Due: Friday, September 23rd (in class)

Reading

Chapter 23 of Vanderbei's *Linear Programming: Foundations and Extensions (4th edition)*. SFU Library link.

From the textbook, Sections 1.1, 1.2 and 1.3.

You may also enjoy reading Chapter 1 of Applegate, Bixby, Chvátal and Cook's *The Traveling Salesman Problem : A Computational Study*. SFU library link.

Please make sure you understand the translation of combinatorial optimization problems into integer programs, for instance knapsack, set covering and travelling salesman.

Also, convince yourself that it is straightforward to model most natural constraints using mixed integer programs. LP is already powerful, but doesn't easily capture integer variables. IP gives those and a way to model constraints of the form "A or B", "satisfying k of n constraints" or even "takes a value from a specified discrete set".

Problems for Math 408 and Math 708

0. (Not graded.) Download and install the demo version of `AMPL` that is available at `http://www.ampl.com/try-ampl/download-a-free-demo/`. You can use either the command-line or graphical version. Note that demo solvers are included with the download. Work through the diet examples in Chapter 2 of the `AMPL` book: `http://ampl.com/resources/the-ampl-book/chapter-downloads/`, so that you are familiar with the software.

1. Take your nine digit student id number and add 10 to each digit to get a sequence of nine numbers $a_1, a_2, \ldots, a_9$ between 10 and 19. Take $b_1, b_2, \ldots, b_9$ to be the first nine digits of π. Your personal knapsack problem is:

$$\text{Maximize } \sum_{i=1}^{9} b_i x_i \quad \text{subject to } \sum_{i=1}^{9} a_i x_i \leq \frac{1}{2} \sum_{i=1}^{9} a_i \quad \text{and } x_i \in \{0,1\} \text{ for } i = 1, \ldots, 9.$$

Solve this integer program using the `AMPL` demo version that with the `Gurobi` solver. Submit a screen shot of the final solution.

2. Consider a situation where your model includes binary variables $x_1, x_2, \ldots, x_9$, each representing the potential purchase of a particular investment. Show how to represent each of the following constraints as a single linear constraint:

- a You cannot choose all of them.
- b You must choose at least two of them.
- c If you choose investment 6, you must also choose investment 4.
- d If you choose investment 3, you may not choose investment 8.
- e You must choose either both investments 7 and 9, or neither of them.
- f You must choose either one of the first 3 investments, or two of the last 3 investments.

3. Given that x_1, x_2, x_3, x_4 are binary variables, show that the feasible set defined by $2x_1 + x_2 + x_3 + x_4 \leq 3$ is the same as the one defined by the three inequalities $x_1 + x_2 + x_3 \leq 2$, $x_1 + x_2 + x_4 \leq 2$ and $x_1 + x_3 + x_4 \leq 2$.

4. Textbook Exercise 1.4.

5. Textbook Exercise 1.10.

Additional Problems for Math 708

6. Suppose that you are building a model which contains variables x_1 and x_2, and you have that $0 \leq x_1, x_2 \leq C$. Show how you can use an additional binary variables to represent the functions $u = \max\{x_1, x_2\}$ and $v = |x_1 - x_2|$.

7. Textbook Exercise 1.17.

8. Consider the problem of allocating storage (memory) dynamically in a computer. Model the memory as a simple array indexed by the positive integers. Suppose we are given a series of n requests to use an array of size s_i from arrival time r_i to departure time d_i. We would like to find the minimum memory size that will accommodate these requests (and a way to do it). Formulate this problem as an integer program.