Due: Monday, November 28th (9:30 a.m.)

The final exam will take place Monday, December 12th at 8:30 a.m. in SUR 3010.

## Reading

Chapter 8 through Section 8.2

Chapter 4 of Applegate, Bixby, Chvátal and Cook reviews some topics that we have seen in class, and a few that we haven't, in their historical context.

## Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw5.pdf` containing all your written work, along with files `hw5.dat` and `hw5.mod` for the AMPL question (question 5); for this exercise separate files for part (b) or (c) would be helpful, which you can call `hw5b.dat` and `hw5b.mod` or `hw5c.dat` and `hw5c.mod`. Please make sure to write your name on the first page of `hw5.pdf` and in the comments of `AMPL` files.

1. Returning once more to your personal knapsack problem, or your amended personal knapsack problem in case your personal knapsack relaxation was integer, you should see that your relaxed optimal solution has exactly one non-integer coordinate.

   a. Explain why in this situation, the cover inequality corresponding to the non-zero terms in your solution will cut your relaxed optimal solution.

   b. Add this new cover cut to your formulation and resolve.

   c. What is the dimension of the new face that you have found?

2. If $C$ is a cover for a 0-1 knapsack inequality, the *extended cover* $E(C)$ of $C$ is $C \cup \{j \mid a_j \geq a_i \text{ for all } i \in C\}$.

   a. Show that if $C$ is a cover for a given 0-1 knapsack inequality, then the *extended cover inequality* given by $\sum_{i \in E(C)} x_i \leq |C| - 1$ is valid for conv$(S)$, the set of integer solutions to the 0-1 knapsack problem.

   b. Can your cover inequality from the previous question be replaced by an extended cover inequality?

   c. For each variable that is not in your extended cover inequality (or simply your cover inequality if it did not extend), try to lift the inequality in that variable. If you succeed in lifting a variable, if you like, you can work with the new inequality for subsequent liftings. This will lead to stronger cuts, but requires solving more complicated knapsack subproblems.

   d. Compute the dimension of any new cuts you have found.

   e. Make a table that shows the optimal solution and objective values of:

      - The initial knapsack relaxation (from assignment 1).
      - The relaxation given by adding any single cut (alone) found on this assignment to the initial formulation.
      - The relaxation given by adding all cuts found on this assignment to the initial formulation.
      - The integer knapsack problem (from assignment 1).

3. List the feasible (integer) points $(\vec{x}, \vec{u})$ to the Miller-Tucker-Zemlin formulation of the TSP (pages 62-63 of the text) on the complete directed graph with 4 vertices. You should take the variables $u_i$ for $i = 2, 3, 4$ to be integers between 2 and 4.

4. Exercise 20.5 from the AMPL book, available at:

`https://ampl.com/learn/ampl-book/`.

5. Finish question 5 from the previous assignment by lift the inequality you found in part (a) in variable 5, and then lifting the resulting inequality in variable 6.

## Additional Problems for Math 708

6. Textbook Exercise 3.13.

7. Compute the Chvátal closure of $P := \{(x, y) \in [0, 1]^2 \mid 2x + 2y \leq 3\}$.