MATH AND 708 ASSIGNMENT #3

Due: Saturday, October 24th (11:30 p.m.)

Reminders

The midterm will take place in class on **Monday, November 9th, 2:30-4:20 p.m.**.

It will cover material from class up to Wednesday, November 4th.

Reading

From the textbook, reread Section 1.2.2, and then read Section 5.2.4 followed by Sections 3.4 through 3.9.

Problems for Math 408 and Math 708

Solutions to be submitted via `Crowdmark`, with code to `Canvas`. Please submit a single file names `hw3.pdf` containing all your written work, along with files `hw3.dat` and `hw3.mod` for the AMPL question (question 3). Please make sure to write your name on the first page of `hw3.pdf` and in the comments of `hw3.dat` and `hw3.mod`.

1. Let A be a totally unimodular matrix, and let B be a matrix obtained from A by duplicating the last row. Is B necessarily totally unimodular? Justify your answer.

2. Consider solving a maximization problem by branch-and-bound. Suppose that Figure 1 represents part of the branch-and-bound tree. The value inside each node represents the solution to the associated relaxation of the problem; nodes are coloured blue if the relaxed solution is non-integer, green if it is integer, and red if no relaxed solution exists because the problem is infeasible.

   a. Which node(s) still need to be expanded?

   b. Which node(s) are candidate for the optimal solution?

   c. Suppose that before beginning the search, we obtained an integer feasible solution with value 30. Which node(s) would we no longer have to expand?

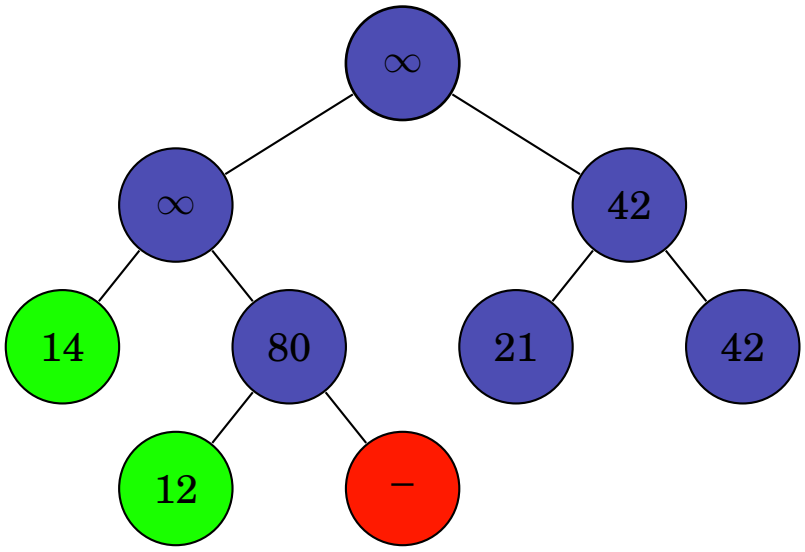


Figure 1: Branch and bound tree for problem 2.

3. Exercise 20.3 (a) and (b) from the AMPL book, available at:

`https://ampl.com/learn/ampl-book/`.

4. Consider the complete graph on 4 vertices. Use the last 6 digits of your student id to weight the 6 edges of the graph.

a. How many Hamiltonian cycles (i.e. cycles connecting all 4 vertices) are in this graph? Give the weight of each cycle.

b. Does your weighting satisfy the triangle inequality?

c. Run the nearest neighbour heuristic on the graph, beginning by traversing the minimum weight edge. What is the approximation ratio that you get from this algorithm?

5. Using the complete graph on 4 vertices, show that if the weights on the edges do not satisfy triangle inequality, then Christofides heuristic can return a solution with approximation ratio 5.

Additional Problems for Math 708

6. Textbook Exercise 4.5.

7. Consider the integer program

$$\min x_{n+1} \quad \text{subject to} \quad 2x_1 + 2x_2 + \ldots + 2x_n + x_{n+1} = n \quad \text{and} \quad x \in \{0,1\}^{n+1}$$

Prove that if n is odd, a branch and bound algorithm (without using cutting planes) will have to examine at least $2^{\lfloor \frac{n}{2} \rfloor}$ candidate problems before it can solve the main problem.

8. Returning to question 4, what is the worst approximation ratio you personally could get over all possible assignments of weights to edges? What is the worst approximation ratio that any student could get?