Due: Friday, November 27th (noon)

The final exam will take place Sunday, December 13th at 3:30 p.m. remotely in the our Zoom classroom.

Reading

Chapter 8 through Section 8.2

Chapter 4 of Applegate, Bixby, Chvátal and Cook reviews some topics that we have seen in class, and a few that we haven't, in their historical context.

Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw5.pdf` containing all your written work, along with files `hw5.dat` and `hw5.mod` for the AMPL question (question 5); for this exercise separate files for part (b) or (c) would be helpful, which you can call `hw5b.dat` and `hw5b.mod` or `hw5c.dat` and `hw5c.mod`. Please make sure to write your name on the first page of `hw5.pdf` and in the comments of `AMPL` files.

1. Returning once more to your personal knapsack problem, or your amended personal knapsack problem in case your personal knapsack relaxation was integer, you should see that your relaxed optimal solution has exactly one non-integer coordinate.

   a. Explain why in this situation, the cover inequality corresponding to the non-zero terms in your solution will cut your relaxed optimal solution.

   b. Add this new cover cut to your formulation and resolve.

   c. What is the dimension of the new face that you have found?

2. If C is a cover for a 0-1 knapsack inequality, the *extended cover* $E(C)$ of C is $C \cup \{j \mid a_j \geq a_i \text{ for all } i \in C\}$.

   a. Show that if C is a cover for a given 0-1 knapsack inequality, then the *extended cover inequality* given by $\sum_{i \in E(C)} x_i \leq |C| - 1$ is valid for $\text{conv}(S)$, the set of integer solutions to the 0-1 knapsack problem.

   b. Can your cover inequality from the previous question be replaced by an extended cover inequality?

   c. For each variable that is not in your extended cover inequality (or simply your cover inequality if it did not extend), try to lift the inequality in that variable. If you succeed in lifting a variable, if you like, you can work with the new inequality for subsequent liftings. This will lead to stronger cuts, but requires solving more complicated knapsack subproblems.

   d. Compute the dimension of any new cuts you have found.

   e. Make a table that shows the optimal solution and objective values of:

      - The initial knapsack relaxation (from assignment 1).
      - The relaxation given by adding any single cut (alone) found on this assignment to the initial formulation.
      - The relaxation given by adding all cuts found on this assignment to the initial formulation.
      - The integer knapsack problem (from assignment 1).

3. List the feasible (integer) points $(\vec{x}, \vec{u})$ to the Miller-Tucker-Zemlin formulation of the TSP (pages 62-63 of the text) on the complete directed graph with 4 vertices. You should take the variables u_i for $i = 2, 3, 4$ to be integers between 2 and 4.

4. Exercise 20.5 from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

5. Textbook Exercise 3.16.

Additional Problems for Math 708

6. Finish question 6 from the previous assignment by lifting the 5-cycle inequality to a facet by adding a term representing the variable x_{v_0}.

7. Textbook Exercise 3.21.

8. Compute the Chvátal closure of $P := \{(x, y) \in [0, 1]^2 \mid 2x + 2y \leq 3\}$.

Schedule of graduate presentations

Each graduate student will present a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading. The tentative schedule and topics are as follows:

Wednesday, December 2nd **Danielle Rogers**, on *Semi-definite Programming and Stable Set*.

Friday, December 4th (early) **Einar Gabbassov**, on *Quantum Annealing for Combinatorial Optimization*.

Friday, December 4th (late) **Alborz Namazi**, on *the Ellipsoid Method*.

Monday, December 7th (early) **Sajeththa Thavayogarajah**, on *the Quadratic Assignment Problem*.

Monday, December 7th (late) **Brett Wiens**, on *Optimization on Gray Codes*.

The criteria for evaluating the presentation will include:

- The presentation highlights critical aspects of the report, and is suitable for the audience.
- Ideas are presented clearly and logically.
- Live presentation is well prepared, accurate, and professionally delivered. Questions are answered appropriately.
- Overheads are clear, well-formatted, and have few errors.