Due: Friday, January 26th (in class)

1. Problem 1-1 from the AMPL book (available as .pdf here). For your solution, please hand in typeset or pen-and-paper solutions and formulations for the mathematical parts of the problems. For the computations, please send me by e-mail the AMPL model and data files, along with a screen shot of the output.

2. Problem 1-3 from the AMPL book. Again, please provide the mathematical parts on paper, and e-mail me the files related to the computation. Please group all the submission files for this assignment into a single e-mail.

3. Linear programming is very widely used tool to the point that it is incorporated into basic software packages like Excel and taught to MBA students. Formulate and solve your linear program from question 1 (a) in Excel. For this problem, you submit just the .xls file in the same e-mail as your AMPL files.

If you would like to see some well written sample Excel linear programs, you can look for instance at the supplemental files related to *Optimization Modeling with Spreadsheets* by Kenneth R. Baker, which is used in SFU's *Introduction to Operations Research* course. The text is available on-line through SFU's library.

4. We have seen that it is possible to change a problem in equality standard form to an equivalent problem in inequality standard form by adding m slack variables, one for each equation. Does this also work if we use the same variable in every equation, thus using only one additional variable?

5. We have seen how to put linear programs into standard equality form. Any polyhedron in standard equality form has an extreme point. Does this mean that every polyhedron has an extreme point?

6. Prove that $(\mathrm{conv}(A) \cup \mathrm{conv}(B)) \subseteq \mathrm{conv}(A \cup B)$ for any $A, B \in \mathbb{R}^d$.

7. Show that the set $P = \{x \in \mathbb{R}^d \mid |x_1| + |x_2| + \ldots |X_d| \leq 1\}$ is a polytope.

8. Show that the disc $P = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leq 1\}$ is not a polytope.

9. The convex hull of a set S of points in $\mathbb{R}^n$ can be defined either as the set of all convex combinations of points in S, or as the intersection of all convex sets containing S. Show that these two definitions are equivalent.

10. Consider a polyhedron $P = \{x \mid Ax = b,\ x \geq 0\}$, where A is $m \times n$ and has linearly independent rows. Determine whether each of the following statements is true or false, providing a proof or counter-example as necessary.

a. If $n = m + 1$, then P has at most two basic feasible solutions.

b. The set of all optimal solutions is bounded.

c. At every optimal solution, no more than m variables can be positive.

d. If there is more than one optimal solution, then there are uncountably many optimal solutions.

e. If there are several optimal solutions, then there exist at least two basic feasible solutions that are optimal.

f. Consider the problem of minimizing $\max\{c^t x, d^t x\}$ over P. If the problem has an optimal solution, it must have an optimal solution at an extreme point of P.

Students will give presentations of recent research papers related to this course at the end of the term. Please choose a paper. I am happy to discuss with you which papers may be suitable. I would like to finalize the choices by February 1st. The ideal situation would be to choose papers that are relevant to your own research. I recommend consulting with your advisor.