and hence is false?*
3. *A self-inconsistent set of three propositions of which every proper non-empty subset is self-consistent is called an* antilogism. *An example would be:*

   {*Lorna has three brothers,*
   Sylvia has two brothers,
   Sylvia has twice as many brothers as Lorna}.

   Find three examples of antilogisms.
4. *Find a set of three* contingent *propositions such that each pair of propositions drawn from that set constitutes a self-inconsistent set. Example:*

   {*Norman is shorter than Paul,*
   Norman is the same height as Paul,
   Norman is taller than Paul}.
5. *Explain why one should* not *adopt the following definition of "necessary falsehood" for a set of propositions: A set of propositions is necessarily false if and only if every member of that set is necessarily false.*

* * * * *

Modal relations between proposition-sets

Two sets of propositions will be said to stand in the relation of *consistency* if and only if there exists some possible world in which all the propositions in both sets are jointly true.

Two sets of propositions stand in the relation of *inconsistency* if and only if there does not exist a possible world in which all the propositions of both sets are jointly true.

One set of propositions stands in the relation of *implication* to another set of propositions if and only if all the propositions of the latter set are true in every possible world, if any, in which all the propositions of the former set are true.

And two sets of propositions stand in the relation of *equivalence* if and only if all the propositions in one set are true in all and just those possible worlds, if any, in which all the propositions of the other set are true.

To illustrate these definitions, we cite the following examples.

Example 1:

The set of propositions

(1.28) {Ottawa is the capital of Canada, All men are mortal}

is *consistent* with the set of propositions

(1.29) {Snow is white, Today is Tuesday, Some dogs meow}.

Example 2:

The set of propositions

(1.30) {April is older than Betty, Betty is older than Carol, Carol is older than Diane}

is *inconsistent* with the set of propositions

(1.31) {Diane is older than Edith, Edith is older than April}.

Example 3:

The set of propositions

(1.32) {Mary invited Brett to act in the play, Gresham invited Sylvia to act in the play}

implies the set of propositions

(1.33) {Sylvia was invited to act in the play, Mary invited someone to act in the play}.

Example 4:

The set of propositions

(1.34) {Today is Wednesday}

is *equivalent* to the set of propositions

(1.35) {It is later in the week than Tuesday, It is earlier in the week than Thursday}.

As we can see, some, although not all, of these sets of propositions contain more than one member, i.e., more than one proposition. Does this mean that the relations of consistency, inconsistency, etc., are not always dyadic, or two-placed relations? Not at all. For a dyadic relation is a relation which holds between two *items* and each of the above sets may be counted as a single item even if some of them have two or more members. Hence a relation which holds between two *sets* of propositions is still a dyadic relation even if there is more than one proposition in either or both sets.[31]

Insofar as modal relations can obtain between sets of propositions as well as between single, individual propositions, certain consequences follow which we would do well to explore.

31. Note that although *(1.35)* is *equivalent* to *(1.34)*, the set *(1.35)* does not itself constitute an equivalence-class, i.e., not all its members are equivalent to one another. One should be careful not to suppose that the relation of equivalence can hold only between equivalence-classes.

It needs to be pointed out that whenever a modal relation R holds between two individual propositions, P and Q,[32] there will always be an infinity of non-identical propositions belonging to the same equivalence-class as P, and an infinity of non-identical propositions belonging to the same equivalence-class as Q, and that *any* proposition belonging to the former class will stand in the relation R to *any* proposition belonging to the latter class. This is easy to prove.

Remember, first, that for any proposition P, whether contingent or noncontingent, there is a set of propositions each of which is true in precisely the same set of possible worlds as P; that is to say, any proposition P, of whatever modal status, is a member of an equivalence-class.

Secondly, the equivalence-class to which any given proposition P belongs is a set of propositions with an infinite number of members. How may we establish this latter claim? For a start, we may note that the set of natural numbers is a set with an infinite number of members. Now for each of these natural numbers there exists a proposition which asserts that the number has a successor. Hence the number of such propositions is itself infinite. Moreover, each of these propositions is not only true, but necessarily true. It follows that there is an infinite number of necessary truths. Now, as we saw before, since every necessary truth is true in precisely the same set of possible worlds as every other necessary truth, the set of necessary truths forms an equivalence-class. And, as we have just seen, this equivalence-class must have an infinite number of members. In short, every necessarily true proposition belongs to an equivalence-class which has an infinite number of members. But if this is so, then the same must be true also of every necessarily false proposition. For it is obvious that there must be an infinite number of necessarily false propositions: to each natural number there can be paired off a necessarily false proposition, e.g., the proposition that that number has no successor, and it is equally obvious that this infinite set of propositions constitutes an equivalence-class with all other propositions which are necessarily false. Hence every necessarily false proposition belongs to an equivalence-class which has an infinite number of members.

How about contingent propositions? The same result holds for them too. As we saw before (p. 37), for any contingent proposition whatever, there exists another non-identical but equivalent proposition which asserts the joint truth of both that proposition and some necessarily true proposition. But there is an infinite number of necessarily true propositions any one of which may be asserted to be true conjointly with a given contingent proposition. Hence for any contingent proposition whatever there exists an infinite number of non-identical but equivalent propositions each of which asserts the joint truth of that proposition and some necessarily true proposition. Hence every contingent proposition belongs to an equivalence-class which has an infinite number of members.

Consider, in the light of all this, two individual propositions, A and B, which stand in some modal relation R. For instance, let us suppose that A is the contingent proposition

(1.3) The U.S. entered World War I in 1914

and B is the contingent proposition

(1.21) The U.S. entered World War I before 1920.

There is an infinite number of non-identical propositions which are equivalent to A, and an infinite number of non-identical propositions which are equivalent to B. Thus it follows from the fact that *(1.3)* (i.e., A) implies *(1.21)* (i.e., B) that there is an infinitely large number of propositions equivalent to A each of which implies an infinitely large number of propositions equivalent to B.

32. Or, we might equally say, "between two unit sets {P} and {Q}. . . ."

Parallel conclusions follow for each of the other modal relations of consistency, inconsistency, and equivalence.

In sum, the point may be put this way: Whenever two propositions, P and Q, stand in any modal relation R, all those propositions which are equivalent to P, of which there is necessarily an infinite number, will similarly stand in the modal relation R to each of the infinite number of propositions which are equivalent to Q.

An interesting, neglected corollary may be drawn from this principle. In section 4 we argued that there are only two species of the modal relation of inconsistency: either two inconsistent propositions (and now we would add "proposition-sets") are contraries or they are contradictories. Now while it has long been acknowledged, indeed insisted upon, that no proposition has a unique (i.e., one and only one) contrary, it has often been as strenuously insisted that every proposition does have a unique contradictory, i.e., that there is one and only one proposition which stands in the relation of contradiction to a given proposition. But in light of the distinction between propositional-identity and propositional-equivalence and in light of the fact that modal relations hold equally well between sets of propositions as between propositions themselves, these claims need to be re-examined. Let us begin with some examples. Consider the proposition

(1.36) Today is Wednesday.

Among its contraries are

(1.37) Today is Monday;

(1.38) Today is Saturday.

Now let's look at some of its contradictories. These will include

(1.39) Today is not Wednesday;

(1.40) Today is not the day after Tuesday;

(1.41) Today is not the day before Thursday, etc.

What *difference* can we detect between the contraries of the proposition *(1.36)* and the contradictories of that same proposition? Just this: the contradictories of a given proposition form an equivalence-class (e.g., *(1.39)*, *(1.40)*, and *(1.41)* are all equivalent to one another), while the contraries of a given proposition are not all equivalent to one another. Thus while the claim that every proposition has a unique contradictory cannot be supported, it can be superseded by the true claim that all the contradictories of a proposition are logically equivalent to one another, i.e., that the set of contradictories of a proposition is itself an equivalence-class. No such claim can be made for the contraries of a proposition. The set consisting of all the contraries of a given proposition is *not* a set of equivalent propositions.

Minding our "P"s and "Q"s

Insofar as the kinds of properties and relations we are concerned to ascribe to single propositions may, as we have just seen, be ascribed to sets of propositions, we would do well to point out that both propositions and sets of propositions may equally well be represented by the same sorts of symbols in the conceptual notation we use. More specifically, when we write such things as "P stands in the relation R to Q if and only if . . . ", etc., we should be understood to be referring, indiscriminately, by our use of "P" and "Q", both to single propositions and to sets of propositions.

In the next section we shall introduce what we call "worlds-diagrams" and will label parts of them with "P"s and "Q"s. For convenience and brevity we often treat these symbols as if they referred to single propositions. In fact they ought to be thought to refer either to single propositions or to proposition-sets.

EXERCISES

1. *Which proposition-sets (a. – e.) are inconsistent with which proposition-sets (i. – v.)? Which proposition-sets (a. – e.) are consistent with which proposition-sets (i. – v.)? Which proposition-sets (a. – e.) imply which proposition-sets (i. – v.)? And which proposition-sets (a. – e.) are equivalent to which proposition-sets (i. – v.)?*

a. *{Today is Tuesday, Bill has missed the bus, Bill is late for work}*	*i.* *{Bill has missed the bus}*
b. *{Someone returned the wallet, Someone lost his keys}*	*ii.* *{Someone who lost his keys returned the wallet}*
c. *{The Prime Minister is 6′ tall, The Prime Minister is exactly 5′ 2″ tall}*	*iii.* *{Mushroom omelets are not poisonous, No mushroom omelet is poisonous}*
d. *{Some mushrooms are poisonous, Some mushrooms are not poisonous}*	*iv.* *{John is 15 years old, John is 5′ 3″ tall}*
e. *{John is 15 years old and is 5′ 3″ tall}*	*v.* *{Although Bill has missed the bus, he is not late for work}*

2. *Which one of the ten sets of propositions in exercise 1 is a set of equivalent propositions?*

3. *Construct an equivalence-class of three propositions one of which is the proposition that Sylvia is Diane's mother.*

4. *Construct an equivalence-class of three propositions one of which is the proposition that two plus two equals four.*

6. MODAL PROPERTIES AND RELATIONS PICTURED ON WORLDS-DIAGRAMS

Worlds-diagrams have already been used: figure *(1.b)* introduced our basic conventions for representing an infinite number of possible worlds, actual and non-actual; and figures *(1.d)*, *(1.e)*, and *(1.f)* gave graphic significance to our talk of the different sorts of modal status that propositions have according to whether they are contingent, necessarily true or necessarily false, respectively. So far we have given these diagrams merely an *illustrative* role: our *talk* of possible worlds could have sufficed by itself. However, these diagrams can also be given an important *heuristic* role: they can facilitate our discovery and proof of logical truths which might otherwise elude us.

In order that we may better be able to use them heuristically we adopt the following two simplifying conventions:

(a) We usually omit from our diagrams any representation of the distinction between the actual world and other (non-actual) possible worlds. When the need arises to investigate the consequences of supposing some proposition to be actually true or actually false, that distinction can, of course, be reintroduced in the manner displayed in figures *(1.d)*, *(1.e)*, and *(1.f)*, or as we shall see soon, more perspicuously, simply by placing an "×" on the diagram to mark the location of the actual world among the set of all possible worlds. But, for the most part, we shall be concerned primarily with investigating the relationships between propositions independently of their truth-status in the actual world, and so shall have infrequent need to invoke the distinction between actual and non-actual worlds.

(b) We omit from our diagrams any bracketing spanning those possible worlds, if any, in which a given proposition or proposition-set[33] is false. This means that every bracket that we use is to be interpreted as spanning those possible worlds only in which a given proposition (or proposition-set) is true. In the event that a given proposition is not true in any possible world, i.e., is false in all possible worlds, we 'locate' that proposition by means of a point placed outside (and to the right of) the rectangle representing the set of all possible worlds. In effect we thus 'locate' any necessarily false proposition among the impossible worlds.

In light of these simplifying conventions, let us first reconstruct the three basic worlds-diagrams depicting the modal properties of contingency, necessary truth, and necessary falsity (figures *(1.d)*, *(1.e)*, and *(1.f)*), and then consider how they might be supplemented in order to depict modal relations.

Worlds-diagrams for modal properties

A single proposition (or proposition-set) P, may be true in all possible worlds, just some, or none. There are no other possibilities. If, then, we depict the set of all possible worlds by a single box, it follows that we have need of three and only three basic worlds-diagrams for the modal properties of a proposition (or proposition-set) P. They are:

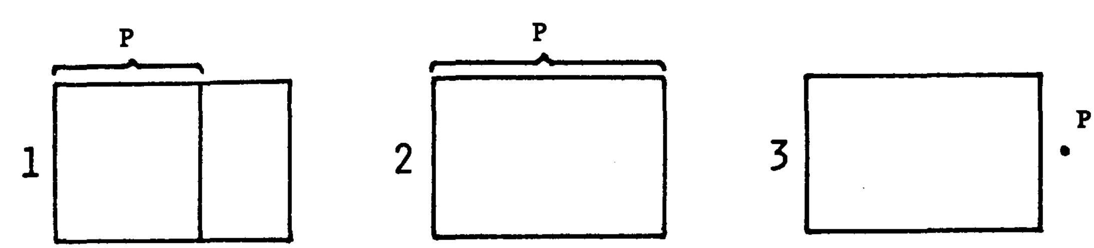

FIGURE *(1.h)*

33. See the subsection "Minding our 'P's and 'Q's", pp. 47–48.

Diagram 1 in figure *(1.h)*, p. 49, depicts the contingency of a single proposition (or proposition-set) P. The proposition P is contingent because it is true in some possible worlds but false in all the others. In effect, diagram 1 is a reconstruction of figure *(1.d)* made in accordance with the two simplifying conventions specified above. Our diagram gives the modal status of P but says nothing about its actual truth-status (i.e., truth-value in the actual world).

Diagram 2 depicts the necessary truth of a single proposition P. The proposition P is necessarily true, since it is true in all possible worlds. In effect, diagram 2 is a reconstruction of figure *(1.e)* made in accordance with our simplifying conventions.

Diagram 3 depicts the necessary falsity of a single proposition P. Here P is necessarily false since it is false in all possible worlds. In effect, diagram 3 is a simplification of figure *(1.f)*.

Worlds-diagrams for modal relations

In order to depict modal relations between two propositions (or two proposition-sets) P and Q, we need exactly fifteen worlds-diagrams. In these worlds-diagrams (see figure *(1.i)* on p. 51), no significance is to be attached to the relative *sizes* of the various segments. For our present purposes all we need attend to is the relative *placement* of the segments, or as mathematicians might say, to their topology. For the purposes of the present discussion our diagrams need only be qualitative, not quantitative.[34]

EXERCISE

Reproduce figure (1.i) *and add brackets for "∼P" and for "∼Q" to each of the fifteen worlds-diagrams.*

* * * * *

Interpretation of worlds-diagrams

Diagrams 1 to 4 depict cases where both propositions are noncontingent. Diagrams 5 to 8 depict cases where one proposition is noncontingent and the other is contingent. The final seven diagrams (9 to 15) depict cases where both propositions are contingent.

Now each of these fifteen diagrams locates two propositions, P and Q, with respect to the set of all possible worlds, and thence with respect to one another, in such a way that we can determine what modal relations one proposition has to the other. How can we do this?

The modal relations we have singled out for consideration so far are those of inconsistency, consistency, implication, and equivalence. Recall, then, how each of these four relations was defined: P is *inconsistent* with Q if and only if there is no possible world in which both are true; P is *consistent* with Q if and only if there is a possible world in which both are true; P *implies* Q if and only if [definition (b)] there is no possible world in which P is true and Q is false; and P is *equivalent* to Q if and only if in each of all possible worlds P has the same truth-value as Q. Recall, further, that our device for depicting a proposition as true in a possible world is to span that world by means of a bracket labeled with a symbol signifying that proposition.

34. Later, when we come to discuss the concept of "the contingent content" of a proposition, we shall suggest how one might want to reinterpret these worlds-diagrams so that the sizes of the segments do take on significance. (See chapter 6, section 11.)

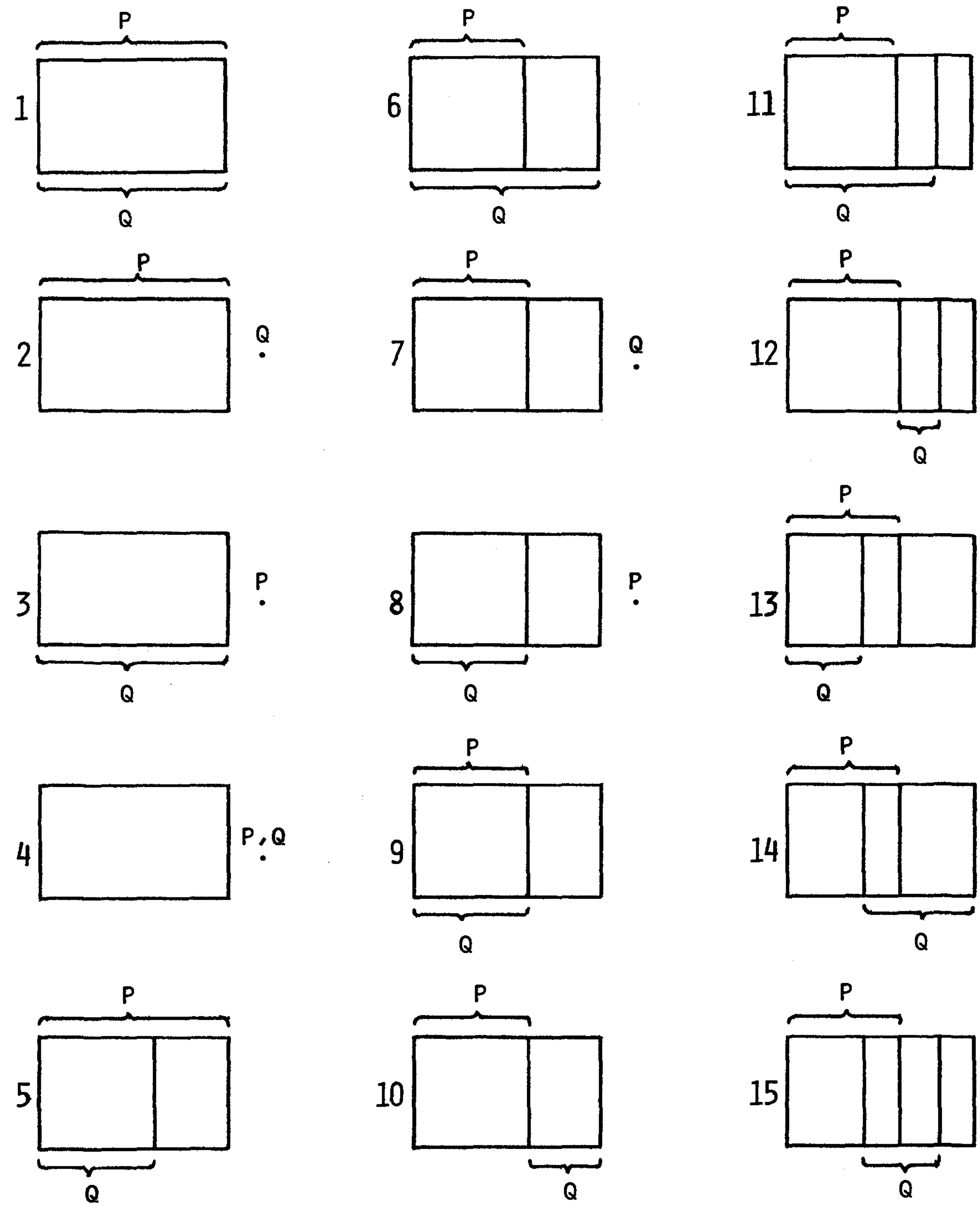
1
P
Q
2
P
Q
3
P
Q
4
P, Q
5
P
Q
6
P
Q
7
P
Q
8
P
Q
9
P
Q
10
P
Q
11
P
Q
12
P
Q
13
P
Q
14
P
Q
15
P
Q

FIGURE *(1.i)*

The requisite rules for the interpretation of our worlds-diagrams follow immediately:

Rule 1: P is *inconsistent* with Q if and only if there does not exist any set of possible worlds which is spanned both by a bracket for P and by a bracket for Q;

Rule 2: P is *consistent* with Q if and only if there does exist a set of possible worlds which is spanned both by a bracket for P and by a bracket for Q;

Rule 3: P *implies* Q if and only if there does not exist any set of possible worlds which is spanned by a bracket for P and which is not spanned by a bracket for Q (i.e., if and only if any set of possible worlds spanned by a bracket for P is also spanned by a bracket for Q);[35]

Rule 4: P is *equivalent* to Q if and only if there does not exist any set of possible worlds which is spanned by the bracket for one and which is not spanned by the bracket for the other (i.e., the brackets for P and for Q span precisely the same set of worlds).

It is the addition of these rules of interpretation that gives our worlds-diagram the heuristic value that we earlier claimed for them. By applying them we can prove a large number of logical truths in a simple and straightforward way. Consider some examples:

(i) Diagrams 2, 3, 4, 7, and 8 comprise all the cases in which one or both of the propositions P and Q is necessarily false. In none of these cases is there any set of possible worlds spanned both by a bracket for P and by a bracket for Q. Hence, by Rule 1, we may validly infer that in all of these cases P is inconsistent with Q. In short, *if one or both of a pair of propositions is necessarily false then those propositions are inconsistent with one another.*

(ii) Diagrams 1, 2, 3, 5, and 6 comprise all the cases in which one or both of the propositions P and Q is necessarily true. In each of these cases, except 2 and 3, there is a set of possible worlds spanned both by a bracket for P and by a bracket for Q. Hence, by Rule 2, we may validly infer that in each of these cases, except 2 and 3, P is consistent with Q. But diagrams 2 and 3 are cases in which one or other of the two propositions is necessarily false. We may conclude, therefore, that *a necessarily true proposition is consistent with any proposition whatever except a necessarily false one.*

(iii) Diagrams 3, 4, and 8 comprise all the cases in which a proposition P is necessarily false. In none of these cases is there a set of possible worlds spanned by a bracket for P. Hence in none of these cases is there a set of possible worlds which is spanned by a bracket for P and not spanned by a bracket for Q. Hence, by Rule 3, we may validly infer that in each case in which P is necessarily false, P implies Q no matter whether Q is necessarily true (as in 3), necessarily false (as in 4), or contingent (as in 8). By analogous reasoning concerning diagrams 2, 4, and 7 — all the cases in which a proposition Q is necessarily false — we can show that in each case in which Q is necessarily false, Q implies P no matter whether P is necessarily true (as in 2), necessarily false (as in 4), or contingent (as in 7). In short, we may conclude that *a necessarily false proposition implies any and every proposition no matter what the modal status of that proposition.*

(iv) Diagrams 1, 3, and 6 comprise all the cases in which a proposition Q is necessarily true. In none of these cases is there a set of possible worlds which is *not* spanned by a bracket for Q. Hence, by Rule 3, we may validly infer that in each case in which Q is necessarily true, Q is implied by a proposition P no matter whether P is necessarily true (as in 1), necessarily false

35. This means that P implies Q in three cases: (i) where the bracket for P spans no possible worlds at all (i.e., P is necessarily false); (ii) where the bracket for P is included within the bracket for Q; and (iii) where the bracket for P is coextensive with the bracket for Q.

(as in 3), or contingent (as in 6). By analogous reasoning concerning diagrams 1, 2, and 5 — all the cases in which a proposition P is necessarily true — we can show that in each case in which P is necessarily true, P is implied by a proposition Q no matter whether Q is necessarily true (as in 1), necessarily false (as in 2), or contingent (as in 5). In short, we may conclude that *a necessarily true proposition is implied by any and every proposition no matter what the modal status of that proposition.*

The special heuristic appeal of these worlds-diagrams lies in the fact that, taken together with certain rules for their interpretation, we can literally *see* immediately the truth of these and of many other propositions about the modal relations which propositions have to one another. The addition of still further definitions and rules of interpretation later in this book will enable us to provide more perspicuous proofs of important logical truths — including some which are not as well-known as those so far mentioned.

A note on history and nomenclature

So far we have given names to only a few of the modal relations which can obtain between two propositions. We have spoken of *inconsistency* (and its two species, contradiction and contrariety), of *consistency*, of *implication*, and of *equivalence*. Within the philosophical tradition, however, we find logicians talking also of modal relations which they call "superimplication" (sometimes called "superalternation"), "subimplication" (sometimes called "subalternation"), "subcontrariety", and "independence" (sometimes called "indifference").

By "superimplication" and "subimplication" (or their terminological alternates), traditional logicians meant simply the relations of *implication* and of *following from* respectively. To say that P stands in the relation of *superimplication* to Q is simply to say that P *implies* Q, while to say that P stands in the relation of *subimplication* to Q is simply to say that P *follows from* Q (or that Q *implies* P). In short, subimplication is the converse of superimplication, i.e., of implication.

By "subcontrariety" we mean the relation which holds between P and Q when P and Q *can both be true together but cannot both be false.* That is to say, *subcontrariety* is the relation which holds between P and Q when there is at least one possible world in which both are true (P and Q are consistent) but there is no possible world in which both are false ($\sim$P and $\sim$Q are inconsistent).[36]

By "independence" we mean the relation which holds between P and Q when no relation other than consistency holds between the two. That is to say, *independence* is the relation which holds between P and Q when there is at least one possible world in which both are true (P and Q are consistent), there is at least one possible world in which both are false ($\sim$P and $\sim$Q are also consistent), there is at least one possible world in which P is true and Q is false, and there is at least one possible world in which P is false and Q is true.

Of the various modal relations we have distinguished, only one is uniquely depicted by a single worlds-diagram. This is the relation of independence. Independence is depicted by diagram 15 and by that diagram alone. Each of the other modal relations is exemplified by two or more of the fifteen worlds-diagrams. Historically, this fact has not always been recognized. Within traditional logic,[37]

36. The term "subcontrariety" reflects the fact, well known to traditional logicians, that subcontrary propositions are contradictories of propositions which are contraries and stand in the relation of *subimplication* to these contrary propositions. Thus P is a subcontrary of Q if and only if (a) $\sim$P and $\sim$Q are contraries, and (b) the contrary propositions, $\sim$P and $\sim$Q, imply Q and P respectively.

37. By "traditional logic" we mean the logic which was founded by Aristotle (384–322 B.C.), enriched by the Stoics and Megarians, and effectively canonized by sixteenth- and seventeenth-century logicians.

the terms "equivalence", "contradiction", "superimplication", "contrariety", "subimplication", and "subcontrariety" were often used *as if* each of them, too, were relations which, in terms of our worlds-diagrams, we would uniquely depict by a single diagram. Traditional logicians tended to think of equivalence as that relation which we can reconstruct, by means of our worlds-diagrams, as holding in diagram 9 alone; of contradiction as if it could be depicted in diagram 10 alone; of superimplication (and hence of implication) as if it could be depicted in diagram 11 alone; of contrariety as if it could be depicted in diagram 12 alone; of subimplication as if it could be depicted in diagram 13 alone; and of subcontrariety as if it could be depicted in diagram 14 alone. That is to say, they tended to think of these relations as if they could hold only between propositions both of which are contingent. (It is easy to see, by simple inspection, that diagrams 9 through 15 depict the only cases in which both propositions are contingent.) One consequence of the traditional preoccupation with relations between contingent propositions was, and sometimes still is, that someone brought up in that tradition, if asked what logical relation holds between a pair of *contingent* propositions (any of those depicted by diagrams 9 through 15), had little difficulty in invoking one of the standard repertoire of "equivalence", "contradiction", "superimplication", "contrariety", "subimplication", "subcontrariety", or "independence"; but if asked what logical relation holds between a pair of propositions one or both of which is *noncontingent* (any of those depicted by diagrams 1 through 8), simply would not know what to say.[38]

A second and more significant consequence of this preoccupation with relations between contingent propositions was, and still is, that the "intuitions" of some (but not, of course, all) persons under the influence of this tradition are not well attuned to the analyses, in terms of possible worlds, of relations like equivalence and implication; they tend to consider these analyses as "counter-intuitive", and certain consequence of these analyses as "paradoxical".[39]

Capsule descriptions of modal relations

It may be helpful to gather together the descriptions we have given in various places of those modal relations which are our principle concern.

P is *inconsistent* with Q:	there is no possible world in which both are true
P is a *contradictory* of Q:	there is no possible world in which both are true and no possible world in which both are false
P is a *contrary* of Q:	there is no possible world in which both are true but there is a possible world in which both are false
P is *consistent* with Q:	there is a possible world in which both are true
P *implies (superimplies)* Q:	there is no possible world in which P is true and Q is false
P *follows from (subimplies)* Q:	there is no possible world in which Q is true and P is false

38. Answers can, however, be given. Worlds-diagram 8, for example, depicts a relation which satisfies the descriptions given of *two* modal relations, viz., implication and contrariety. That is, two propositions, P and Q, which stand in the relation depicted in diagram 8, are such that (1) P implies Q and (2) P and Q are contraries.

39. In chapter 4 we develop this point further and suggest also that some resistence to these analyses has its source in a failure to distinguish between valid inference and demonstrability.

P is *equivalent* to Q:	in each possible world, P and Q have matching truth-values
P is a *subcontrary* of Q:	there is a possible world in which both are true but there is no possible world in which both are false
P is *independent* of Q:	there is a possible world in which both are true, a possible world in which both are false, a possible world in which P is true and Q is false, and a possible world in which P is false and Q is true

EXERCISES

Part A

1. *Which worlds-diagrams (in figure* (1.i)*) represent cases in which P is necessarily true (i.e., in which* $\Box P$ *obtains)?*
2. *Which worlds-diagrams represent cases in which Q is necessarily true?* [$\Box Q$]
3. *Which worlds-diagrams represent cases in which P is contingent?* [∇P]
4. *Which worlds-diagrams represent cases in which Q is contingent?* [∇Q]
5. *Which three worlds-diagrams represent cases in which we may validly infer that P is actually true (i.e., true in the actual world)?*
6. *Which seven worlds-diagrams represent cases in which P implies Q?* [$P \rightarrow Q$]
7. *Which seven worlds-diagrams represent cases in which Q implies P?* [$Q \rightarrow P$]
8. *Which eight worlds-diagrams represent cases in which P is consistent with Q?* [$P \circ Q$]
9. *Which seven worlds-diagrams represent cases in which P is inconsistent with Q?* [$P \not\circ Q$]
10. *Which three worlds-diagrams represent cases in which P and Q are contradictories?*
11. *Which four worlds-diagrams represent cases in which P and Q are contraries?*
12. *Which four worlds-diagrams represent cases in which P and Q are subcontraries?*
13. *Which three worlds-diagrams represent cases in which P and Q are equivalent?* [$P \leftrightarrow Q$]
14. *Which worlds-diagrams represent cases in which P is possible and in which Q is possible?* [$\Diamond P$ *and* $\Diamond Q$]
15. *Which worlds-diagrams represent cases in which P is possible, Q is possible, and in which P is inconsistent with Q?* [$\Diamond P$ *and* $\Diamond Q$ *and* $(P \not\circ Q)$]
16. *Which worlds-diagrams represent cases in which neither P implies Q nor Q implies P?* [$\sim (P \rightarrow Q)$ *and* $\sim (Q \rightarrow P)$]
17. *Which worlds-diagrams represent cases in which both P implies Q and P is consistent with Q?* [$(P \rightarrow Q)$ *and* $(P \circ Q)$]

18. *Which worlds-diagrams represent cases in which both P implies Q and P is inconsistent with Q?* [$(P \rightarrow Q)$ *and* $(P \phi Q)$]

19. *Which worlds-diagrams represent cases in which both P implies Q and Q does not imply P?* [$(P \rightarrow Q)$ *and* $\sim (Q \rightarrow P)$]

20. *Which worlds-diagrams represent cases in which P and Q are consistent with one another but in which P does not imply Q?* [$(P \circ Q)$ *and* $\sim (P \rightarrow Q)$]

21. *Which worlds-diagrams represent cases in which P and Q are inconsistent with one another and in which P does not imply Q?* [$(P \phi Q)$ *and* $\sim (P \rightarrow Q)$]

22. *Which worlds-diagrams represent cases in which P implies P?* [$P \rightarrow P$]

23. *Which worlds-diagrams represent cases in which P is contingent, Q is necessarily true, and in which P implies Q?* [∇P, $\Box Q$, *and* $(P \rightarrow Q)$]

24. *Which worlds-diagrams represent cases in which P is necessarily true, Q is contingent, and in which P implies Q?* [$\Box P$, ∇Q, *and* $(P \rightarrow Q)$]

25. *Which worlds-diagrams represent cases in which P is equivalent to Q and in which P and Q are inconsistent with one another?* [$(P \leftrightarrow Q)$ *and* $(P \phi Q)$]

26. *Which worlds-diagrams represent cases in which P implies* $\sim P$? [$P \rightarrow \sim P$]

27. *Which worlds-diagrams represent cases in which* $\sim P$ *implies P?* [$\sim P \rightarrow P$]

28. *Which worlds-diagrams represent cases in which P is consistent with* $\sim P$? [$P \circ \sim P$]

29. *Which worlds-diagrams represent cases in which both P implies* $\sim Q$ *and P is consistent with Q?* [$P \rightarrow \sim Q$ *and* $P \circ Q$]

30. *Which worlds-diagrams represent cases in which P is contingent and* $\sim P$ *is necessarily true?* [∇P *and* $\Box \sim P$]

31 – 45. *For each of the fifteen worlds-diagrams in figure* (1.i) *find any two propositions which stand in the relation depicted by that diagram.*

Example: diagram 14

Let "P" = There are fewer than 30,000 galaxies.
Let "Q" = There are more than 10,000 galaxies.

Part B

The expression "It is false that P implies Q" is ambiguous between saying "P does not imply Q" and "P's being false implies Q" which may be symbolized unambiguously as "$\sim (P \rightarrow Q)$*" and "*$(\sim P \rightarrow Q)$*" respectively.*

46. *Which worlds-diagrams represent cases in which P does not imply Q?* [$\sim (P \rightarrow Q)$]

47. *Which worlds-diagrams represent cases in which P's being false implies Q?* [$\sim P \rightarrow Q$]

48. *Which worlds-diagrams represent cases in which both P's being true implies Q and P's being false implies Q?* [$(P \rightarrow Q)$ *and* $(\sim P \rightarrow Q)$]

Part C

The expression "Q is a false implication of P" means "Q is false and Q is an implication of P." In order to represent this relation on a set of worlds-diagrams we shall have to have a device for depicting the actual world (i.e., for depicting that Q is false). Rather than persisting with the convention of figures (1.d), (1.e), and (1.f), we shall hereinafter adopt the simpler convention of representing the location of the actual world among the set of possible worlds by "×". In worlds-diagrams 1 through 4, the "×" may be placed indiscriminately anywhere within the rectangle. But when we come to diagrams 5 through 15 we are faced with a number of alternatives. To show these alternatives we will label each of the internal segments of the rectangle from left to right beginning with the letter "a". Thus, for example, worlds-diagram 11 containing an "×" in the central segment would be designated "11b". A worlds-diagram on which the location of the actual world is explicitly marked by an "×", we shall call a "reality-locating worlds-diagram".

49. *Taking account of all the different ways the actual world may be depicted on a worlds-diagram, how many distinct reality-locating worlds-diagrams are there for two propositions?*

50. *Using the convention just discussed for describing a worlds-diagram on which the actual world is depicted, which worlds-diagrams represent cases in which Q is a false implication of P?* [$(P \rightarrow Q)$ *and* $\sim Q$]

51. *Assume P to be the true proposition that there are exactly 130 persons in some particular room, 2B. Let Q be the proposition that there are fewer than 140 persons in room 2B. Draw a reality-locating worlds-diagram representing the modal relation between these two propositions.*

(For questions 52–55) Assume that there are exactly 130 persons in room 2B. Let P be the proposition that there are exactly 135 persons in room 2B, and Q the proposition that there are at least 133 persons in room 2B.

52. *Is Q implied by P?*

53. *Is Q true?*

54. *Is P true?*

55. *Draw a reality-locating worlds-diagram representing the modal relation between the propositions, P and Q.*

* * * * *

Appendix to section 6

It is an interesting question to ask, "How many different ways may *three* arbitrarily chosen propositions be arranged on a worlds-diagram?" The answer is "255".

It is possible to give a general formula for the number (W_n) of worlds-diagrams required to depict all the possible ways of arranging any arbitrary number (n) of propositions. That formula is[40]

$$W_n = 2^{2^n} - 1$$

40. Alternatively, the formula may be given recursively, i.e.,

$$W_1 = 3, \text{ and}$$

$$W_{n+1} = (W_n + 1)^2 - 1$$

Using this latter formula it is easy to show that the next entry in table *(1.j)*, i.e., W_7, would be 39 digits in length.

We may calculate the value of W_n for the first few values of n.

n	W_n
1	3
2	15
3	255
4	65,535
5	4,294,967,295
6	18,446,744,073,709,551,615

TABLE *(1.j)*

7. IS A SINGLE THEORY OF TRUTH ADEQUATE FOR BOTH CONTINGENT AND NONCONTINGENT PROPOSITIONS?

In this chapter we have introduced one theory of truth, the so-called Correspondence Theory of Truth which has it that a proposition P, which ascribes attributes **F** to an item *a*, is true if and only if *a* has the attributes **F**. Sometimes this theory is summed up epigrammatically by saying that a proposition P is true if and only if 'it fits the facts'.

Now some philosophers who are perfectly willing to accept this theory as the correct account of the way the truth-values of contingent propositions come about, have felt this same theory to be inadequate or inappropriate in the case of noncontingent propositions. We would do well to review the sort of thinking which might lead one to this opinion.

Suppose one were to choose as one's first example a contingent proposition, let us say the proposition

(1.42) Canada is north of Mexico.

When one asks what makes *(1.42)* actually true, the answer is obvious enough: this particular proposition is actually true because of certain peculiar geographical features of the actual world, features which are shared by some other possible worlds but definitely not by all.

Following along in this vein, we can ask a similar question about noncontingent propositions. Let us take as an example the noncontingent proposition

(1.43) Either Booth assassinated Abraham Lincoln or it is not the case that Booth assassinated Abraham Lincoln.

If we now ask what makes this proposition true, some persons have felt that the answer cannot be of the same sort as that just given in the case of the example of a contingent proposition. For these persons rightly note that whereas the truth-value of *(1.42)* varies from possible world to possible world, the truth-value of *(1.43)* does not. No matter how another possible world may differ from the actual world, no matter how outlandish and farfetched that world might seem to us in the actual world, the truth-value of *(1.43)* will be the same in that world as in the actual world. In the actual world, Booth did assassinate Lincoln and *(1.43)* is true. But there are other possible worlds in which Lincoln did not go to Ford's Theater on April 14, 1865, and lived to be re-elected to a third term; yet in those worlds, *(1.43)* is true. Then, too, there are those possible worlds in which Booth did shoot Lincoln, but the wound was a superficial one and Lincoln recovered; yet in those worlds, too, *(1.43)* is true.

In the eyes of some philosophers these latter sorts of facts challenge the correspondence theory of truth. In effect these philosophers argue that the truth of noncontingent propositions cannot be accounted for by saying that 'they fit the facts' since they remain true whatever the facts happen to be. How could the truth of *(1.43)* in one possible world be explained in terms of one set of facts and its truth in another possible world in terms of a different set of facts? The very suggestion has seemed to these philosophers to constitute a fatal weakness in the correspondence theory of truth. In their view there just doesn't seem to be any proper set of facts to which true noncontingent propositions 'correspond'. And so, they have been led to propose *additional* theories of truth, theories to account for the truth-values of noncontingent propositions.

One of the oldest supplemental theories invoked to account for the truth of necessary truths holds that their truth depends upon what were traditionally called "The Laws of Thought", or what most philosophers nowadays prefer to call "The Laws of Logic". Precisely what is encompassed within these so-called laws of thought has been an issue of long dispute among philosophers. Nevertheless, it seems fairly clear that whatever else is to be included, the laws of thought do include the 'Laws' of Identity, of The Excluded Middle, and of Noncontradiction: roughly, the 'laws' that an item is what it is, that either an item has a certain attribute or it does not, and that an item cannot both have a certain attribute and fail to have it.

But this supplemental theory turns out, on examination, to be wholly lacking in explanatory power. For if there is controversy over the matter of what is to be included among the laws of thought, there has been even greater controversy, with more far-reaching implications, over the matter of what sorts of things these so-called "laws of thought" *are*. The account which seems to work best in explaining what the laws of thought are, is that which says of them that they are themselves nothing other than noncontingently true propositions. But if this account is adopted — and it is by most contemporary philosophers — then it is very hard to see how the laws of thought can serve as an explanation of what it is that the truth-values of noncontingent propositions depend on. The trouble is that if a certain class of noncontingently true propositions, the honored "laws of thought", are to account for the truth (or falsity) of other (the run-of-the-mill) noncontingent propositions, then some further, presumably still more honored propositions must account for *their* truth, and so on and on without end. In short, invoking some propositions to account for the truth or falsity of others leads one to an infinite regress or lands one in a circularity.

A second supplemental theory, which we shall call "the linguistic theory of necessary truth", holds that the truth-values of noncontingent propositions come about, not through correspondence with the facts, but rather as a result of certain 'rules of language'. Capitalizing on the very word "contingent", this theory is sometimes expressed in the following way: The truth-values of contingent propositions are *contingent* upon the facts, while the truth-values of noncontingent propositions are *not* contingent upon the facts but are determined by rules of language. Or put still another way: contingent truths (and falsehoods) are factual; noncontingent truths (and falsehoods) are linguistic.

The linguistic theory of necessary truth does not belong only to the preserve of philosophers. It is enshrined also in commonplace talk of certain propositions being true (or false) "by definition". Thus, it might be said, the proposition

(1.44) All rectangles have four sides

owes its truth to the fact that we human beings have resolved to *define* the term "rectangle" as "plane closed figure with four straight sides all of whose angles are equal". For, given such a stipulative definition, the truth of *(1.44)* follows immediately.

This talk of "definitional truths", and similar talk of "verbal truths", suggests that necessary truths are not grounded — as are contingent truths — in correspondence between propositions and states of affairs, but are grounded rather in some sort of correspondence between propositions and the arbitrary conventions or rules for the use of words which we language users happen to adopt.

Yet, despite its initial plausibility, this theory seems to us defective.

Its plausibility, we suggest, derives — at least in part — from a confusion. Let us concede that when we introduce a new term into our language by defining it (explicitly or implicitly) in terms of already available expressions, we make available to ourselves *new ways of expressing* truths. But this does not mean that we thereby make available to ourselves new truths.

Once we have introduced the term "rectangle" by definition in the manner sketched above, the sentence

(1.45) "All rectangles have four sides"

will express a necessary truth in a way in which we could not have expressed it earlier. But the necessary truth which it expresses is just the necessary truth

(1.46) All plane closed figures with four sides all of whose angles are equal have four sides

and this is a proposition whose truth is far less plausibly attributable to definition. For this proposition, one is more inclined to say, would be true even if human beings had never existed and had there never been any language whatever in the world.

Talk about necessary truths being "true by definition" may also derive some plausibility from the fact that our *knowledge* that certain propositions are true sometimes stems from our *knowledge* of the meanings, or definitions, of the terms in which they are expressed. Thus a person who comes to know that the expression "triac" *means the same as* the expression "bidirectional triode thyristor" may thereby (even without knowing independently what either means) know that the sentence

(1.47) "All triacs are bidirectional triode thyristors"

expresses a necessarily true proposition. But this does not mean that the necessary truth of the proposition so expressed is itself something that stems from the meanings of these expressions.

Perhaps the gravest defect in the linguistic theory of necessary truth has to do with its inability to explain how it is that necessary truths, such as those of logic and mathematics, can have significant practical applications in the real world. In such applied sciences as aeronautics, engineering, and the like, arithmetical truths may be applied in ways which yield important new inferential truths about the world around us: about the stress tolerances of bridges, the efficiency of airplane propellers, etc. But if these necessary truths are merely the result of arbitrary human conventions for the use of mathematical symbols, all this becomes a seeming miracle. Why should the world conform so felicitiously to the consequences of our linguistic stipulations?

The explanation of the success of logic and mathematics in their applications to the world, we suggest, lies elsewhere: in the possible-worlds analysis of necessary truth. Necessary truths, such as those of mathematics, apply to the world because they are true in all possible worlds; and since the actual world is a possible world, it *follows* that they are true in (i.e., apply to) the actual world. This account of the matter, it should be noted, does not require a different theory of truth from that which we have given for contingent propositions. To say that a proposition is true in the actual world is, we have claimed, to say that it fits the facts in the actual world, i.e., that states of affairs in the actual world are as the proposition asserts them to be. And likewise, to say that a proposition is true in some other possible world — or, for that matter, in all possible worlds — is just to say that in that other world — or all possible worlds — states of affairs are as the proposition asserts them to be. One and the same theory of truth suffices for all cases: the case when a proposition is true in all possible worlds and the case when it is true in just some (perhaps the actual one included).

In order to see how this single account of truth suffices for all cases, let us return to the example of the noncontingent proposition *(1.43)* to see how this theory might be invoked to explain its truth.

But first, let us reflect for a moment on another proposition, viz., the *contingent* proposition

(1.48) Either George Washington was the first president of the United States or Benjamin Franklin was the first president of the United States.

This proposition, being contingent, is true in some possible worlds and false in all the others. What feature is it, in those possible worlds in which *(1.48)* is true, which accounts for its truth? The answer is obvious: *(1.48)* is true in all and only those possible worlds in which either Washington or Franklin was the first president of the United States. But note — as in the case of the noncontingent proposition *(1.43)* — how much variation occurs between the worlds in which the proposition is true. In the actual world, for example, Washington, not Franklin, was the first president of the United States. Thus in the actual world, *(1.48)* is true. But in some other possible worlds, Franklin, not Washington, was the first president of the United States. But in that world, too, *(1.48)* fits the facts, i.e., is true.

Here, then, we have a parallel to the situation we discovered in the case of the noncontingent proposition *(1.43)*; that is, we have already noted that *(1.43)* is true in some possible world in which Booth assassinated Lincoln and is true in some possible world in which he did not. The relevant difference, in the present context, between these two cases — the noncontingent proposition *(1.43)* and the contingent proposition *(1.48)* — lies in the fact that the former proposition is true in every possible world in which Booth did not assassinate Lincoln, while the latter is true in only some of the possible worlds in which Washington was not the first president. But this difference clearly has only to do with whether the set of possible worlds in which each proposition fits the facts comprises all, only some, or none of the totality of possible worlds. It in no way challenges the claim that the truth of both *(1.43)* and *(1.48)* is to be accounted for in the same way, viz., in their fitting the facts.

Summing up, we may say that there is no *special* problem about the truth-values of noncontingent propositions. They are true or false *in exactly the same sort of way* that contingent propositions are true or false; i.e., depending on whether or not they fit the facts. The supposition that there are two kinds of truth, or that there is a need for two theories of truth, is misconceived. *Propositions* are true or false; also they are contingent or noncontingent. And these various properties of propositions (two for truth-status and two for modal-status) can combine in various ways (four, to be exact). But it should not be thought that in saying of a proposition that it is, for example, noncontingently true, we are saying that it exemplifies one of two types of *truth*. Rather, this expression ought to be construed as "noncontingent *and* true". Viewing the matter in this fashion, it is easy to see that the question about the special way in which the truth (or falsity) of noncontingent propositions is supposed to come about *does not even arise.* Logic does not require more than one theory of truth.

8. THE "POSSIBLE-WORLDS" IDIOM

Throughout this book we speak often of other possible worlds. Why have we, in common with many other philosophers and logicians, adopted this idiom? The answer lies in its enormous heuristic value.

Many contemporary philosophers believe that the possible-worlds idiom provides a single theoretical framework powerful enough to illuminate and resolve many of the philosophical problems surrounding such matters as

1. The logical notions of necessity, contingency, possibility, implication, validity, and the like
2. The distinction between logical necessity and physical necessity
3. The adequacy of a single theory of truth
4. The identity conditions of propositions and of concepts
5. The disambiguation of sentences
6. The link between the meaning of a sentence and the truth-conditions of the proposition(s) it expresses
7. The technique of refutation by imaginary counterexamples
8. The epistemic concept of that which is humanly knowable
9. The distinction between accidental and essential properties
10. The concept of the contingent content of a proposition
11. The concept of probabilification

Each of these is dealt with, in the order given, in this book. But the list goes beyond the confines of this book. Recent work couched within the possible-worlds idiom includes work in

12. Ethics
13. Counterfactuals
14. Epistemic logic
15. Propositional attitudes

and much more besides. To cite some examples: C.B. Daniels in *The Evaluation of Ethical Theories:*

> For the purposes of this book, an ethical theory is simply a determination of a unique set of ideal worlds. An ideal world, relative to a theory that determines it as ideal, is a possible world, a fairy world perhaps, in which everything the theory says is ideally true is in fact true. An ideal world relative to an ethical theory is a world in which everything the theory says ought to be the case is the case.[41]

41. Halifax, Nova Scotia, Dalhousie University Press, 1975, p. 1.

David Lewis, in *Counterfactuals:*

> *'If kangaroos had no tails, they would topple over'* seems to me to mean something like this: in any possible state of affairs in which kangaroos have no tails, and which resembles our actual state of affairs as much as kangaroos having no tails permits it to, the kangaroos topple over. I shall give a general analysis of counterfactual conditions along these lines.
>
> My methods are those of much recent work in possible-world semantics for intensional logic.[42]

And Jaakko Hintikka, in "On the Logic of Perception":

> When does *a* know (believe, wish, perceive) more than *b*? The only reasonable general answer seems to be that *a* knows more than *b* if and only if the class of possible worlds compatible with what he knows is smaller than the class of possible worlds compatible with what *b* knows.[43]

The extraordinary success, and the veritable explosion of research adopting the possible-worlds idiom was not the motivation or even the expectation in its adoption. As a matter of fact, the idiom is not especially new; only its widespread adoption is. Talk of possible worlds can be found throughout the writings of the great German mathematician and philosopher, Gottfried Leibniz (1646–1716). Leibniz was quite at home in the possible-worlds idiom. He liked to philosophize in terms of that idiom, asking such questions as, "Is this the best of all possible worlds?" and "Why did God create this particular world rather than some other possible world?" And writing of those truths which we (and he on occasion) have called "necessary", he penned these germinal lines:

> These are the eternal truths. They did not obtain only while the world existed, but they would also obtain if GOD had created a world with a different plan. But from these, existential or contingent truths differ entirely.[44]

Leibniz's style and idiom were in advance of their time. Other logicians, and perhaps Leibniz too, saw no particular *advantage* in this way of talking, merely an alternative. It was not until the early 1960s that philosophers such as Kripke and Hintikka returned to Leibniz's idiom and used it both to illuminate the philosophical bases of logic and to push the frontiers of logic and many other areas of philosophy in new directions.

Nonetheless, it is important to keep this talk of 'other possible worlds' in its proper philosophical perspective. We must never allow ourselves to regard this manner of talk as if it were talk about actually existing parts of this world. Whoever supposes that other possible worlds are basic entities of the physical universe has failed to appreciate the point of our earlier insistence that other possible worlds are not 'out there' in physical space. Other possible worlds are *abstract* entities like numbers

42. Cambridge, Mass., Harvard University Press, 1973, p. 1.

43. *Models for Modalities: Selected Essays,* Dordrecht, D. Reidel, 1969, p. 157.

44. "Necessary and Contingent Truths" translated from *Opuscules et fragments inédits de Leibniz,* ed. Courturat, Paris, Félix Alcan, 1903, pp. 16–22, reprinted in *From Descartes to Locke* ed. T.V. Smith and M. Grene, Chicago, The University of Chicago Press, 1957, pp. 306–312. We are grateful to our colleague, David Copp, for calling this passage to our attention.

and propositions.[45] We feel driven to posit them because we seem unable to make ultimate sense of logic without them. Positing the existence of these abstract entities allows us to unify logic, to rigorize it, to expand it, and most importantly in our eyes to *understand* it.

45. This distinction between abstract and non-abstract (i.e., concrete) entities will be examined at greater length in the next chapter.